Troughton's Improved

Fig. 1

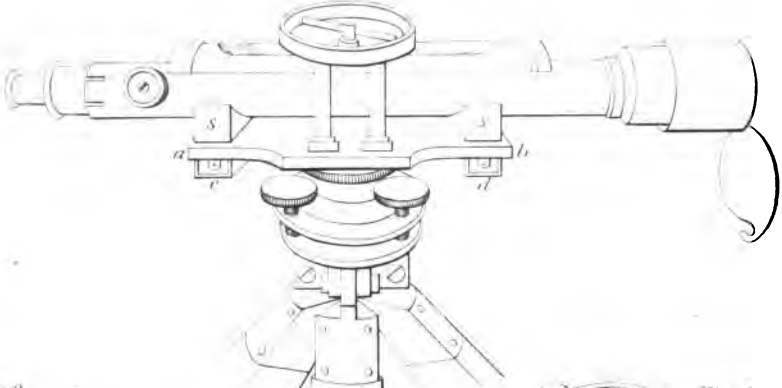


Fig. 3.

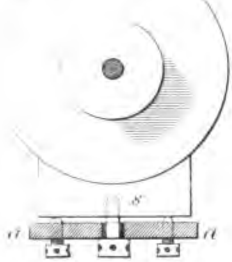
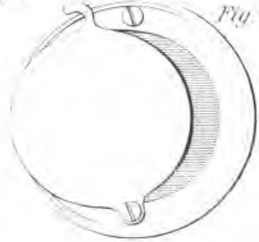


Fig. 4.



Gravatt's Dumpy

Fig. 2

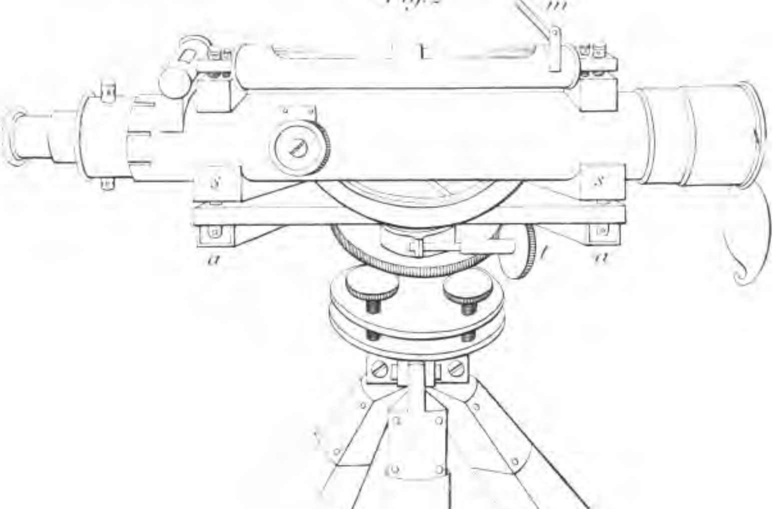
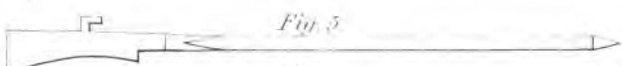


Fig. 5



F. Mansell, Sc.

ENGINEERING FIELD NOTES

ON PARISH AND RAILWAY

SURVEYING AND LEVELLING

With Plans and Sections,

BEING

A SEQUEL TO HIS ELEMENTARY TEXT BOOK.

WITH PRACTICAL FORMULÆ FOR THE CALCULATION
OF EARTH-WORK, THE THEORY AND PRACTICE OF RUNNING OUT CURVES
AND PUTTING DOWN SIDE STAKES ETC.

AND

A TRAVERSE TABLE.

BY HENRY JAMES CASTLE,

SURVEYOR AND CIVIL ENGINEER,

LECTURER ON PRACTICAL SURVEYING AND LEVELLING TO KING'S COLLEGE, LONDON.

Second Edition.

LONDON:

SIMPKIN, MARSHALL, & CO., STATIONERS' HALL COURT.

1847.

TO

LIEUT. COL. SIR R. BONNYCASTLE, R.E.,

ETC., ETC.

COMMANDING OFFICER OF ENGINEERS,
UPPER CANADA,

AS AN HUMBLE TOKEN OF RESPECT FOR HIS VIRTUES,

OF ADMIRATION FOR HIS TALENTS,

AND

AS A SINCERE, THOUGH INADEQUATE, OFFERING OF ACKNOWLEDGMENT

OF THE MANY

PROFESSIONAL SERVICES RECEIVED AT HIS HANDS,

WHILE EMPLOYED UNDER HIM IN THE FIELD,

THIS

The Second Edition

IS, WITH EVERY WISH FOR HIS HEALTH AND HAPPINESS,

RESPECTFULLY INSCRIBED BY HIS SINCERE FRIEND,

AND OBLIGED SERVANT,

THE AUTHOR.

PREFACE.

It was from the want of any sufficient treatise, that I could put into my pupils' hands, on the subject of Land Surveying and Levelling, and the inconvenience I experienced in consequence, while engaged in my professional duties, as Lecturer at King's College, London, that I was induced to compile the following pages.

I found many parts of the subject ably discussed, scattered indifferently among several authors, but none sufficiently consecutive, or in detail, to suit my purpose.

Most of them were too elementary—confined to Chain Surveying—and sometimes not referring, even in that, to the modern system of "tyeing in," as it is called, or triangulating.

Others, on the contrary, were either purely military, or were so completely analytical, that, though of invaluable assistance to the professional man of science, they were, from the omission altogether of the more humble details of operation, both of the Chain and Theodolite, unfitted for the use of the civil surveyor; and not one of them contained any information on the subject of the Circumferentor, or seemed sufficiently illustrated with plans and field notes.

From the direction also, that education had now taken towards

the arts and sciences of life, *in most public and private schools,* I was induced to think that the present would be found a desirable *school book,* among the higher forms, as a full and corrected treatise of the theory and practice of Surveying. At the same time I trusted, that, by supplying the previous deficiencies, I should be furnishing the Surveyor with a complete *vade mecum* of reference.

To fulfil both these objects, preliminary chapters on the most useful Problems and Theorems—on the Nature and Use of Logarithms—on Mensuration of Planes—and sufficient Trigonometry were introduced into the first edition, to enable the reader to understand fully any of the subsequent Trigonometrical Problems.

Having, however, since the publication of the first edition, written another and more elementary work upon this subject, *which was prepared expressly for the use of the upper forms in Schools and Colleges,* called the "ELEMENTARY TEXT BOOK." I have thought it unnecessary in this, the second edition of the larger work, to repeat the elementary portion; I have been by this means enabled to devote more space to the higher parts of the subject, and to make the examples and illustrations more numerous and complete.

The work has been divided into five parts. The first three confined to Surveying; the fourth, to Levelling; and the fifth, to Engineering Field Work.

That portion, which refers to Surveying, has been, for facility of reference, divided into the Chain, Theodolite, and Circumferentor—containing copious field notes, plans, and diagrams, of each kind, together with drawings of the Theodolite and Circumferentor, and ample descriptions of their uses and adjustments.

In the first part, on Chain Surveying. The difference between the methods requisite for Parish and Railway Surveying has been fully explained, and PLAN and FIELD NOTES have been given of each.

In the second part, on Trigonometrical Surveying; all the indispensable practical problems have been both theoretically explained and practically illustrated by actual field notes; the calculations in most cases being fully worked out.

In the *third part, on the Circumferentor*—an instrument, which, in this country, has been hitherto almost exclusively confined to Mining Surveying, a full and practical account with ample Field Notes, Illustrations, and Calculations, has been given of the method of using this instrument in surveying in new countries, being the result of the Author's five or six years practice and experience under Government, in the province of Upper Canada. Under this head have been inserted, though equally necessary for Railway Surveying and Levelling, a few practical methods of determining a MERIDIAN.

The mode of obtaining the LATITUDE has also been given, as necessary, in some cases, to the preceding.

In *part the fourth,* which is devoted to Railway Levelling, a full description has been given of the mode of using and adjusting the best kinds of Spirit Levels of the day, such as, the Y, Gravatt's Dumpy, and Troughton's Improved.

The various kinds of Levelling, viz., Trial Levelling and Cross and Final Levelling, have been fully explained, and illustrated, and the whole of this part has been considerably enlarged.

The information contained in it is of the most comprehensive character and will be found invaluable to the young Leveller.

Field notes of both the Main and Cross Levels have been given, of *some miles of Railway* with *Sections* complete. The method of computing the lowering and raising of the approaches has been also given; and Drawings of the sections and cross sections, conformably to the regulations required for plans, that are intended for parliamentary deposit, have been added for illustration.

A special Example has been added of a portion of a Line of Section taken over the same Plan as is given for an Example of Railway Surveying, in order to explain the different methods of calculating the superficial and solid quantities required both *before* and *after* the Act.

The several methods, whether correct or incorrect, adopted in practice, for calculating the CUTTINGS AND EMBANKMENTS, have been carefully investigated and examined, and corrections given for those that are wrong; and practical examples have been annexed, fully worked out, by the *Prismoidal Formula*—by Bidder's tables—and checked by a new Formula, which is specially adopted for cuttings of *any* length and height, for which I am indebted to Professor Moseley, late of Kings College, London.

In the fifth part, on Engineering Field work, have been explained the theory and practice of running out Railway Curves, with their several checks and corrections, together with the mode of calculating the widths of the line and putting down the side stakes.

And *lastly,* there is an Appendix of Field Notes of two Railway Surveys and a Traverse Table of Latitudes and Departures to any distance and to minutes of bearing.

I have now to acknowledge my obligation to the works of several authors, to whom I am indebted for much valuable assistance: among others, to Hall, Bridges, Keith, Hutton, Gummere, &c. &c.

Having intended these pages, as much for those who may have no opportunity of obtaining the assistance of a master, as for those who have, I have, in every case, annexed examples for practice;—to most of them have been given answers; to a few, when the same result could be obtained by two or three methods, as additional practice for the Student, the answers have been omitted.

Before concluding, I have only to appeal to the kind indulgence

of my readers for any mistakes that may have crept into the work, reminding them, that as there is a greater number of examples given in this work than in any other upon the same subject, some slight indulgence in that respect may be reasonably conceded.

Adelphi, London,
March 2, 1847.

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CHAPTER I.

ITS NATURE AND OBJECT.

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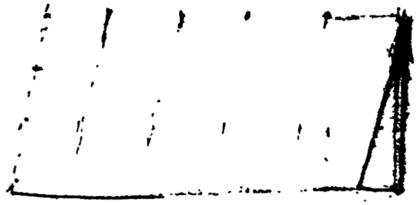

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ILLUSTRATIONS.

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LAND SURVEYING.

Part the First.

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CHAP. I.

ON THE CHAIN.

As every measurement, whether of work, solidity, or superficial extent, must be measured by some unit, to which it can bear constant relation, it has been concluded in the case of land measurement, to make that unit an acre; the acre, or arpent, is the generally recognised unit of land measurement; it varies, however, considerably in different counties.*

The Statute Acre in England, however, to which all the others must be reduced, consists of ten square chains, or one chain front by 10 chains deep, or of its equivalent rectangle ax, where a can be any number, and $x = \frac{10}{a}$ square chains;

* For the different value of the acre in different counties of England, and for the method of ranging and measuring with the chain, see my Elementary Text Book, pages 29 and 94.

each chain containing 22 yards, or 4 poles, or 66 feet, or 100 links, and each link 792 inches.

The acre, therefore, is equal to 10 square chains.

or $(4)^2$ poles	$\times$	10	=	160	square poles.
or $(22)^2$ yds.	$\times$	10	=	4840	square yards.
or $(66)^2$ feet	$\times$	10	=	43560	square feet.
or $(100)^2$ links	$\times$	10	=	100,000	square links.

The Offset Staff.

Is a narrow slip of wood about 1½ inch wide by 1 inch thick, and generally 10 links long, divided into links; it is made of deal or some light wood, and should be furnished at one end with a small notch or hook, to put the chain through the hedges, and at the other with a dibber to make holes for the flags. It ought to be numbered on both sides, from different ends. The use of it is to measure short distances, called *offsets,* from the line to the hedges, &c.

NOTE.—[These offsets should never, unless under peculiar circumstances, exceed one chain.]

As these offsets must be measured at right-angles to the chain, the surveyor should stand on the opposite side of the chain to the hedge or object to be measured to, and walking along the chain, looking at either end, mark where a perpendicular from the given object would fall upon the chain. These are but approximations, but practice will soon make them as practically accurate as is necessary.

The Cross Staff.

This is an instrument used solely to lay off right angles. It has been fully described in the Text Book. In addition to what has been there said about it, it may only be necessary to add here, that, in the measurement of straight-sided fields, when the perpendiculars are taken from the diagonal to the

opposite corners, considerable trouble is practically experienced in determining the position of the perpendiculars on the diagonal; and two or three trials are often required, when accuracy is wished for. To remedy this, a little portable instrument was invented by a friend, which can be carried in the hand, and which at once determines the required position of the perpendicular. It is a small oblong piece of wood, of about two inches thick, with two pins placed vertically, having a small mirror, at an angle of 45°, which being held in the line of the diagonal, and turned towards the flag, which is set in the corner of the field, gives you, when your hand is at the required position, the reflection of the flag in the same line with the pins.

By holding this in your hand, keeping the pins in the direction of the diagonal, and walking along it till you perceive the reflected flag immediately in the line of the pins, the true position of the perpendicular can be correctly determined.

It is a simple but very useful instrument.

This little instrument might sometimes be advantageously called in by the *Railway Surveyor*, when he is much pressed for time—as in running his base line, he has but to observe carefully, where perpendiculars, from the corners of the opposite hedges, would fall upon it, and put down stakes. The distances subsequently measured from these stakes to the corners, would give him the shape of the fields—roughly it is true, but not more roughly than, I am sorry to say, is in many cases unavoidable, from the unreasonable demands that engineers make upon surveyors, and the amount they insist upon as *their* measure of a fair day's work—an amount, that, borrowing several hours every day from the night, and sending in work of such quality as may reasonably, under the circumstances, be expected, the surveyor is scarcely even able to keep up to. Why, even this year (1846), when there is little work doing, I have known instances, in the short month of November, of surveyors having been asked to do two miles of

survey in the day. What can fairly be expected, but that, if there is any opposition to the line, such work must inevitably be thrown out on standing orders?

CHAP. II.

THE FIELD BOOK.

The field book should be of a convenient size for the pocket, having the left page ruled with a central column, and the right page left blank for remarks. The central column should be headed "*Chains*," on either side "*Offsets*," and the right page "*Remarks*."

The central column is intended for all actual lines measured, and, by commencing *from the bottom of the page*, the page becomes a smaller representation of the reality with the line measured from you, and the offsets, at their respective distances on that line, taken at so many links to the right or to the left, as are actually on the ground. In keeping the field book, it *first* should ever be remembered, *that the central column is virtually but one line, representing the chain, the space within the column being merely required for the several distances on the chain, whence the offsets are taken;* and, *secondly*, that all offsets, read either way, *outward from the central* column, in the same way as they are measured *outwards from the chain.*

To preserve uniformity, as it is more natural to measure from left to right, the place measured *from*, is put on the left of the central column, at the bottom of the line, and the station measured *to*, is put at the top, to the *right;* the points of commencement and termination of the line can thus be immediately seen.

The book should be interleaved with blotting paper, and the entries, if possible, should be made in ink. The pages should also be numbered, before beginning, for facility of reference.

If the direction of the line is determined by an angle taken by the theodolite, or the bearing of the line be given by the circumferentor, the angle of the former or the bearing of the latter is placed in the central column, immediately above the starting point.

When the line crosses a road, or hedge, &c., make corresponding lines in the field book, as, in the example of the West End Hampstead Survey, at the several distances, 0·13; 0·40; 10·90; 10·95 (*page* 12).

In taking offsets to corners of fences, houses, &c., mark the relative position of the corner as to the chain line, see distance (in the same example) 0·13 on base line, where there is an offset of 6 links on the left, to corner of pond; 0.40, where the other side of pond crosses, there being an offset, on the right, of 20 to the corner of the pond, and 15 on the left; and generally, be careful to make the field book, as much as possible, a *fac simile* of the ground itself, with each post, hedge, house, &c., placed on the book, in the same position with respect to the centre column, considered as a line, as they stand to the chain on the ground.

Stations are generally expressed in the field book by the following character △, which, in the plan, is represented by a circle in pencil, drawn round the station point, which should be always that of a needle.

I would never recommend the use of letters for stations in Chain Surveying. In the first place, they are soon exhausted; in the second, they in no way assist the memory. The BASE LINE, perhaps, had better, when referred to, be termed the *base line* AB, in contradistinction to the secondary lines, which are required in surveys of some extent, and are virtually base

lines to their own portions of survey, and may be lettered CB, EF, &c., &c.

In all other cases, distinguish the lines by their lengths, and the points upon them, by the distances of those points from the zero end of the lines, thus, in the same example before noticed. From 11·89 on 1189 to A on base line; the line 21·08 running from the *end* of the line 11·89 to the beginning of the base line, &c.

In *theodolite* surveying, it is better perhaps to use letters (see example), as the stations are but few, and mostly come within the exception above referred to, letters, in this case, as usually applicable to trigonometrical stations, may be, therefore, more characteristic and distinctive.

Surveyors sometimes take the bearing of the base line at the commencement of a survey, and enter it at once in the field book; this enables them to plan the estate in reference to the meridian line. It must be remembered, that this is but the *magnetic* bearing, and must be entered as such.

Note.—The usual boundary of the field, where the ditch is between it and the hedge, is the brow of the ditch, or that edge of the ditch which is furthest from the hedge. This, however, is not always the case, as sometimes it is the stem of the quickset, or the roots of the hedge; depending upon local custom.

The common allowance from the quick root, for the brow of the ditch, varies, in different places; being 5 links, 8, and sometimes as many as 10 links.

When between fields, it is ordinarily 5 links; when the ditch separates two contiguous properties, 6; and adjoining waste lands, moors, commons, roads, &c., generally 7 links. A wall is generally the division line between the properties, on which side soever the ditch may be placed. When there is a boarded fence between two fields, the fence belongs to the occupant of the other side to where it is clap-boarded, as the nails are considered to be driven home.

The *centre* of a stream running between two properties, is usually the boundary line.

In most parishes in England, a parish ditch is the boundary; the course of this ditch having been altered by time and circumstances, will account for the apparent freaks of the division of properties, which is so striking in the map; running across fields, dividing a pond, and putting one end of a street in one parish, and the other end in another.

It is particularly important in taking the notes of any survey, whenever any of the measured lines cross, or come near any of the division lines of the adjoining property, that these division lines should be noted in the field book. It at once localises the estate surveyed, and is oftentimes of great use to the surveyor afterwards.

CHAP. III.

SURVEY OF A SMALL FARM.

THE following example of the Field Notes and plan of a farm near Hampstead are subjoined for the practice of the reader.

The work should be plotted from the notes, the areas calculated, and the whole compared with the original.

Farm, near West End, Hampstead.

Check Line From 400 on 2065	6·46	to B on Base Line
	10·77	to B on Base Line
to G. P. 32	4·50	
path —	2·20	— ×
to stile 30	2·10	
From 20·50 on 20·65		
High	Barnet	Road
Hedge —	20·65	— ×
D	20·50	△
15 + 15	20·35	
	11·00	△
+ 25	10·00	
28	6·00	
23	3·00	
D 17	1·00	
15 + 20	0·30	
to G. P. 8	0·24	8 to G. P. —
D —	0·20	— ×
From 0·36 on 4·40		

	4·40	to 16·24 on B. L.
D + 19	4·00	
45	2·00	
29	1·00	
△	0·36	
14	0·15	
D + 7	0·00	
From 6·16 on 6·16		
	6·16	△
D + 16	6·00	
22	5·00	
24	4·00	
48	2·00	
50	1·00	
From 3·57 on 6·89		
D	6·89	to 10·58 on B. L.
10 + 22	3·60	
	3·57	△
D + 12	3·50	
19	3·00	
D + 20	2·00	
D + 16	1·00	
From 9·97 on 10·20		
Hedge —	10·20	— ×
D ——	10·11	— ×
D 15 + 40	10·10	
	9·97	△
D 53	9·00	
15 + 55	8·00	
58	7·00	
60	6·00	
D 50	5·00	
15 + 35	4·00	
20	3·00	
15	2·00	
D 12	1·00	
15 + 10	0·50	
From A on Base Line		

End of Right Side of Base Line

Check		Line
	1·63	to 8·00 on 9·63
From 6·03 on 6·04		
	1·64	to 9·34 on 9.63
D + 23	1·55	
2	1·00	
D + 0	0·13	
From 6·03 on 6·04		
D	6·04	— ×
Barnet Road. 6	6·03	△
From 2·80 on 9·63		
D	5·49	to 5·66 on 21·48
15 + 11	5·13	
D 22	4·00	
15 + 0	2·60	
D 10	2·00	
15 + 34	1·00	
From 800 on 963		
D —	9·63	— × 0 + 15 D
△	9·34	
	8·00	△ 70 +
Road + 21 △	2·80	
to G. P. 14	2.33	— × } Occ. Road
to G. P. 12	2·15	— × }
Road + 5	1·00	
From 0·00 on 21-48		
	4·17	to 1·34 on 4·56
	1·88	4 + 35
	1·45	
	0·89	8 + 56
D —	0·15	— ×
From 3·59 on 7·00		

	1·70	to 112 back from 700 on 700
	1·40	4 + 8
	1·17	10 + 20
D —	1·03	— ×
H—	0·92	— ×
	0·85	16 + 20 Shed.
	0·50	7 + D
From 1·17 on 2·50		

	2·50	to 5·66 on 21·48
	2·00	45 + 20
	1·17	△
	1·00	13 + D
	0·60	7 + D
From 4·56 on 4·56		

	5·09	to 10·00 on 11·89
D + 30	4 00	
63	1·60	
D 40	0·55	
From 4·56 on 4·56		

D —	4·56	— ×
H —	4·43	— × D
	4·00	10 + 15
	3·00	20
	2·80	20
H —	2·00	— ×
D —	1·50	— ×
Fence	1·34	— ×
D + 7	1·00	
D + 15	0·40	
From 16·24 on B. L.		

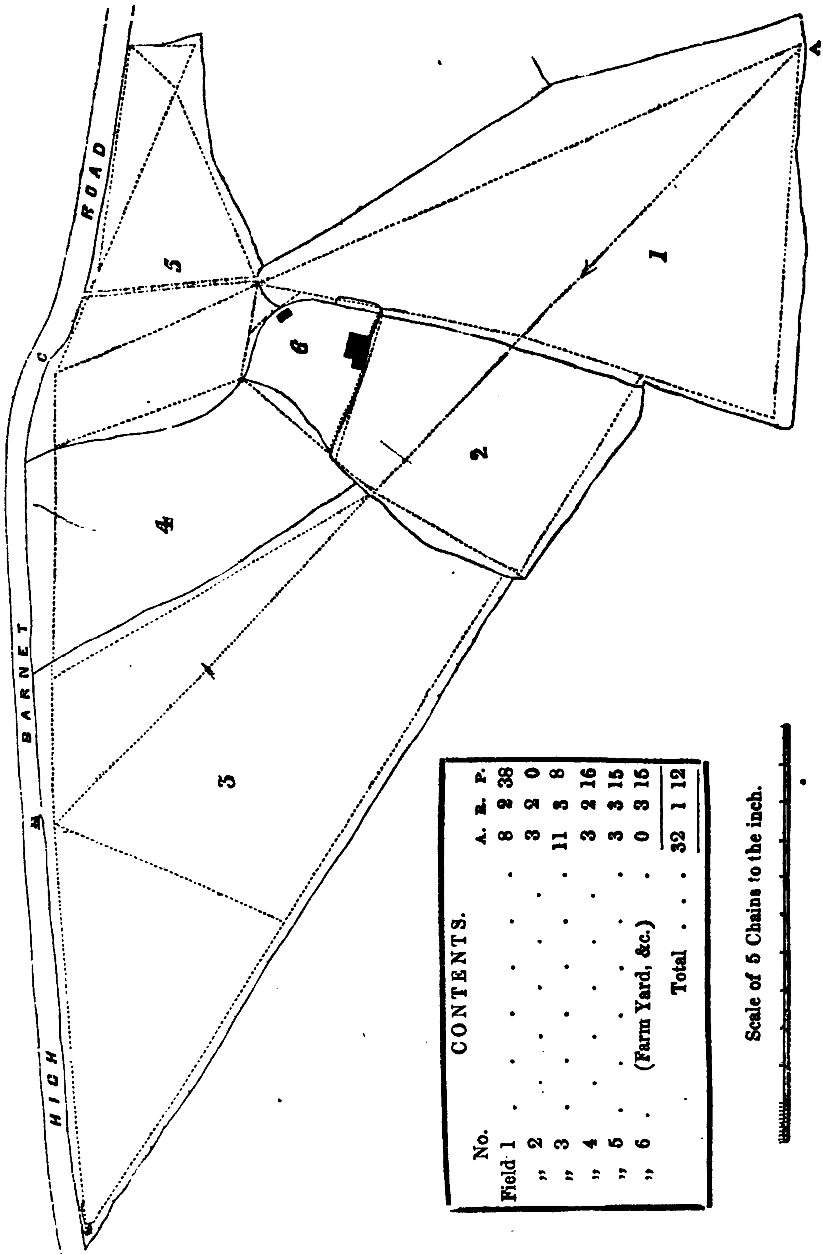
	9·46	to 3·90 on 11·89
	8·90	25 + 10
	8·00	18
	7·00	20
	6 50	15
	6·00	2
	4·00	12
	3·00	10
	2 00	20
	1·00	30 + 10
H —	0·15	— ×
D —	0·10	— ×
From 16·24 on B. L.		

		7·00	to 5·66 on 21·48
	50	6·30	
	22	6·00	
	18	5·30	
Farm	12 + 4	5·11	
		4·00	3
Yard	D	3·80	
	10 + 3	3·72	
	D	3·59	△ o + Tree
	10 + 20	1·00	
From 10·58 on B. L.			

D	60	21·48	
		21·22	—
	H —	21·12	— ×
		21·08	to A. on B. C.
	76	20·00	
	160	14·60	
	73	9·00	
	58	8·00	
	42	7·00	
	D + 30	6·00	
G. P.	16	5·66	△ 0 to G. P.
From 11·89 on 11·89			

	10	11·89	△
Barnet		10·00	△
Road	D —	9·85	— ×
	H —	9·60	— ×
	90	9·50	△
	85	7·00	
Barnet	D	4·23	— ×
Road.	H	4·05	— ×
	60	3·93	
		3·90	△
From 28·20 on B. L.			

	High	Barnet	Road.
	H —	28·55	— ×
		28·20	△ to B
		20·00	
	H —	16·45	— ×
	D —	16·30	— × 42
		16·24	△
	D —	10·95	— ×
	H —	10·90	— ×
		10·58	△
		10·00	
		0·50	10
		0·40	
	15	0·30	20 Pond
	6	0·13	
From A on Base Line.	Base		Line.



CONTENTS.

No.		A.	R.	P.
Field 1		8	2	38
,, 2		3	2	0
,, 3		11	3	8
,, 4		3	2	16
,, 5		3	3	15
,, 6	(Farm Yard, &c.)	0	3	15
	Total	32	1	12

Scale of 5 Chains to the inch.

General description of the Notes and mode of plotting.

AB is the base line, which runs through the survey. It is so selected as to run through, as near as possible, the centre of the farm; on each side of this are based the various triangles by which the hedges and buildings upon the farm are obtained.

It commences from a point, marked A on the plan, near the corner of a field, and has the following offsets upon it: at 0·13 the near side of a pond crosses and extends 6 links to the left; at 0·30 the left offset to corner of pond is 15; and on the right 20 links; at 0·40 the other side of pond crosses; at 0·50 it is 10 links to further angle of pond; at 10·58 we come to a station; at 10·90 the hedge; and at 10·95 the ditch crosses; at 16·24 we arrive at another station. At 16·30, where ditch crosses, it is 42 links to the corner formed by junction of cross ditch on the right; at 16·45 hedge crosses; at 28·20 is the station B, the other extremity of the base line, and at 28·55 the hedge of the high road to Barnet crosses. This being the termination of a line, a single line is drawn above this.

The next line turns to the right, denoted by the mark ⌈; it is 11·89 long, and has various stations upon it, viz., at 3·90 we come to a station; at 3·93 it is 60 links on the left to corner of field; at 4.05, hedge and, at 4·23, ditch crosses; at 7·00 an offset on the left of 85 to ditch; at 9·50, which is 90 links to ditch on the left, is a station; at 9·60, hedge; and at 9·85, ditch crosses; at 10·00 is a station, and at 11·89, being 10 links on the left to ditch, is another station. A single line is drawn above this.

From this point we again turn to the right; and at 5·66 we come to a station, at a gate post; the other gate post is 16 links off on the left. At 6·00 chs. it is 30 links to ditch on the left; at 7·00 chs. 42; at 8·00 chs. 58; at 9·00 chs. 73; at 14·60 chs. the offset on the left is 1·60 to ditch;

at 20·00 chs. it is 76; and at 21·08 we come back to the starting point A, on base line; at 21·12 hedge crosses; at 21·22 ditch crosses; and at 21·48 it is 60 links on the left, to corner of a bend. This being a return to an old station, a double line is drawn above to denote it.

We have now three lines, the base line, and two other lines, viz., 11·89 and 21·08; the former running from B, the latter from A; from B and A, therefore, as centres, describe the distances 11·89 and 21·08, intersecting each other in C; BC, and CA will be the direction of these two lines, which are now become "fixed;" this line is now produced to 21·48 chs.

The next line runs from 10·58 on base line to a station 5·66 on 21·48. It is consequently a check line, though, as it is not measured expressly for that purpose, it is not so termed in the notes. Let the reader, therefore, mark off these two points upon their respective lines, on his plan, and measure with his scale the distance between them. If this distance is 7·00 chs. the work is thus far correct, and the offset may be put in. The offsets upon this line are these, viz.—At 1·00 chs. there are 20 links to the hedge, and 10 beyond that to the further brow of the ditch; at 3·59 is a station, by the side of a tree; at 3·72 it is 0 to hedge, and 10 to ditch and corner of field; at 3·80 the line crosses the ditch of a small enclosure, which, at 4·00, is 3 links from the chain line; at 5·11 the enclosure ends; the distance, on the left, to the fence of farm yard, here, is 16 links; at 5·30 the offset is 18; at 6·00 chs. 22; at 6·30, 50 to corner, and at 7·00 chs., where ditch crosses, we come to an old station, 5·66 on 21·48; a double line is also put above here to signify this.

It will be found that the following line also, running as it does from a known point, 16·24 on the base line, to another known point, 3·90 on 11·89, is a check line; let the reader try this also by his scale. The offsets upon this are so simple,

they need not be referred to, being taken merely for the sake of the ditch which runs by the side of it.

The next line starts from 16·24 on the base line, and turns to the right; it runs to no previous point, and is, therefore, a loose line, as until some other point upon it is connected with a previous known point it may be drawn anywhere. By looking at the following line it will be found to be tied in at the end to a previous point, 10·00 on 11·89, by a line 5·09. Now it is no longer a loose line. From 16·24, on the base line, and from 10·00 on 11·89, as centres, describe the two distances, 4·56 and 5·09; and these two lines will become fixed; their intersection is at the corner of the farm yard (marked No. 6 in the plan). Offsets might now be laid upon them, were it not that it were better to wait for some *check* line to prove their correctness.

On examining the field notes, the following line will be found to answer the purpose, as it runs from the point of intersection to the old point, 5·66 on 21·48. Place the scale between these two points, both of which are upon the paper, and try them; the distance between them should be 2·50.

The offsets of these three lines can now be put in. As to the first line 4·56, the first distance 0·40 having an offset of 15 to the left, shews that there is a *cross* hedge there, being the corner of field No. 3. At 1·34 the line crosses a straight fence belonging to the farm yard. This point is also made a station. At 1·50 the line crosses the ditch between the field 4, and the farm yard 6, running now through the former; the offsets which were previously on the left being subsequently on the right.

The offset on the two lines need no explanation.

With reference to the next line, on inspection of the plan, a small shed will be seen in the farm yard, contiguous to the line, 2·50, but not near enough to take an offset correctly. The line, 1·70, was, therefore, taken; it runs from 1·17 on 2·50 to 1·12, back from 7·00 on 7·00; that is, to a point, which is

1·12 back from the end of the line 7·00, in other words to a point 5·88 in that line.

NOTE.—It might be considered (and by beginners generally is) far more easy to write down "to 5·88 on 7·00" at once, but practically, this objection is found to obtain, that, though in the field the distance 112 may be correctly measured, yet, in the hurry of field work, the subtraction of that number from 7·00 may be incorrectly done, and as the only entry then made in the book would be the result, the actual measurement would be forgotten, and there would be no correcting it. The insertion of the measured distance in the notes, and the leaving of the work to the office prevents the possibility of this.

The offsets of this line, 1·70, seem so clear, as scarcely to need explanation. It may, however, be mentioned, perhaps, that the length of the shed is from 0·85 to 1·17, and the width 20 links; not being parallel to the chained line, but at 0·85, being 16 links off, and at 1·17, being only 10 links.

The line, 4·17, which comes next, runs along the railing of the farm yard. It is taken to get the farm-house in. The dimensions given in the notes will sufficiently explain the mode of obtaining the position and shape of it. At 0·89 it is 8 links from the line; at 1·88 it is 4; at the former it is 56 links deep; at the latter only 21, having a "set in" as it is termed. The distance upon the chained line, 1·45, will shew the extent of this; 21 and 35 will make the 56.

NOTE.—It is generally desirable, where, as in the present cases, there is a block of buildings together, to score the lines in different directions. This at once distinguishes one part from another, a matter of considerable moment, where there may be different ownerships and occupations; in the present case, the house is the smaller portion on the right, the remainder is the barn.

This is also another check line, and therefore has two lines over it.

We now come to field No. 5, which requires two or three triangles to complete it; 8·00 chs. and 5·49 are the two sides of the larger triangle, and 6·03 and 1·64 those of the smaller, the larger triangle having no check, but the smaller; the line 1·63 running from the corner of the field to 8·00 on the line 9·63.

This closes the *right* side of the base line.

The measurements on the *left* are merely a succession of triangles, beginning at the field No. 1, each based upon the preceding; the first upon the beginning of the base line, and the last checked by a check line to the end of the base.

The first triangle is 9·97 and 6·89; the second 6. 16 and 4·40; the last, the larger one in field No. 3, being 20·50 and 10·77, having a check line running from 11·00 on the former to B on the base line, and being 6·46 long.

There are no offsets of any moment upon these lines.

Calculation of Areas.

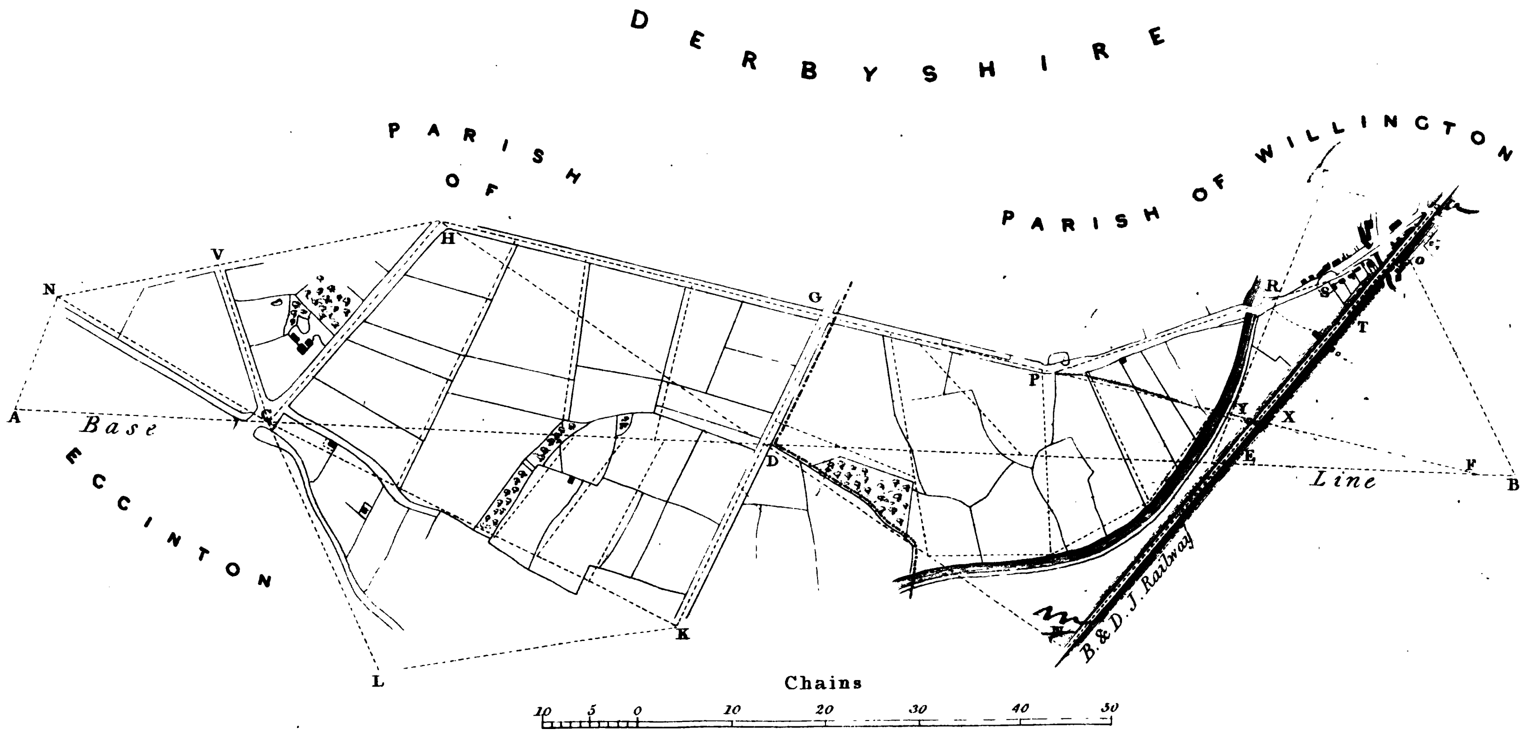
Take each field separately, and treat them as at page 52 of the Text Book, by dividing them into triangles. The "second method", as given at page 54, "*by giving and taking*", is the plan universally adopted in practice, in changing the irregular lines of the hedges into the regular sides of the triangles, from which the calculations are made.

The areas thus found should correspond with the areas on the plan.

The plan in the book is plotted to a scale of 5 chs. to the inch. It would, however, be better for the student to plot his to a larger scale, say of one or two chains, and compute his own areas.

In offices, where there is much work, it is usual for two parties always to be engaged in this computation, and for separate sheets to be used; the one for the scaling, the other for the working.

A column is drawn in one sheet for the numbers of their


DERBYSHIRE
PARISH OF
PARISH OF WILLINGTON
ECCINTON
Base
Line
B. & D. J. Railway
A
B
C
D
E
F
G
H
K
L
N
P
R
S
T
V
X
Y
Chains
10 5 0 10 20 30 40 50

F. Mansell. sc.

fields, and another column for the various triangles, in which each field is divided; in this column are entered the base and perpendicular, thus for example:

Field	Perpendicular. Base. sq. chains.	Area.
No. 3.	$\frac{2{\cdot}40 + 4{\cdot}60}{2} \times 3{\cdot}20 = 11{\cdot}2000$	A R P
	$\frac{14{\cdot}40}{2} \times 2{\cdot}40 = 17{\cdot}28$	2. 3. 15.

On the other sheet is entered the working of them, having also one column devoted to the numbers of the fields, so that, in case of any error being suspected, the one sheet will at once shew on examination, whether *the lines* have been properly *scaled;* the other, whether the *calculations* are correct.

The whole of the areas are entered in this way in corresponding sheets, and filed, as office documents.

CHAP. IV.

EXAMPLE OF THE USUAL ARRANGEMENT OF LINES OF A PARISH SURVEY WITH THEIR CHECKS.

THIS example is that of a larger portion of country, comprising some 60 or 80 fields, 7 or 8 roads, a village, river, canal, &c., and a line of rail road running through a corner of it, towards the village. It is an actual survey of a portion of the county of Derby, in the parish of Willington.

A base line AB has been selected, contiguous to an inten-

ded line of railway, and running through the whole of the survey, intersecting the road HC at C; the road GK at D; and the road MO at E, which distances are carefully marked in the field book. Where the base line crosses the hedges also, at the most favourable place for running cross lines along the hedges, stakes must be put in, and the points carefully noted, taking offsets *en route* to any hedges or corners of hedges, fences, or other objects, that may be within an offset distance.

NOTE.— The methods as given below, the student must bear in mind, have been supposed *free* from those *local obstructions* which will often materially change the arrangement. He must keep in view the difference he will find in laying down a method of survey, *when all the plan is before him, and he has a bird's eye view of the whole, whieh is supposed to be perfeetly flat and free from every obstruction;* and, when, ignorant of the locality, unable to comprehend its content at a glance, he has also to contend with the local difficulties of hills, vallies, woods, rivers, houses, &c.; in fact, of all that constitutes, in all cases, the difference between theory and practice.

The method actually adopted would be, in order to avoid any needlessly going over the same ground twice, to commence at A, measure AN, NVH, then HC, CV, and the fields within the block VCH and NC.

From H, the next line measured would be HG, observing, carefully where the best stations could be taken for the cross hedges, on the same side of them, as the stations were selected in the base line.

Then measure GD and HD, observing, in measuring HD, to have the range of the line carefully defined, where the several hedges cross; so as accurately to define the several points in the line CD, where the cross hedge-lines, from AB to HG, intersect. By this plan, all these cross-lines are check-lines.

Produce GD, to K, in the same straight line; measure KC, taking notice as before, where the cross hedges come, and on their proper side, and *complete* the block HGKC.

Then measure CL and LK.

Now return to C, and produce HC to F, where it intersects the base line; marking the several points P, Y, and X, upon it, and the several cross hedges.

Then from D produce HD to M; and from M, measure a line in range with EX, which produce to O; join OB; then complete the block GXMD, and the triangular piece KDM.

There now remains but the part adjoining the village.

From P measure PRS, and join ST; then produce VR to *b*, and join *ba*. The lines, *a*T, TS, SR, R*b*, will tie the whole of the houses in.

This must always be the plan adopted in the survey of a village, or farmhouse, or homestead; to confine all the areas within one triangle, whose three sides should severally pass through the principal points of the place.

Having given the method, adopted in practice, for saving time in the survey of the plan, we will proceed to explain the nature and use of the several main lines.

The line ME, of the triangle DME, is the measure of the angle MDE: but CH, in the triangle CHD, is the measure of the opposite and equal angle CDH. Therefore, the measured and the determined distance, agreeing or disagreeing, of either the side CH or ME, is a proof of the correctness or incorrectness of the angle at the vertex D.

Produce ME, the *fixed* line, to O; any points upon this production are also *fixed*. The point X, which is in a range with the road HP, is fixed; and H, being a fixed point, the length of HX is determined.

Its measurement becomes a line of verification to the oppo-

site angle HMX, or (MDE, being supposed correct,) of the supplemental angle DEM: which is the angle that this new line MD makes with the base line. *The line OB, if the nature of the ground will permit its being measured, measures the opposite angle BEO and is another check upon its correctness.*

Again, MX is the measure of the angle MHX; GD is also the measure of the same angle.

The actual distance of GD, compared with its computed or determined distance, is a check upon the correctness of the length of MX.

Having determined the correctness of these triangles, there can be no error of any moment in the *filling in.*

In fact, all the lines used for the measurement of the offsets to the cross hedges, are only so many additional check-lines to the triangles, or measures of the angles at their vertices.

EK being *determined* by the previous measurements, its measured distance is a check upon the angle CDK, and therefore, upon the direction of the line KC, relative to the base line AB.

The correctness of the triangle, CLK, is secured by the common check line to its vertex L*c*.

The triangle AHC, having in AC a portion of the base line, depends upon the correctness of the measured distances AN and NC.

A being thus a fixed point, as well as H, measure the line NH, and as H has been previously assumed correct, HN is a measure of the angle NCH, which is the supplemental angle to the two known angles HCD, ACN; the length VC is a check upon the distances CN and CH.

Now returning to the other parts of the survey, the line HY, produced to the base line at F, is an additional verification *of the whole of the triangulation.*

To ensure a correct survey of the village, observe that the line MOa passes close to one side of it.

From P, drawing PRS through R, and joining ST, you have *known* lines *close to the village* on *another* side; producing YR to a point b, such that a line ba shall pass close to the third side of the village, you surround the whole with a fixed triangle. All errors must be confined *within* this limit; and all lines, for the measurement of the streets or lanes, carried through to either of the sides of this triangle, are, as in the case of the cross hedge-lines, in the first part of the survey, virtually but so many corrobative checks of its accuracy.

CHAP. V.

RAILWAY SURVEYING.

Preliminary Remarks.

As in all matters connected with surveying as well, in fact, as in the ordinary affairs of every day life, the express object of the survey should always be borne in mind; it must be remembered, that the special object of Railway surveying is a strict compliance with the standing orders of Parliament. These orders involving not so much a faultless representation of this or that hedge, of the width to a few links of this or that river, but a positive insertion of every thing required by those orders, and generally such an accurate representation of the boundaries, &c., as may enable an owner or occupier to distinguish his individual property or occupation.

This is the grand object of a railway survey. While on tithe or parish surveying, it is a general and particular accuracy in delineating every portion of a parish, such, that not only one part of a field should be the exact distance from any other part, but that the field itself should be correctly placed with respect to any other field. In the former, the purpose is answered, if the base line through it be correctly measured and the fields on either side, within the "limits of deviation" stand relatively in the right position—beyond them, errors are of no importance; while the latter requires, both in the base line, and its subsidiary lines and triangles, equal correctness.

In parish surveying, first class plans have been returned for one or two poles of error. In railway surveying, on the contrary, (I am strictly speaking of a parliamentary survey), it is a matter of little moment whether a field contains 5 acres and 25 perches or 5 acres and 1 rood. In permanent railway surveys, of course there ought to be the same correctness as in parish surveys.

In the insertion of a right of way, it is generally considered sufficient, although its position may not be strictly correct, if the deviation from the true spot does not exceed some 10 or 20 links.

I am quite aware these things should not be, but they do exist; the indecision of companies, the difficulty of raising a sufficient sum to commence the survey upon, the necessity of waiting until the crops are off the ground, and various other practical impediments, have always had the effect of delaying the commencement of the work, until it becomes impossible to complete all the requirements for the standing orders, and at the same time do it with the accuracy of a tithe commutation survey.

There is, therefore,—and it necessarily follows, that such should be the case—a great difference, both as to the results, and also as to the mode of conducting the operations, of these

two kinds of survey. It may be useful, therefore, to give the student an example or two, of railway surveying as distinct from the other.

These examples, I will endeavour to make as practical as possible; having always in view this special characteristic of railway surveying, or rather, as I before observed, of parliamentary surveying, viz, that of leaving nothing out which is required by the standing orders, "and of throwing away no unnecessary care upon, (to them) immaterial details."

CHAP. VI.

MODE OF CONDUCTING A RAILWAY SURVEY.

The most important part of railway surveying, is the *selecting* and running the several *base lines.*

In consequence of the ordnance surveys, one half the trouble is saved in this country (at least south of the northern part of Yorkshire) which is unavoidable in most others.—The ordnance maps are invaluable to the engineer. Furnished with the maps of that portion of the conntry through which the railway is intended to be made, he has only to drive over the ground and personally examine the locality — a few days will enable him to mark the direction of the line upon the maps. These maps have only to be handed to the surveyor, who can at once proceed with the survey.—

Although, not immediately in my province here, I may be permitted, in writing a book on surveying and levelling, to address a few words to engineers themselves.

Some engineers, eager to commence operations, do not generally take sufficient time to obtain *for themselves* sufficient information. During the last season (1845—46) I have seen serious consequences resulting from this, and these in several cases.

They receive instructions to get up a line from A to B, order maps, drive over the ground, making a few local enquiries on the way, draw the line in upon the map, as they pass through, return home, send copies of the line to the surveyor with orders, to put some one upon a trial level, but to go on at once with the survey, only taking care to make it double the Parliamentary width on each side, so as to allow them plenty of room to deviate, should they at any time during the survey desire to do so.

What the consequences may be, I am not now speculating upon; what they actually were, the numerous cases that occurred last year, will amply testify.

The surveyor, having double the work to do, (by having double the width) ought really to have double the usual sum per mile; but this would be unjust to the company. If then he is only paid half what he ought to be, when he has double the work to perform, he must necessarily pay the same in proportion to those he employs, who, if they are good and honest hands, hurry through the work, to complete their contract, but pre-engage themselves to other parties, to perform some other work, after their completion of this, and if any deviations are required, decline undertaking them.

If they are dishonest persons, (speaking morally) they slur the work over, knowing they are not fully paid, by finding it necessary to do double the work in the same time, in order to realize the same profit from it; and as they cannot tell where, within this double width, the base line will really go, they take the chance of their inaccuracies being beyond the limits of deviation.

Besides these, there are other evils that indirectly, but

necessarily accompany this practice. The attention of the surveyors is unavoidably divided, being spread over double the width, so that, instead of concentrating all their energies to prevent inaccuracies, or omissions, within the "limits of deviation", (and beyond them they are not cognizable) they know not how far these limits will extend, nor where to direct their more particular attention to.

The uncertainty of the position of the line is also most seriously injurious, when it crosses high roads. Were the line fixed, so that the distance upon it, where it crossed the road, was known, any error in the angle it made with the road, would be a matter of little moment; but having the line unfixed it might cross it at a point which, from the error alluded to, would make the distance between that and the preceding road different upon the plan and section, a discrepancy, which forms a favourite allegation of non-compliance with standing orders.

It is most important that the base line of the survey should be as near as possible to that of the proposed line of railway; as all distances upon the base line, if carefully measured, must necessarily be correct, and whatever errors of direction may exist away from it, must consequently increase as the distance from it increases.

To enable the surveyor to run his base lines as near as possible to the rail line; this should be decided upon, if possible, before the survey is commenced.

To do this, instead of the engineer giving directions to the surveyor, to commence the survey of the line, and the trial levels simultaneously, he should invariably have two or three trial levels taken, so as to satisfy himself of the best course for the line, and then give the orders for the survey.

If this plan were universally adopted, the results of the parliamentary ordeal of standing orders, would be very different. This is one thing that gives the advantage to old companies in obtaining their branches. Their engineer has learned the

necessity of this plan by experience; and they themselves know better what branch lines are really required, and at the same time, both where the country is best fitted for a line, and when they are least likely to meet with opposition, and also where they may have greater weight and influence to overcome any attempts to oppose them.

CHAP. VII.

RUNNING OF BASE LINES.

1. "*As a general rule, run your base lines as close upon the direction of the proposed railway as you possibly can.*"

There are, of course, many obstacles in the way of your doing so practically. Woodland, in this country, is so valuable that woods and coppices must necessarily be avoided.

If you have a wood upon the right as well as on your left hand, run your base line between them, taking care, when you can do so, to keep always WITHIN "*the limits of deviation.*"

Farm buildings, villages, bends of rivers, &c., may also be in the way; these must also be avoided if possible.

2. *Let your base lines be continued as long as the limits will allow.*

The obstacles above referred to, the curved direction of the line itself, &c., may sometimes unavoidably render short lines necessary. You cannot, in all cases, do as you would wish, circumstances must necessarily govern the lengths of

the lines, but the longer the base lines, and the fewer, the angles taken, the better.

In selecting your lines, be careful to ascertain exactly your position upon the ground; and, if you have the means of doing so, use any conspicuous local objects that the country may furnish you with, to determine the direction of your course.

Examine the Ordnance map, and see whether there are any churches, windmills, or other conspicuous objects near the direction of the proposed line of railway. Should there be any of these, even though they be not exactly in the rail line, make towards them, being careful, however, not to get beyond the limits of deviation. The spires of churches, windmills, telegraphic stations, &c., are objects deemed invaluable in a survey.

A country slightly undulating is far better for base lines, than one perfectly level. In the former you may generally select your angular station from one rising ground to another, commanding as you do, a long distance from each; but, in the latter, you frequently have no point whatever before you for reference; the very field, from the height of the hedges, which generally prevail in flat, pasture land, in which you may be standing, perhaps, bounds your view. In this latter case you have nothing but the compass in your instrument to guide you. You may, perhaps, be fortunate enough, sometimes, to have an intelligent chainman with you, who is well acquainted with the country, and may afford you some information as to the locality of the neighbouring villages, woods, churches, &c., but this is rarely the case. In hilly ground too, moreover, you are sometimes seriously at a loss to find any known signs by which you can determine precisely the locality. There may be nothing of note in the neighbourhood, nothing to refer to.

In this state of things your needle is your only help. The bearing of the base line must be carefully determined

upon the map (all ordnance maps being north and south), and due allowance being made for the variation, a similar line to the same bearing must be run upon the ground. This line, if straight, will be sufficiently correct if found, at the close, not to have exceeded the limits of deviation.

NOTE.—A subsequent chapter will be devoted to determining the variation of the compass above referred to, and to the method of running a base line in a given direction.

CHAP. VIII.

MODE OF CARRYING ON THE RANGE OF THE LINE, WHEN THE DIRECTION HAS BEEN DECIDED ON.

PLACE a flag at the starting point, and another as far as possible onwards in the required direction. Produce this range thus: do not stand at the first flag in order to range another person in a line with the second flag beyond it, but go further on the line yourself and determine the position of third flag, by placing it so as to cover the first two. If this third flag be at some considerable distance from the other two, it will be requisite to use a telescope; it is, perhaps, the quickest plan to take the theodol ite with you, and without opening the legs, hold it perpendicular and use its telescope; a difference of a couple of inches in its position will cover, or render distinctly visible, the separation of the two flags. Proceed onwards to the fourth flag, and do the same, and continue to advance in this way, as far as it may be

necessary to carry the line. In case of any slight error creeping in, as can scarcely perhaps be avoided in all practical problems; suppose, for example, at the 7th flag, the 1st, 2nd, 3rd, and 6th flags are in line; the 4th and 5th are slightly out: fix this 7th flag in line with the 1, 2, 3, and 6, taking no notice of the 4th and 5th. By carrying on the range in this way, between the 6th station and the 1st (if visible), whatever errors may have occurred in the 4th and 5th flags; they are corrected at once and confined to themselves.

Generally, at any point of the line, by carrying on the range between a near and a distant flag, the line at its termination will be found approximately correct, as, for instance, any line, AB, which, connecting the two points, A and B, considered as the beginning and end of any base line whatever, and running between the whole of the several points actually taken, is not half a foot distant from any one point, and, therefore, limits every error to that distance.

In going over a hill, however, this method of ranging onwards with the telescope is impracticable: then the properties of the theodolite are brought into operation. To obtain flag 3, for instance, the theodolite must be placed at No. 2, set to zero, and reversed to 180 degrees, a flag being placed in that direction. This method, however, is liable to error, and requires to be performed very carefully. When the onward flag is intended to be placed at some distance, it had better be done in two ways: in the one just mentioned, by reversing the instrument to 180 degrees, taking special care that the bubble at right angles to the vertical arc, is perfectly level, otherwise in the case of flag 1 and flag 3 not being in the same horizontal plane, that is, of the one being higher than the other, serious errors will result; in the other way, which serves as a check upon the former, by setting the telescope to flag 1 and clamping it tight, having the clips of the telescope previously slackened, then by reversing the telescope itself in the Ys, if

the intersection of the web covers the onward flag by both ways, you may safely consider the work to be correct.

The ranging of the base line is perhaps one of the most important parts of railway surveying: it is quite as important as the chaining it. If the base line be correctly done in both cases, the filling in afterwards is very simple. One of the difficulties in ranging a base line, is in the objection that farmers naturally have to the cutting of their hedges. To lighten this objection as much as possible, great care should be taken in getting a correct range at once, and not, as I have known many young hands do, cutting the hedge a second and a third time, from their not having got the right place, or not cutting it through in a proper direction. This is more especially the case when the line crosses the hedge obliquely: the chainmen or axemen are often too apt to cut every hedge through "square" without considering how the line crosses it. The way to avoid this is very easy, suppose you have a flag (6) on one side of a field; flag 5 or 4 being visible: go to the other side of the field, but within it, and close to the hedge, place flag 7 carefully in range. In the next place, range yourself between flags 6 and 7, about 10 yards from flag 7; then send a person round the other side of the hedge with two flags, if necessary, tied together where the hedge is high, or the ground is much lower on that side, and range him in the right direction beyond No. 7. You may see the double flag over the hedge, and may place it in line. You have a flag now on each side the hedge, and the men can see exactly which way to cut, after having done which, the flag No. 7 is to be taken up, and fixed beyond the hedge, where the double flag stood, being careful, however, now that the hedge is clear, to correct any slight inaccuracy in ranging. Another evil with young hands and careless men is, that they are apt to cut too low. It is the unnecessary havoc that these youngsters have made, coupled with the prejudice that many ignorant farmers still

entertain against Railways, that renders them so bitter against Surveyors in general.

Another fault, that young men are too prone to commit, proceeds from their fondness for taking very long sights, in order, no doubt, to save themselves some additional trouble. Sometimes, for instance, a long distance in advance can be perceived, to a clear spot between the trees, where the range will come; but more time is frequently lost in unsuccessfully trying to find this spot, than would have been occupied in carrying on the range by short lines at a time.

This is generally the case, when the course of the line runs across a valley, with a brook and trees at the bottom.

The best, safest, and shortest plan is: to carry on the range down the hill, across the brook, and up the opposite side, by short distances, and when the top is reached, to correct any inaccuracies of range, that the shortness and number of the observations may have occasioned; the range, imperfect as it may be, will bring the surveyor pretty near to where the line should come. No time, certainly, would be lost in this way in seeking it. When placed on one side of a hill, and looking to the opposite hill, the apparent line of sight is often materially deceptive, causing objects to appear differently under a different point of view; so that it is not so easy, as a young surveyor might fancy, to find any distant point, seen under such circumstances.

Ranging.

In ranging a base line for a railway survey, I have always found it the best plan, to have plenty of assistance; economize as much afterwards as you please, in the filling in, but ample material in the former is the truest economy. One or two extra hands may enable the surveyor to do double the work, and thereby gain more, than he will pay for a dozen men.

It is seldom that any, but a first rate hand, is entrusted with the base line; the value of *his* time bears no relation to that of the men. To do the work, therefore, quickly and effectually, the surveyor ought necessarily to require the assistance, first: of two men with *slash hooks* *, to cut through hedges; two men, to help him with the instrument, and to carry flags; and two men, or boys, to keep behind and bring up the flags, that are done with. The last two should be furnished with bundles of pointed willow twigs, as many as they can carry, to put down in the place of the flags. These flags should be slit at the top, into which small pieces of paper should be fixed; and one of the boys should occasionally be sent forward with the flags taken up, in order to keep the ranging party always going. The two flagmen should take with them as many flags, as they can manage to carry; some small ones, of 6 feet in length; a smaller number of others, 12 feet long; say: a dozen of the longer ones, and two dozen of the smaller, furnished with red and white flags, and shoed and pointed with iron.

The surveyor should take plenty of white paper with him, cut into squares of about 3 or 4 inches, to fix in the hedges where the line comes.

It is the duty of the axemen, besides cutting down the hedges, wherever they find a flag near the hedge, to cut a station on the ditch side, thus: and fit the flag in the hole again. This saves the willow twig at every hedge. The surveyor puts a piece of paper in the hedge, and as the station mark shews him where the line passes, when the boy comes up for the flags, he takes that one away, without putting a willow twig in its place, as few hedges are so far apart, as to be invisible from the next, fewer willow twigs will thus be wanted; and the quantity they take out

* The slash hook is an instrument, made very much after the pattern in the frontispiece.

with them, generally, by this means, lasts them the whole day, even though 3 or 4 miles of ground may be got over.

Another important purpose is answered also in this way; the papers in the hedge are not so easily noticed; flags are offensive to the farmer's eye, and occasion sometimes disagreeable interruptions to the progress of the survey. It cannot be too strongly impressed on the mind of young surveyors, that the quieter, in any work of this kind, the operations are conducted, the more courteous their demeanour to farmers and occupiers of the land, they are invading, and the more a disposition to avoid doing unnecessary damage is evinced, the better, in every point of view, will it be to themselves. I must not omit to mention here (and it is a precaution often neglected) that the surveyor and flagmen should take plumb bobs with them, for the purpose of seeing that the flags, when fixed, are perpendicular. Many a long line has been sadly out of range from neglect of this.

When there are two or three base lines, forming angles between them, instead of commencing the second line at any fixed point upon the first, unless there be some special reason for doing so, consider the first line an indefinite one. By so doing, you may select the direction of the second, on any part of it, whether in the beginning, or middle; and various local obstructions may compel you to do so. You can then run it back, to cross the first one, wherever it may fall. This should also be done with the third, or fourth, and with as many base lines, as the nature of the survey may require. Where you can do it conveniently, at each point of intersection, a 20 foot pole should be placed.

Chaining the base line.

It is necessary that the chain shuld be of the proper length, by comparing it before hand with the standard measurement,

and in using it, care should be taken that it is properly stretched, and perfectly straight every time.

Let the base line also be measured twice, this must in no case be neglected. Should any errors recur in the side lines those especially that run parallel to the base line, the surveyor is enabled frequently to detect and remedy them. In measuring each chains length, let the pin be placed outside the handle, it is seldom that the chain from its weight will not occasion a loss in length of the thickness of the handle.

In chaining through hedges, be sure and pass the chain through in the right direction, and as low down as the thickness of the roots will permit, this can be easily done with the slash hook, or with the offset staff, if it be furnished, as it should be, with a hook at one end, and a dibber at the other, and it had better be made round and coloured *black and white.* Let every tally be invariably inserted in the field book, beginning again at every mile.

In chaining over hilly ground, the best practical method is, to keep the end of the chain close down to the ground on the the higher side, and to hold it up at the lower end, holding the point of the pin in the hand, with the ring downwards, and drop it.

If the ground is very steep, measure it by 25 or 50 links at a time, or even less if necessary.

Let every thing be done carefully, and do not hurry over the base line, nothing can be gained by it. Take all offsets, that are near to the base line, and as you go along, *sketch where you can, the position of* the *fields on each side of it.* You will find this useful in selecting your subsequent lines and in filling in. Above all things do not forget to mark all parish boundaries.

A station must also be marked at every hedge close to the ditch, and inserted carefully in your field book. Be provided with slips of white paper to write the length of the line measured, wrap them up carefully and put them into the

station holes, in one of the corners, covering them with a piece of turf. If the ground is at all soft, the station might be cut out at once, the paper put in, and the mound thrown in again, the hedge being cut, the gap made will enable the surveyor easily to find it afterwards.

In crossing a road obliquely, the station should be within the road, but on one side of the beaten track. Where the hedges are sufficiently low, as not to require cutting, in order to preserve the direction of the base line across the road, which, when the hedges are high, is known by the gaps cut on either side, another station should be made on the other side of it. As each road generally requires a line to be run down it, its point of intersection with the base line can afterwards be determined in the filling in.

Much useful information may be obtained, invaluable to the reference men afterwards, by the surveyor inserting at the time, in his field book, as he goes out of one field into another, which is arable, or which is pasture land: it would be very little trouble to him, and would materially add to the general good.

CHAP. IX.

TAKING ANGLES ON THE BASE LINE.

In taking the angle between an old line and a new one, let the instrument be placed at the end of the old line; take the angle made between the *old* and the *new*, and not between the new and the old; that is, the instrument being set to zero at the old line, is turned from left to right till it coincides with

the new, when the angle is read off. If the angle reads less than 180°, the new line runs to the left of the old, that is, if the old line were produced, it would pass to the right of the new; if more than 180°, it runs to the right. By this means, the very frequent mistake of not knowing which way the new line turns is prevented. As a check also upon the correct working and reading off this angle, be careful to repeat it three times (see the part on the theodolite); taking the furthest flag that can be seen by it upon both lines; thus, in the example of a portion of a line near Wigston Magna, the angle taken is 227° 7′, shewing that the new line, BC, turns to the *right.*

CHAP. X.

HENTON AND FINCHLEY SURVEY.

RAILWAY SURVEY IN THE PARISHES OF HENDON AND FINCHLEY.

(*The field notes will be found at the end of the book.*)

The base line being ranged, and the bearing, from A to B, found to be three degrees east, the chaining was commenced.

If the student wishes to derive any benefit from this example; he must plot it himself from the notes, referring as he goes on to the accompanying explanations, which, of course, are confined to those things where difficulties may be experienced and which may retard his progress.

At 0·70 and 1·60 a road crosses, and at 0·85 a station is taken, for the purpose of subsequently measuring along

the road, and determining the position of any objects to be noted upon it. At 1·56 there is a cross hedge. At 1·83 there was a station taken, but it was not used; this is frequently the case; it does not occasion much delay, and should no station happen to be near, when you have occasion to run a side line to the base line, some time is generally lost in finding the exact point of intersection with the base line. At 9·70 is a △, close by the next hedge; 17·27 and 17·47 are the sides of an occupation road; at 18·10 and 18·40 the line runs through a pond; at 18·68 there is a △, and an offset of 35 to the side hedge; at 23·90 and 26·07 are stations taken in the middle of a large field, where the ground rises in the centre, so that you cannot see from one hedge to the other. It was here especially requisite to retain the direction, and to have a station, in case of the closing side lines running to the base line, near to these points. At 29·30 and at 38·20 hedges cross; at 38·20 being 1·10 from the corner; at 39·72 it crosses another hedge into an old lane; at 40·00 is a station taken, in case of finding it requisite to triangulate along the old line, this was, however, dispensed with; at 40·35 you get out of the lane into a piece of waste; at 41·86 you leave the waste and enter the lane again, which, at 43·00, joins the High Barnet Road, lying on the left of the survey. You are now crossing the road; at 44·00 there is an offset of 40 to the right, and at 44·95, of 60 to a cross hedge on the left; at 45·00 you get out of the road; at 50·00 is an offset of 50 on the left to a bend of the brook, which forms the parish boundary between Hendon and Finchley. At 54·85 a hedge crosses, and being a very high and thick one, stations were taken (54·00 and 55·00) on either side, *as it was not known along which side of the hedge the side lines would come;* at 56·13 is another station, and hedge crossing; at 63·08, 65·73, 68·57, and 76·70, there are hedges crossing, having offsets, at 63·20 and 67·55 to the adjoining corners; at 76·43 is the last

station (B), which, for our present purpose, we will suppose to be the end of our base line.

It must be remembered that in all these stations, there are placed small pieces of paper with the measured distances marked upon each, and also that some white paper is stuck in every hedge, so as to shew at once the direction of the line, and the position on it, of every important point.

As for the selection of the line, it was taken from a point where through a gap in the hedge, the straight trunk of a particular tree could be seen as a conspicuous object in the distance; the gap was on a hill, and so was the tree with a valley between them, and one or the other of these points was visible at almost every station; these served as excellent checks upon the ranging.

It is in the judicious selection of these local objects that great saving of time is effected and greater correctness attained.

Filling in in Railway Surveys.

Take the *right* side of the base line first. It must not be forgotten, that in the selection and order of measuring the side lines in the field, the object is to get over the ground as quickly as possible, and to save the necessity of going over the ground twice; that under favourable circumstances in the triangulation upon the base, the sides of the triangles, upon which the other lines depend, and which should not be more than a mile apart, should alone be strictly independent lines, that is, not subsidiary to obtaining the position of an adjoining hedge. All the other lines should be strictly subordinate, depending upon the shape of the fields, and yet, at the same time, collectively and individually acting as checks upon each other, and upon the whole. There is no objection to even the sides of these triangles, also running along the side of a

hedge, to get the hedge; but still the triangle must not be sacrificed to that, so that, should there happen to be a hedge running near, in such a position, that the triangle will not, by chaining alongside of it, become too oblique, let one of the sides run along the hedge. This is the case in the accompanying example, which, if the reader will carefully attend me through a few practical details, I will proceed to explain.

The first line measured after the base line, started from the commencement of the base: viz. at 23 links, and turned to the right. This line, which was intended to be one of the sides of the triangle A B C, ran along by the side of a hedge of a large field, and crossed the lane at 20 links, and 90 links; the offset was not taken at 90, because the corner of the field had been (and more correctly) determined on the base line, (see 1·56 on base line) where an offset was taken of 3 to the right; at 2·50 and 3·00 chs., offsets are taken to the pond, but none to the ditch, till the end of the field, where by the offset (D + 0) *read from right to left, being a left hand offset;* the reader will perceive, the line measured came close to the ditch. The length of the line was 13·98. The next line ran from 13.98 *towards* the base line. It must be remarked here, that the direction of this new line was not taken to any particular station upon the base line. It would have been too great a waste of time to have done so; the direction was *towards* the base, but the angle of it was altogether arbitrary, dependent upon experience: the object of it being to make the triangle as nearly equilateral as possible. The points, where this line intersected the base, were afterwards determined by measuring onwards to, or backwards from, the nearest known point; thus, this line which was 19·27 long, was found to be 82 links, back from 17·87 on the base line. At 4·95 the hedge crosses; at 5·75 there is an offset of 35 to a cross hedge, which runs straight to 9.78 on the last line. *This saved the measuring of a line along the hedge,* and enabled a better station (d) to be selected, whence to run a line along the hedge in the next field, to the

base line; this station was 7·80. At 10·79 another station was selected, for the purpose of carrying the survey *onwards;* the hedge which this line crossed at 11·40, being *within* the limits.

Having reached the base line, the next line measured was from (9.70) on the base line, station (e), to 7·80 on the preceeding line 19·27, taken for the purpose of getting the irregularities of the hedge. At 4·35, this line also crosses a hedge, which runs straight to a previous point; this also saves a line. The length of this line is 8·11.

Before proceeding with observations on the field practice, it might be useful, at once to explain the object of these lines, as regards the plotting or conveying of their position to paper, which is the sole object of the survey. Between the points 23 and 9·70 on the base line you have the base of a triangle, whose other two sides are 13·98 and 19·27; construct this triangle and lay off the offsets. At present, there is no check upon it. The next line (8.11), which you are obliged to measure for the sake of getting the hedge, becomes a check. The triangle being constructed, the lines become fixed, and the line (*de*) connecting the point 9·70 upon one of the sides, with the point 7·80 upon the other, becomes a fixed line. The length of this can be determined without measuring; its measurement proves its correctness.

These lines being determined, we may resume the field work. The next line (*fh*) runs from 10·79 on 19·27, keeping along the hedges of three fields, being 23·17 long, and tied into the base line (*h l*) by the following line 17·25, which runs to 38·20 on the base line; the line 23·17 crosses the first hedge at 4·10, obliquely, the corner of it being 40 links, perpendicular from the end of 5·00 chs. At 5·07 it crosses the cross hedge, 100 links from the corner on the left. It crosses the next hedge at 14·72, the corner being 10 links on the left, and 25 links on the right. It crosses the third hedge at 23·20; but the station is at 23·17, within 3 links of it. On

the next line 17·25, the first hedge crossed is at 7·40; but the corner of it is 40 links, perpendicular from 7·24. At 11·60, there is a cross hedge |— thus; the rest of the line is regular.

Upon the long line 23·20, stations are taken at 4.85 (g) and at 15·73 (h) for the purpose of measuring the lines gn and hm, which run along side of the hedges. Had the hedge near n been straight to the hedge near g, the line gn would not have been measured; this would have been the case also with hm; had they both been straight, but one or the other would have been measured as a check. Having now reached the base line, the next line measured is from m to h, 13·78 chs.; that is from 45 onwards from 29·52 on the base, to 15·73 on 23·20; this is also a check line. At 5.48 this line crosses a hedge; the same hedge, which the preceeding line 17·25 ($k\,l$) crossed at 7·40. This hedge not being straight, must have a line along it, and a station is taken at 6·55 for that purpose. Before this, viz. at 5·65, there is a cross hedge |— to the right, 25 links off, which runs straight to offset 35 at 18·68 on the base line. The next line 7·52 is for the purpose of getting the hedge; at 6·00 and 6·30 the ditch changes sides.

The following line is from n to g, taken to get the irregular hedge between them: it is 8·69 chains long. This line crosses an occupation road, which must be carefully noted. The various turnings in the hedge must be sketched in the field book.

The next line ($r\,o$) turns to the right, and runs to the corner of the brook, or Parish Boundary between the Parishes of Hendon and Finchley; the line is 9·67 chains long. The line next in order runs from 89 on from 55·00 on base line, to the end of the last line at (o): this line was taken for the purpose of getting the brook with its several bends and windings: the trees by the side of the brook prevented the line being taken closer.

The next line ($o\,s$) is from 9·67 on 9·67, turning to the left, and is 15·28 chains long, having a station on it of 13·40, whence

a line 13·70 is taken to the same point (p) on the base line as before.

The ensuing line is from 11·12 on 15·28, turning to the left, at a very acute angle, running backwards along the side of two hedges and crossing the brook again back to an old line, viz. to 11·48 on 17·25.

This line is a check line. The line 15·28 would have been produced, but it was found to cross the hedge several times. Another direction, therefore, a little to the left was taken, and the line 5·70 was measured, with a station on it at 5·57, whence the following line 12·02 was measured to 22 on from 63·08 on base line.

There was then taken from 2·06 on 12·02 a line 10·00, having upon it a station at 9·83, whence another line 16·46 was taken to 76·43 on the base line. The following line 19·82, running from 6·65 on this last line to 55·00 on the base line, for the purpose of getting the brook, but which served also as a check upon the previous work, completed the survey on the right.

Left side of base line.

FILLING IN CONTINUED.

The first line measured was from 0·70 on the base line, 10·46; to *b*; *a loose line.*

The next line was from 10·43 upon this last, running along the High Road to Barnet, and ending at *e*, being 26·16; this is also *a loose line.*

There are several stations taken upon this line; at 9·57, at 16·04, and at 24·00. The left side of the High Road, together with such hedges and cross roads, as adjoin it, or are connected with it, are taken in, and the POSITION of such houses as are near the road, is shewn by dotted lines; being beyond the limits of deviation, they are merely put in as "lands marks", to distinguish the locality.

From 26·16 the end of the preceding line, is taken the

next line 5·50, also *a loose line*; and from the end of 5·50 another line is taken (5·22 long,) to 25 back from 29·52 on base line. *This is another loose line.*

The next line measured is from the end of the preceding one to 26·16 on 26·16, the line lettered (*te*) in the plate: it is 9·17 chains long. This is afterwards used as a CHECK line.

The object of not having yet taken any tye lines has been to save going over the ground twice or thrice, which should always be avoided.

The next line (*wc*) is from 85 on base line along the lane to 16·04 on 26·16. There are several offsets taken upon this line; but they will be easily understood from the plan. There are stations at 5·76 and 9·04; at 5·76, to obtain the orchard and premises in the rear; at 9·04, to run a line in front of the dwelling house; being the line marked 12·10, running to *d*. This line is 13·88 chains long, *and is also a loose line.*

The line following which is 9·21 chains long, starts from the end of this last (*c w*) line, and runs to a certain point upon the base line: it is lettered *c v* upon the plan. This line *tyed in* the whole of the preceding *loose lines.*

To plot this portion, it will be necessary to reverse the order in which they were chained, thus; upon *vw,* on the base line, construct the triangle *vcw*; *vc* being equal to 9·21 chains, and *cw* to 13·88 chains: then upon *cw,* construct another triangle *cbw*; the sides *cb* and *bw,* being respectively 16·04 and 10·46.

The line 1604 is then produced through *d* to *e,* 26·16 chains long: *e* is then joined to *t, et* becomes a check line, it is 9·17 chains long; and upon *et* is constructed the triangle *eft*; the sides *ef, ft,* being 5·50 and 5·22 chains long respectively. This brings us to where we left off.

The next line measured is from 17·87 on the base line to 24·00 on 26·16. As the point 24·00 is a fixed point, and also the point 17·87, the measured length 7·24 chains becomes a check. This is the case with the next line, which runs from the end of 7·24 (*d*) unto 9·04 on 13·88, and is 12·10 chains long.

This line gives you all the points, where the cross hedges and fences abut upon it, and completes the front of the farm.

The next line 442 is also a check line, running from one, to another known point. And so also is the following line 10·40 which gives the corners of the garden, orchard and rick yard, having a station at 5·31 upon it, whence a line is afterwards taken along side of the rick yard, and through the stable doors to a point upon the line 12·10 *in front* of the farm, viz. from 5·31 on 10·40 to 9·40 on 12·10; this line measures 4·22 chs.; from 3·21 upon this line, an arc is then struck of 1·03, and from 3·64 on a previous line 10·40, at the corner of the rick yard, another arc of 2·67 is drawn, their point of intersection gives the rear of the offices and division fence between the orchard and rick yard; this triangle is on the *left* of the line 4·22.

The two next lines, 1·90 and 1·34, form a triangle on the right of it; this triangle gives the kitchen garden. The arc 1·90 is struck from 3·21 on 4·22, and the arc 1·34, from 3·00 on 9·21.

Theoretically, this triangle, and the preceding one, might have been constructed on the other side of their imaginary bases; but, *practically*, this could not be done, as, had the hedge, which these lines were intended to obtain, been on the other side, the lines must have been altogether differently arranged.

The three following lines lie *between* the high road, and the lane in front of the farm house; they run from a given point in each of the three sides 13·88, 10·46, and 16·04, to a certain point *within* the triangle. The lines are 1·80, 5·42, and 4·68. This closes this portion of the survey.

The next line is a continuation of 5·50, running along the road; it is 16·55 chs. long, and having the direction given, it becomes a double check upon the previous portion. The line measured next runs from 29·52, on the base line, to a station 4·00 chs. upon the preceding line, and is 4·48 chs. long; this line also acts as a check.

The base line having now crossed the high road, it was found requisite to commence the survey upon the other side of the road.

The first field taken was a large field, and the first line along it was measured from 1·40 on the line 16·55.

The next line was 12·44, both loose lines, and from the end of 12·44 it was intended to take a line across the field to 4·00 on 16·55: circumstances, however, prevented this; the owner met our party while occupied in measuring the line from 12·44 on 12·44 to 11·40 on 16·55, and forbade them going on with it. They had, however, measured as far as 2·45 in that direction, viz., *l k*; the line *k r* was then measured 25·18, the stations upon it being 0·93, 6·29, 13·00, and 24·92. The following line ran from 24·92 on 25·18 to 18 back from 63·08 on base line, and measured 9·62 chs. The next lines measured were the lines 8·88 and 6·76, running from this long line 25·18 to the base line. The tye line, 10·19, was then taken.

To plot this part of the work.

From 4·12 back from 16·55 on 16·55, describe an arc of 6·76 radius, and from 45·00 on base line describe another arc of 10·19; their point of intersection will be a fixed point; from this point describe an arc whose radius is equal to 24·92—93 = 23.99; and from 18 back from 63·08 describe an arc 9·62; their point of intersection will limit their directions, produce the line 23·99 backwards 93 links, till it becomes 24·92 chs. long; this will be the point (*k*); from this point produce the next line (*k l*) to *l*, making it 2·45 chs., then, from *l*, describe the arc 12·44; and from 1·40 on 16·55 describe the arc 6·16; their intersecting point will give the other two sides.

The only difference occasioned by the interruption and opposition of the farmer was, that the angle made by the

... from a to l, and the main road was measured ... the line 10·19. instead of on the right, by the ... from l to 1·40 on 16·55. The direction of the ... n the plan.

... are so simple, having, in fact, nothing to ... as to their measurements in the field or their ... at home, that it is not deemed necessary to ... they will be understood by any one who has ...

CHAP. XI.

GREAT WIGSTON SURVEY.

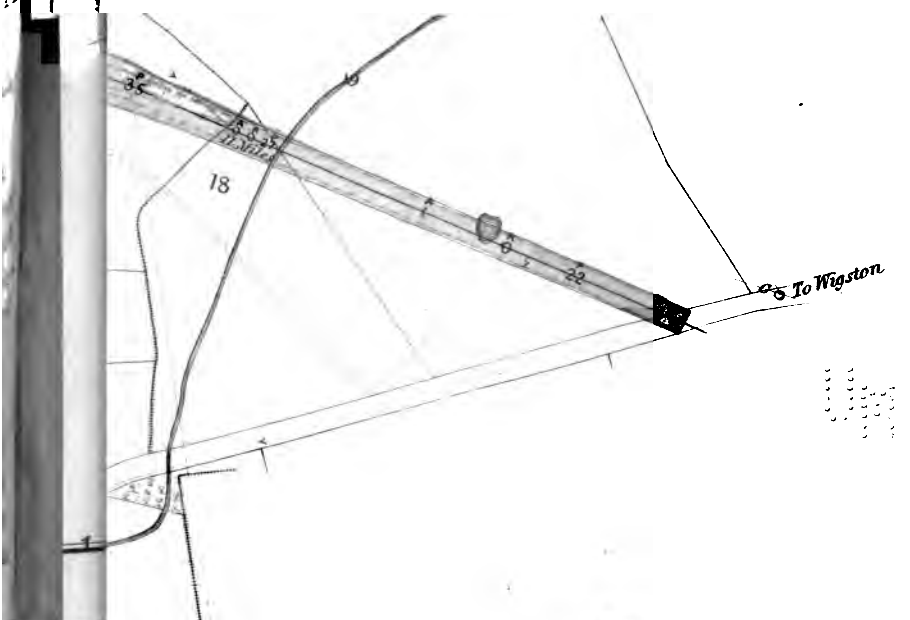
... SURVEY BETWEEN NEWTON HARCOURT, AND GREAT WIGSTON.

This example exemplifies the case of two Base Lines, forming an angle between them.

(The notes are at the end of the book.)

The first base line, AB, is 114·38 chs. long; the second, AC, is 81·07. The first portion of the line, AB, as far as 59 chs. is not shewn in the plan (the latter part only being required to shew the mode of tying the angle in), though the distances are all considered to be measured from A.

There will be no further occasion to speak of the offsets, unless of a peculiar character, as they are pretty much the same in all cases. In considering this first base line, the peculiarity to be noticed is, that observations are made throughout, whether the land through which the line passes is arable, meadow, or pasture land. It is very easy to do this



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as you go on, and it becomes a good check upon the references afterwards, besides being of great use perhaps to yourself in distinguishing between one field and another in plotting.

Observe first, that at 107·00 chains, is the division boundary between the parishes of Newton Harcourt and Great Wigston: 114·38 chains is the end of the line.

At B, the angle ABC is taken; that is, the angle measured between the line BA, round (from left to right) to BC; viz., 227° 7′; shewing therefore that the new line BC turns to the *right*. This angle was taken three times; the *result* only has been left in.

This new Base BC, 81·07 chs. long, has nothing upon it worth noting, except that every "tally" is inserted in the notes, and that at every tally there should be a station put down in the field.

There is a water course at 63·90 and 63·95, and a pond at 70·80; 79·60 and 81·07 are the two sides of the road, from Great Wigston to Newton Harcourt. Be careful in distinguishing in the book between high roads, turnpike roads, and occupation roads, as they materially affect the section; and an incorrect designation becomes a serious allegation (*at least, may be made so, if the committee are so disposed*), of non-compliance with Standing Orders.

Filling in on the right side of the Base BC.

The first line measured was a loose line, through B, viz., from B southwards, and from B northwards in the contrary direction: viz., 16·43 chs. to the south; and 20·30 chs. to to the north. This latter line, crosses out of the parish of Great Wigston, into that of Newton Harcourt, viz. from 18·40 to 19·80 chains.

This loose line is now tyed in, by the next line 15·44, which runs from 13·70 on the last line to 9·70, &c., on the Base Line BC.

The next line runs from 17·00 on the line 20.30 to 25·57 on

the Base Line. These lines form a triangle, viz. 25·57 is the base, 17·00 one side, and 23·84 the other; having as a check the line 13·03 that runs from 13·70 on the side 17·00 to 9·70 on the Base Line. The following line runs from 20·00 on the base line to 14·10 on the line 23·84 and is therefore a fixed line. This line is produced to 14·10.

Having plotted the above, be careful to mark before you proceed, (and do so always throughout the work,) whenever you have any lines fixed, the direction in which those lines were measured. Thus: ——→——

In the next line, from 3·07 on 28·84 draw a line to the last point, 14·10, and produce it to 14·00; this line is also a fixed line. From this point a line 14·20 chs. long is measured to 25·25 on BC. This also is a fixed line.

With respect to the last three lines, however, the best *plan in the plotting, is to reverse it,* and generally, let your smaller lines check the larger, and not your larger the smaller. *Construct the triangles with the longest lines, and let the small lines prove them,* thus: from 3·07 on 23·84, describe an arc of 14·00; and from 25·25, on BC, another arc of 14·20, join their point of intersection; the distance 6·24 subtracted from the whole length, 14·00, which is equal to 7·76, will give the check for this triangle. These triangles are on the right side of BC. Now, if these smaller lines are found to measure correctly, the larger lines must necessarily be correct. As it is, however, exceedingly important that no error should occur at an angle, or change of direction of the base line, a check line is also taken from the other end of the double line, that ran through the point B, on the left of the base line, so as to verify the supplemental angle. This check line tyes it into the base line, at 15·33. It is called a check line, as it is measured *solely for that purpose,* and has no other use, as the other lines have.

The next line runs from 0·32, on BC, through 11·50 on the last line, to 17·70, having a station at 13·57; to this point there

is, then, drawn a line from 16·43 on 16·43; this line being 10·90 chs. long. In plotting it, however, construct the triangle with the two sides, 13·57 and 10·90, first, and let the 11·50 and the 8·70 become the proofs of its correctness.

The line 14·00 (page 3) that runs from 3.07 on 23·84 is then produced, till it intersects the road from Wigston to Newton Harcourt indefinitely; the stations being taken near hedges as usual. A line is then run from 80·00 chs. on BC, turning to the *right*, along the road, *till it intersects* (which it does at 28·60 chs.) the previous line; and the point, where it cuts this line is 32·60. From the two points, therefore, 14·00 on 14·00 and 80·00 on BC, describe two arcs, 32·60 and 28·60; join their point of intersection, and you obtain the direction of them both. All the subsequent lines, that fill in the work, being measured from points upon the Base Line, to points upon one or the other of these two lines, serve as check lines; thus, the next line (from 58·94 on BC) is a check line.

The second line, however, from 28·60 on 32·00, that is, from the point of intersection to the base line at 50·90, is the best check. It seldom happens, practically, that a line can be taken so conveniently as this has been, from the point of intersection, to get in so many hedges; and it does not follow necessarily, that the line must run from the point of intersection; it might run from any point upon either of the lines; but, the nearer to the point of intersection the better. These are hints that might very profitably be borne in mind, by some very excellent workmen; men, who from practice, make very fair surveys, and produce very good plans; but, who seldom call their reasoning faculties into exercise; who, in consequence of this, soon sink behind the better educated, but less experienced, part of the profession, when these, on their part, avoid running into the other extreme, of considering all "rules of thumb," as they term them, beneath their notice.

The other lines, on the right side of the base line, are only small lines, and unimportant; certainly, requiring no explanation for any one, who has carefully plotted thus far.

We will now proceed, therefore, to the left side of the base line.

Left side of the BASE LINE BC.

Having, in going over the previous portion, now arrived at the *end* of the base line BC, the lines are taken backward.

The first line 32·10 is a loose line, tied in by the next line 24·35, and checked by the following line 27·88, which intersects the base line at 16·20.

The two next lines 16·10 and 19·35 form a triangle, running from 20·00 on the base line BC, and from 5·00 on one of the last lines, 24·35. Upon this there is another triangle based, the two lines 17·40 and 7·40 being the sides of it. This is checked by the line 6·85; the preceding triangle has also two checks; viz. 10·22 and 9·52 (on 10·60).

The next line 22·70 serves to connect the triangles already made (in the beginuing) at B, and those, that have been just completed. It runs from 17·60 on 17·70 (page 4) which is a line measured from the point B towards the left of the BASE LINE to 11·30 on the line 16·10 (page 7).

This also is a good check line, in fact it is not possible for any error to creep in, if all these lines agree.

There are two small lines 10·50 and 9·50, taken next in order, to get the othes two sides of the field.

This completes the BASE LINE BC.

BASE LINE AB.

The subsequent notes refer only to the line AB, and its subsidiary lines.

The first line 22·90, defining the position of several hedges, acts as a further check upon the angle ABC. It runs

from the end of 20·30, the second line measured (see page 3) to 94·00 on the base line AB. The line after 17·03, also answers the same purpose, besides marking out the parish boundary between Great Wigston and Newton Harcourt.

The line 12·00, which follows, is another proof of the correctness of the work, at the same time that it gives the corners of an irregular hedge; this line runs from the last line to the angle at B.

The two lines 18·65 (on 19·50) and 13·20 are triangulated upon the first side line measured, 16·43 (page 2).

The next line is a check line; a very long line, taking the direction of several hedges, and crosses several previous lines, running from 5·00 on an ensuing line (10·90 page 4) to 21·10 on a recent one 22·90 (page 9).

This closes the preceding triangles.

A long loose line is now taken keeping the direction of the road, and making for the bridge over the canal, which is a conspicuous object. This line is subsequently tied in. The point where it crosses the BASE LINE is only ascertained in the course of chaining it; the point of crossing being immaterial. The line is 35·00 chains long, and it crosses the BASE LINE AB at 20·20 of its own length, and 79·30 of the base.

The two lines following fix and check it, the first, starting from 4·00, ties it in on one side of the BASE LINE, by limiting its distance 12·80 (on 14·00) from the fixed point 16·80 on 22·90, and the next line 33·45 checks it, by its measured distance being taken from a fixed point 18·65 on 19·50 (page 10) to a given point in this loose line 25·80, which is on the left side of the BASE LINE AB.

The next two lines 11·00 and 18·90 are subordinate lines. The line following, however, 13·90, joining 33·70 on 35·00 with 64·00 on AB is an important one; as, besides forming another tye line to the loose line 35·00, it gives a direct and well positioned triangle to obtain the direction of it in plotting.

To plot this therefore; from 64·00 on AB, describe an arc 13·90 and from 79·30 on AB, another arc equal to the difference between 33·70 and 20·20, or 13·50: complete the triangle and produce the side 13·50, 20·20, chs. to the left of the Base Line. Draw the two preceding lines 12·80 and 33·45, and see if their plotted distanees agree with the measured ones. The few other lines are so simple and so regular in their arrangement, that they need not be referred to. They have all sufficient checks to enable the surveyor to see and to feel, that his work is correct.

If the reader cannot do this, he certainly cannot have understood what is gone before, and in that case, he had better go through the preceding part again.

CHAP. XII.

PARISH SURVEYING.

The instructions to be obtained from the tithe office at Somerset House are so full and distinct, that very little need be added to them. A few general remarks at all events will be sufficient.

If the parish is *compact*, enclose it by one or two triangles, the sides of which should run as near as possible to the parish boundaries.

If it is a *straggling* parish, divide it into the fewest portions possible, and serve each of those portions as a separate parish, triangulating upon a common base line passing through the whole.

This line may have to pass *out* of the parish, but that is of no consequence. In fact all the sides of the triangles might be outside the parish, the base running through it, if by this arrangement they would fall nearer to the boundaries. The smaller and the fewer the triangles that have to be deducted or added to the large triangle the better.

An old plan is very useful in every survey, as though perhaps incorrect in its details and imperfect as a whole, it still gives the surveyor a birds' eye view of the locality and enables him to determine how to arrange his lines. An intelligent chain-man possessed of local information, will also in this case be an useful assistant to the surveyor.

As to the selection of these triangles the base line should be as near as possible in the centre of the parish; and as long as the shape of it will admit; and the other large lines should intersect it at angles of about 60 degrees.

When any portion of the parish is very narrow, two good triangles one at each end, with connecting lines at the vertices would be far better than one oblique triangle. Should there be any village or object of importance within the parish, it would always be better, to run some one of these large lines through that part, even at some (if not too great) sacrifice of the rules above.

It must be distinctly understood, that these large lines of the triangle are strictly independent lines; and not to be made subordinate to the filling in, at any sacrifice, however slight, of the goodness of their relative position.

As to the *running* of these lines, treat them as the base lines in a railway or any other survey. Make towards any conspicuous object, such as a church spire, tower, windmill, &c., and if, in selecting this onward mark, some backward object could be got in range, so as to *line from*, so much the better.

Suppose, for example, the triangle to be ABC; AB, the base line. Range AB in the way above mentioned, taking

care to plant every flag perpendicularly. In this there is not the same difficulty as in railway surveying, of being in an enemy's country, and having to sacrifice every thing to get through. You are not also now fighting against time, but have ample leisure to do the thing correctly.

First of all, examine your chain well, and before beginning lay down a correct chain's length, against some straight permanent fence; and in the course of your work, try your chain frequently. In chaining, begin some chains length before your station A, in case you should afterwards wish to change it.

Cut stations against every ditch, and, as in railway surveying, put paper with the length of the station upon it, into each hole: and where the fields are wide, make a station in the middle of the field. You are not compelled to use it, if you do not want it. Be sure, also to make stations at every 10 chains *throughout*, and enter them in your book.

With respect to the mode of keeping the field book, sufficient has been said in the earlier part of this work; to those who are working for the tithe commissioners, there is, as regards the keeping it in pencil or ink, no choice upon the subject: they have determined upon the latter; with a very correct conception of the class of men, they sometimes have to deal with.

Whether it is any security against roguery is a matter of opinion. The man, who, finding he has made a mistake, would alter his notes to suit (a thing, which, using the pencil, he might easily do), would not, in my opinion, scruple to copy the whole book again, if he were compelled to put it in ink; and send the copy and not the original to the office.

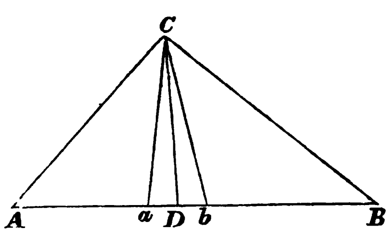
As to the glazed paper and metallic pencils, I cannot say I like them, though in very wet weather, they are useful, in fact indispensable; when writing in ink would be impracticable. Books made of asses' skin, are sometimes used in November surveys, but a good hard pencil, and the common blue ruled

book, are, in my view, the best. A clear head and a quick hand, with a determination to do right on the part of the surveyor; and the many means that science has placed at their disposal, in checking and testing the field work, on the part of the commissioners, are the greatest securities they can have.

For example, let the commissioners insist upon all the angles of the large triangles being taken, as well as a check line. The sides and angles would be mutual checks. Were this done, no surveyor would venture, if he found one of the lines 50 links too long in the plotting, to alter it in his book, for this length might on computation turn out to be correct, taking for data the other sides and the angles returned by him, and *his own plotting*, in some other part to have been wrong.

With the larger lines correct, which this plan would secure, there could be no possibility of error, in any other part, that would not be easy of detection. Most of the lines in the "filling in" would be check lines, which could be calculated if necessary, so that no "fudging" would be safe.

With respect to the usual check upon the triangle, viz., that of measuring a line from the vertex nearly perpendicular to the base, a few words here perhaps, might not be out of place; especially as the goodness of this check has been called in question by persons whose opinion must have some weight.



They say (supposing DC to be the check line), that a mistake might have been made in the measurement from A towards B, and that the length of AD, instead of that given by the chaining should be either Aa, or Ab; and that though D is in consequence in its wrong place, there

would, from its position as to C, be no difference, as to its distance from C, whether taken at *a*, or D, or *b*; and that *therefore, the distance* CD, *as a check line, is good for nothing.* For myself, I differ from them in toto. I admit the premises, but deny the inferences: the very fact, that, (whether the point upon the base, to which a line is drawn from the vertex, is at *a*, of D, or *b*,) that line is of the same length, is its very value; it is that very thing that makes it one of the best of checks. The *object* of the check has evidently been totally lost sight of; it has not been to determine the position of any point *D*, upon the base, whether *it* is in its right place or not, and this by the assistance of another point C, which, by means of two lines AC and BC, is fixed at a determined distance from AB; for which purpose, this new point C, as maintained, would be perfectly useless; *but*, to determine the position of C, by means of the new point *D*, for which, the fact of its not materially affecting the length of CD (by which length the position of CD is determined) whether it be measured from *a*, or D, or *b*, renders it particularly fitting. The point to be ascertained is whether AC, or CB, have been correctly measured. Now, one, or both may be wrong; suppose they have been entered a chain too short on plotting them, the check CD on the plan will be found to be shorter than what it is entered in the field notes. On the other hand, let them be entered more than they really measure, the plotted check will be found more than the real one; and this the more certainly, because, though some slight error may have crept in in measuring the length from A towards B, the distance of the check, DC, will not be materially affected by it,

To resume the field work: having got to the end of the base line, AB, the next thing to be considered is, whether the nature of the country is such, as to make it neccssary for the line BC to run directly to C, or whether any point near C can be taken; in the latter case, a line may be at once run from B,

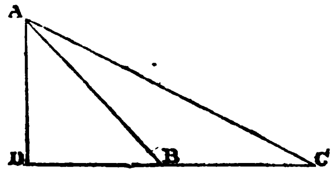
in a direction, as near as possible, towards C; in the former a trial line had better be ranged out *towards* C, so as to obtain the general direction of the line between B and C, and then a line be run *from* C, back again towards B; the intersection of this line with AB, or with AB produced, can be easily determined. [This will be a great saving of time.] If the angle ABC be taken, and the sides AB, BC, be measured, the angles at C and A can be calculated, and the direction of the line at either place, can be laid off by the theodolite.

As for the line CA, whether it is to be started from A towards C, or *vice versa,* must depend upon which of the two can be more easily changed. In addition to these lines, which should be very carefully measured, the three interior angles, if possible, should be always taken, or, at least, as many as can be, and the sides computed.

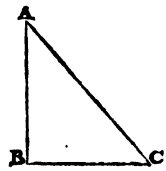
The length of the check line, besides its measurement, in the field, might be computed also. This would serve as a check upon the check line itself; for, if, taking any two sides and an angle, or any two angles and a side, the others were found correct, then these assumed as data, would infallibly determine the correctness of this check. There is no greater reason why the check line should be correct, than any one of the sides; it is the agreement of all that presumes the correctness of each.

The simplest test, by computation of the correct length of this check line, is to be found in Euclid's first and second books, prop. 47, 12, and 13, where it will be seen, that, in every triangle, the square of the sides, subtending any angle, is greater than, equal to, or less than, the square of the sides containing the angle, according as the angle is obtuse, or right-angled, or acute; and the excess or deficiency is equal to twice the rectangle of one of the sides containing the angle, and that portion of it intercepted between the angle and a perpendicular, drawn to it from the opposite angle; *i. e.*—

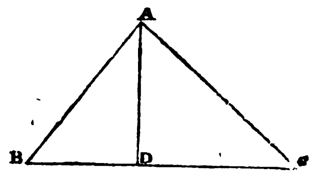
In the *obtuse*-angled triangle ABC.
$AC^2 = AB^2 + BC^2 + 2\ CB \cdot BD.$

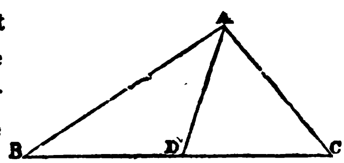


In the right-angled triangle;
$AC^2 = AB^2 + BC^2.$



In the *acute*-angled triangle;
$AC^2 = AB^2 + BC^2 - 2CB \cdot BD.$





Also, in any triangle, the sum of the squares of any two adjacent sides, is equal to twice the square of half the other side, together with twice the square of the line drawn from the vertex to the point of bisection, i. e., $AB^2 + AC^2 = 2(BD^2 + DA^2)$.

The length of any line (aC), therefore, (the angle A B C being known) drawn from the vertex of a triangle to any point upon the base, equals the root of the sum of the squares of either side, and the segment adjacent, *minus* twice the rectangle of that segment a B and the distance (B D) measured on that segment, from the angle at the base to a perpendicular from the vertex, i. e. $= a\,B^2 + CB^2 - 2\,a\,B.\ BD$, where $BD = CB \cos.\ ABC$. (*See Diagram at page* 57.)

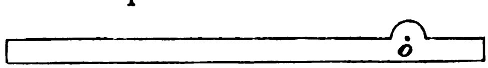
After the large triangles have been properly tested, then let the surveyor, before proceeding to the filling in, throw out such small triangles, as may be necessary to enclose the boundaries outside the main lines, and be sure never to omit the checks to each.

When these are done, then the filling in can be commenced; here, as almost every line will be a check line of itself, few independent lines will be required, as those, that are run to take up the hedges, will generally answer that purpose too.

Little more need, therefore, be said upon the subject, except, that the young surveyor should be careful never to get into arrears with the plotting. I know no other bad habit of the profession, so bad, or so troublesome as this. It is, without exception, the worst habit, a young man can fall into.

Let the sides of the large triangles be laid down and proved always before you base any thing upon them. I admit, there is some difficulty in doing this correctly; but that is only a stronger reason why it should be done at once, and not be delayed, lest, by so doing, it should be hurried at the last. It would be better to keep away from the field a day or two, if necessary.

The best plan, to lay these lines down, is, to draw the base line in carefully *first*; then, with a very long straight edge, having one part of it made projecting for that purpose, describe the required arcs.

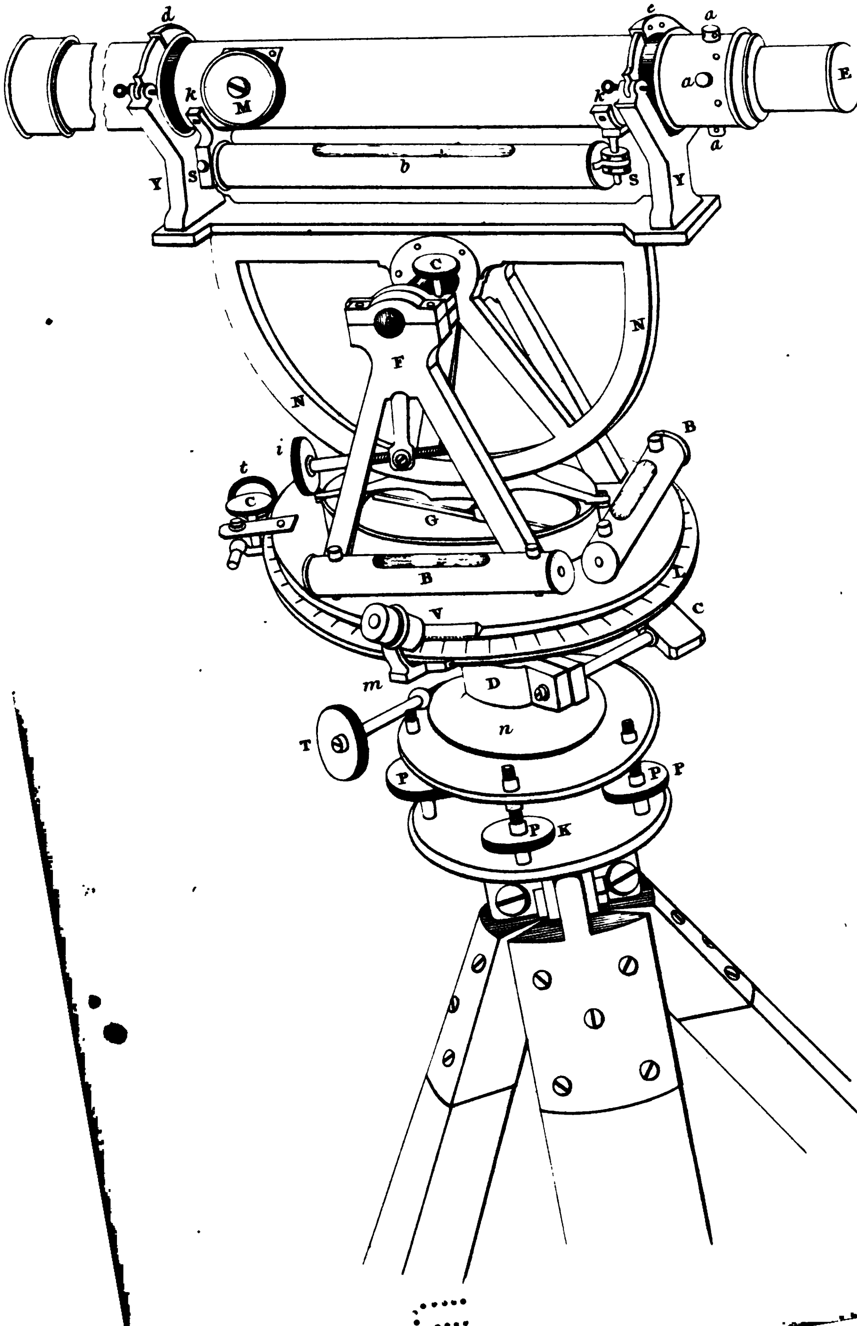


The annexed is the shape of the straight edge; the centre is at *o*, which is on the line of the face. A needle, or pin, can be driven through it into the paper, so that it can be turned round that point as a centre. The proper distances can be laid off upon the bevelled edge, and the required arcs can be struck. Sometimes common Beam Compasses are used, but I do not consider them so accurate as this; lying flat to the paper, as it does, and moving round so small a centre, with the distances capable of being exactly marked, and perfectly rigid (for it might be made, if necessary, of steel) it must necessarily be correct. The straight edge is also useful to draw the lines, when their directions are ascertained.

Sometimes pins are stuck in to each end of the line, and a piece of silk, or thread, fixed between them; this, when the

paper is quite flat, is taken up carefully between the fingers, and being previously blackened with burnt cork, snapped upon the paper. To ensure correctness in this way, the paper must be quite flat, and the thread *taken up perpendicularly*, without which the line is valueless.

Simm's 5 Inch Theodolite.



LAND SURVEYING.

Part the Second.

CHAP. I.

THE THEODOLITE.

Method of taking a horizontal angle.

Adjust, first, the parallel screws (P, P, see diagram), so as to have, as nearly equal lengths of the worm, as possible, above the upper plate.

Extend the three legs, approaching or extending each, until the bubbles in the two levels (B, B) are nearly central, and the plummet, suspended from a hook under the body of the instrument, hangs freely above the centre of the station. The better plan is to move only one leg, which is, of itself, capable of a double motion.

Press the legs firmly in the ground, unclamp the *whole* instrument by means of the large clamp-screw (C), observing to keep the other motions clamped.

It must be remembered, that the two levels, on the

horizontal plate, are conjugate, *i. e.*, at right angles; and, that the opposite screws, also, are conjugate, each pair of them.

Set one level in a plane, vertically parallel to a plane, passing through any one of the screws; and the other level and the other pair of screws, will be also in parallel vertical planes, conjugate to the former.

If both the bubbles of the levels, thus placed, are not in the centre, take the bubble that is not level, and, loosening one of the corresponding conjugate screws, tighten the other, until the bubble is accurately adjusted. Then loosen and tighten the other pair in the same way, till the same result be obtained. This will probably throw the first out; repeat the process to each, until both bubbles are level.

Having these plates (K, *k*) level, clamp the whole instrument, and, unclamping the parallel circles, set the broad arrow of the vernier, which is in the upper plate, to 360°, or zero of the larger circle, and clamp it. This must be done by the magnifying or reading glass (*m*), attached to the horizontal circle. It is a great pity that this magnifying glass is not attached to the upper, instead of the lower circle, there should be one fixed to each of the horizontal verniers. The want of this is the only defect in Simms' otherwise perfect little instrument, the 5-inch theodolite, as shewn in the plate.

This large circle is divided into 360 degrees, and then again, subdivided into half degrees, which are numbered from left to right, and, by means of the vernier, read off to minutes.

Again, unclamp the large clamp-screw, and turn the whole instrument towards the left of the stations, between which you are desirous of taking the angle, until you can cut the object as accurately with the intersection of the cross wires o the telescope, as can be done by the hand. Clamp the screw (C), and slowly turning the milled-head tangent screw (T), you can obtain all possible degree of accuracy.

Now, as the zero points of both the upper and lower circles

are together in the present position of the telescope, and as the lower circle is graduated from left to right, by separating the upper circle, and turning it round, till the centre of the cross wires of the telescope, which is attached to it, cuts exactly the centre of the object at the second station, you obtain the angle between the two, determined by the position of the vernier, and the length of the arc of the circle it has described. This can at once be read off from the plate by the broad arrow of the vernier, which will stand exactly above the number of degrees and minutes of the angle, measured between the two given objects.

When the cross wires, therefore, nearly cover the object, clamp the plates, and use the tangent-screw (t); and, with the magnifying glass (m), read off the angle, the degrees, on the limb; and the minutes, by means of the vernier. (For a description of the vernier, see page 71.)

The angle, thus read off, should always, if great accuracy be required, be read off by each of the two verniers. The common 5-inch theodolite is furnished with two, and the larger instruments have three, equidistant from each other, so that the *mean* of the readings, taken at different points on the circumference, should correct the errors of eccentricity or graduation. In extensive surveys, when the instruments with three verniers are used, other securities are adopted against these errors.

Instead of fixing the broad arrow of the vernier at the zero point of the horizontal limb, at starting, the telescope is directed to the first station, with the broad arrow indifferently placed upon the lower plate, and its position carefully read off by the several verniers, and the *mean* taken. The difference between this mean, and that of the reading of the second station, is the measure of the required angle.

In incorrect graduation, this is perhaps the best check that can be used.

As an additional check, these angles are often repeated;

that is—the angle is not taken again, by separating the upper plate and bringing the vernier back to zero, and then taking it a second time—but, without detaching the two plates after the last observation, turning the whole instrument bodily round to the first station, and, then unclamping the vernier plate, and turning it round to the second station.

The difference between this and the first reading, before starting, will be double the mean angle. Keep the two plates still together, and turn the whole round, repeating the process as before.

Where the instrument shall have made one or two complete revolutions, 360 degrees or 720 degrees must be added to the observed angle, in order to obtain the true.

The difference between this third reading, and the reading at starting, will be three times the angle required.

It is requisite that the verniers should be separately marked, as A, B, C.

To take a vertical angle, or an angle of elevation or depression.

First set the whole instrument level, as was explained before, by means of the bubbles on the vernier plate. Then bring the bubble of the telescope level (*b*) to the centre of the tube, observing whether, at the same time, the zero point of the circular arc coincides with the zero of the vernier. This must be carefully examined by the microscope.

The instrument being thus perfectly level, when the zero point of the circle and the broad arrow are together, raise or depress the telescope, till you distinctly cut the required object with the horizontal wire, or the common intersection of the three wires. The changed relative position of the broad arrow, will give the required angle, which will be an angle of depression, if the broad arrow be found between the zero of the vertical arc and the object-glass of the telescope, and of elevation, if beyond them.

It will be requisite, for particular accuracy, to invert the telescope in its Y*s*, and read off the same angle from the other end; half the difference of these two will be the angle of error, of the vernier.

Adjustments.

THE TELESCOPE.

The accuracy of this instrument, in its application to the purpose of taking angles, depends altogether upon the correctness of the line of collimation. The optical axis of the telescope, which is an imaginary line, joining the centre of the object-glass and eye-glass, should pass through the point of intersection of the cross wires or webs.

These webs are attached, as in the diagram annexed, to a broad flange of an inner tube, within the tube of the telescope, near the eye-piece, with which it is connected by two pairs of conjugate capstan-headed screws (*a*, *a*, *a*), see plate of the theodolite, so as to admit of a double relative motion: thus, *a*, *a*; *b*, *b*; are the conjugate screws, which, passing *through* the telescope, *screw into* an inner tube: this inner tube has a broad flange, with small slits in it, into which are gummed, or glued the cross webs.

1. *To ascertain, whether the line of collimation is in adjustment.*

Place the telescope within the Y*s*, and, having found some point clearly defined, which is cut by the intersections of the cross wires, turn the telescope round on its axis, and observe, whether, during its whole revolution, the centre of the wires remains the same, always covering the same point. If it does, it is in adjustment; if not, turn the telescope round on its axis, and correct for half the error, by means of the small capstan-headed screws (*a a a*), above referred to, loosening one and tightening the other.

2. *Whether the axis of the level is parallel to the axis of the telescope.*

Place the telescope on the Y*s*, and unclamping the large clamp-screw, set the telescope over one pair of conjugate screws, and loosen and tighten them till the level, attached to the telescope, is made perfectly level by the vertical tangent-screw.

Then reverse the telescope in the Y*s*, if the level remains the same, this also is in adjustment; if not, correct for one half the error by the capstan-headed adjusting-screws, at the end of the level (S); and the other half by the vertical tangent-screw.

There is also a side adjustment required.

The level may not always be immediately under the telescope, but a little to the right or to the left; this must not affect the position of the bubbles, or a lateral adjustment, similar to the vertical one, is indispensable, by means of the capstan-headed screw, at the other end of the level (S).

HORIZONTAL.

1. *To make the axis of the bubble on the vernier plate, parallel to that plate.*

Let one bubble be over one pair of the circular plate screw, then the other bubble will be over the conjugate pair; make both bubbles level, turn them half round the circumference, and if the bubbles deviate from the centre, correct one half the error, by the small milled-headed screws above the levels: and the other half error, by the circular plate screws; repeat this, till the bubbles are level, in every position, throughout a whole revolution of the circumference.

HORIZONTAL.

2. *Whether after having duly corrected for the third adjustment, or made the "axis of the bubbles, on the vernier plate, parallel to that plate," the bubbles will remain perfectly level, during a whole revolution of the instrument upon the common axis.*

Clamp the two circular plates, and unclamp the large clamp-screw; set the bubbles perfectly level, as before; when, immediately over each pair of conjugate-screws; reverse them, if they continue level, they are in adjustment; if not, the two circular plates are moving upon different axes, and are not parallel to each other. This imperfection can only be well remedied by an instrument maker.

VERTICAL.

1. *Whether the vertical arc moves in a truly vertical plane.*

Set the vernier plate, or upper horizontal circle, perfectly level.

Direct the telescope to some well-defined angle of a building; or, should there be no building convenient, suspend a string, with a plummet attached, from the top of a high pole, this perhaps is the better plan, and taking care that the intersection of the wires exactly cuts the string, near the plummet, raise the vertical arc, observing whether the cross wires, throughout the whole of the vertical motion of the telescope, cover the vertical string; if they do, this also is in adjustment.

As it is seldom found that two objects, whose horizontal angle is required, are exactly in the same horizontal plane, this adjustment becomes a very important one, aud requires great care. Considerable error has resulted from neglect of it.

VERTICAL.

2. *Whether the vertical vernier is in adjustment, or perfectly central.*

Direct the telescope to some point of elevation, and note the angle. Reverse the telescope in its Y*s*, and raising the telescope to the same object, read off the same angle; if these angles are the same, the vernier is in adjustment; if not, correct the vernier for the error, by means of the small screws, fastening the vernier to the upper plate, which can be loosened, and half the difference of these two angles will be the angle of error; or, which is better, add this angle of error to every angle of elevation, when you use the end that reads off the *smaller* angle, and subtract the same, from that of depression, under the same circumstances.

When you read with the *larger* angle, subtract this angle of error from the angle of elevation, and add it for the angle of depression.

PARALLAX.

Is an error occasioned by the cross wires of the telescope not being at once, in the common focus of the object-glass, and eye-glass.

The existence of parallax is ascertained by moving the eye about, when looking through the telescope, and observing whether the cross wires are clear and distinct.

If they are not, first adjust the eye-glass, by means of the moveable eye-glass tube, till you can perceive the cross wire clearly defined, and sharply marked against any white object.

Then, by moving the milled-head screw (M), at the object-end of the telescope, until you obtain the proper focus, according to the distance of the object, you are enabled at once to see the object distinctly, as well as the intersection of the wires, clearly and sharply defined before it.

The existence of parallax is very inconvenient, and where disregarded, has frequently been productive of serious error. It will not always be found sufficient to set the eye-glass first, and the object-glass afterwards. The setting of the object-glass, by introducing more distinct rays of light, will sometimes affect the focus of the eye-glass, and produce parallax or distinctness of the wires, when there was none before. The eye-piece must, in this case, be adjusted again.

Generally, when once set for the day, there is no occasion for altering the *eye-glass*, but the *object-glass* will, of course, have to be altered at every change of distance of the object.

THE VERNIER.

The vernier is a contrivance for subdividing, to any extent, the smallest division in a graduated scale, varying according to the scale to be subdivided, and the extent of subdivision, but having the same principle in all cases.

The following explanation of that of the 5-inch theodolite, of Simms, which is given in the plate, and which is a very convenient and accurate little instrument, will serve for all.

In this, the lower circle is divided into half degrees or 30 minutes; and the vernier so arranged as to read off to one minute.

Twenty-nine divisions of the graduated scale, which is twenty-nine half degrees of the lower circle, are taken; and are divided, upon the vernier, into thirty divisions: now, as thirty divisions are compressed into the space of twenty-nine, each of these thirty divisions is one-thirtieth less than those of the twenty-nine: or, as the whole arc, in the graduated scale, is equal to 29 half degrees, or 870 minutes, and these are subdivided into 30 divisions, in the vernier scale, each of these subdivisions is equal to 29 minutes—therefore, if the zero point of the limb correspond to the broad arrow of the vernier, the first division line of the vernier is one minute to the right

of the corresponding division on the limb; the second division on the vernier, two minutes to the right; the third, three minutes; and, so on, till the thirtieth division of the vernier, exactly coinciding with the twenty-ninth division, is just 30 minutes to the right of the thirtieth, or corresponding division in the limb.

If the vernier, therefore, be moved, till its first division corresponds to the first division of the limb, the broad arrow of the vernier will be removed one minute from the zero of the limb. If the second division of the vernier be made to correspond with the second division of the limb, the broad arrow of the vernier will be two minutes removed from the zero of the limb. If the third corresponding divisions coincide, the zeros are three minutes removed, and so on.

Hence, *to set the instrument* at any angle, of any number of minutes, between 0 and 30, and 30 and 60, move the vernier, until the broad arrow becomes in such a position, *within the required half degree*, as, that the number of the line, on the vernier, coinciding with the same numbered line on the limb, shall correspond to the number of minutes required. And, to *ascertain the number of degrees and minutes*, that there are in a given angle, observe where the broad arrow of the vernier is; if, between a full degree and a half degree, the angle will be so many degrees and as many minutes, as are denoted by the number of the first division line of the vernier, (reading onwards as the degrees number), that coincides with the corresponding division in the limb; or, if between a half degree and a whole one, so many degrees and 30 minutes, *plus* the broken number of minutes, as denoted by the coincidence of the corresponding lines of the vernier and limb.

The principle of the above subdivision of the vernier of Simms's theodolite, is very simple, and is universally applicable to any subdivision; viz., divide the value of the division in the graduated scale, by the number of divisions in the

vernier, and the quotient will be the value of each of these subdivisions.

Thus, in the Theodolite, the graduated division is 30 minutes; the number of vernier divisions 30; the instrument (29 being subdivided into 30) reads to one minute.

In the Sextant, the graduated division is 10 minutes, or 600 seconds; the vernier subdivisions are 60; the reading of the instrument therefore is to 10 seconds, (59 of these 10 minutes being subdivided into 60.)

In the Circumferentor, in this country, the common graduation of the plate is to one degree, or 60 minutes; the vernier subdivisions are 20, making the reading of the instrument only 3 minutes (19 of these divisions being subdivided into 20); in this case, nine and a half on each side of the zero are divided into 10.

Hence, the value of the graduated division being given, and the extent of subdivision of reading required, we are enabled easily to ascertain the number of requisite subdivisions.

For, by the rule above, if $x+1$ be the required number of the subdivisions in the vernier, and x, that in the graduated arc; V, the value of the graduated division; and v, that of the required subdivision, we have,

$$\frac{V}{x+1} = v, \text{ or } x = \frac{V-v}{v}.$$

As for example:

in the Sextant, where $V=10$ minutes or 600 seconds and $v=10$ seconds

$$\text{then } x = \frac{600-10}{10} = \frac{599}{10} = 59 \text{ divisions.}$$

Again, in the Circumferentor, where $V = 1$ degree or 60 minutes; and $v = 3$ minutes,

$x = \dfrac{60-3}{3} = \dfrac{57}{3} = 19$ divisions of the plate, to be subdivided, in the vernier, into $x+1$, or 20.

CHAP. II.

[*A few general remarks upon the Theodolite—its relative merits, as compared with other instruments—and the respective advantages of its own different modes of application.*]

1. *As compared with other instruments.*

It is an instrument calculated for extreme accuracy. It is an instrument that should always be used, when, as in the Ordinance Survey, quality and not quantity is the desideratum; when the correctness of the result, and not the rapidity of execution, is the object.

I would here beg to caution my *young* readers against falling in this profession, as is often the case in many others, into a mere system of exclusiveness, viz. of advocating this or that particular system of surveying, whether by the Chain, Theodolite, Sextant, or Circumferentor; they are all useful in their way, and each, under certain circumstances, has the advantage; and there is scarcely any survey, of any extent, but what all of the three may advantageously be brought into use.

In broad and extensive flats, though the triangulation were better carried on by the chain, as the chain is indispensable for determining the cross hedges, yet the *long lines of the triangulation must be run in by the theodolite*; the common method of ranging by the poles would not be sufficiently accurate.

In broken and hilly countries, where the chaining could only be obtained by an application of the angles, taken by the theodolite, to the determining of the comparative lengths of

the hypothenusal to the horizontal lines, this instrument is indispensable.

The correct length of one side of a triangle, together with the measures to minutes, half-minutes, obtained with the accuracy of which a good theodolite is susceptible, of its two adjacent angles, will always more certainly determine the position of a third point, when hills intervene, than the incorrectly measured distances of the two other lines. And, in both the above cases, the circumferentor, (the third part will be devoted to the uses and application of this instrument), or the sextant, under certain limitations, can be advantageously introduced for the filling in.

2. *As to its own modes of application.*

There are two methods of using this instrument, generally adopted; the first by the *needle*: the second by the *back angle.*

The first method (*by the needle*):—The broad arrow of the vernier, and the zero point of the horizontal limb, are, by means of the adjusting screw, made carefully to coincide; *the magnifying glass* must be always used for this purpose; the needle is then released, and allowed freely to play upon its agate; and the whole instrument, with the two circles, firmly clasped together, turned round until the north end of the needle coincides, *as nearly as the eye can tell*, to the north point or zero *in the graduated circle in the compass box.* The whole is then clamped, and if in clamping, any error has arisen, it is carefully corrected by the large adjusting-screw (T.) Now, if the two plates be detached, and the vernier plate turned round to the object, the angle read by the vernier, will be the angle made at the station, between the first object and the north end of the needle.

If the vernier plate be again unclamped, and turned round to the second object, the vernier will, in this case, also denote the angle made between this second station and the same

north point. The difference between the first and second reading will be the measure of the angle required.

This is the method of taking an angle *by the needle.*

It is, however, a method that I consider *very objectionable.* It is, after all, only an imperfect circumferentor. The graduation of the inner plate of the compass is too contracted, the diameter too small, and the needle too clumsy, to admit of any accuracy in the setting of it in the first instance.

The second method, *by the back angle.*—In this case, the instrument is placed at the second station, the bearing between the first and second being assumed as a base line, determined in position, and the angles are all based upon that line; thus, supposing the starting point to be A, the first station B, the base line AB, and the several stations C, D, E, F, &c., then the angle at B is that between A and C; the angle at C, that between B and D, and so on.

The disadvantage of this method is, that the error of each angle is not confined to itself, but is increased by the sum of the errors of all the preceding; while in the first method, by the needle, each angle is confined to itself, being the angle made between each line severally, and certain meridian lines, that are considered parallel.

On the other hand, as these meridian lines depend upon so small a needle, and so confined a graduation as that of the theodolite, they can scarcely be considered practically parallel.

Looking, however, at both sides of the question, I am myself disposed to give the preference to the Back Angle method, as I am convinced, that, as far as the mere reading from the instrument goes, there is as great risk of error in two consecutive readings of the needle, as could possibly occur in the reading by the back angle. And with respect to the plotting, there are double the errors in the first method, to what there are in the second; for, in the first, besides the error that may occur in plotting the observed angle, which is common to the two methods, there is the error of the non-parallelism of the meridian lines, which is peculiar to the first.

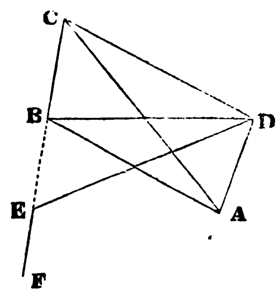
CHAP. III.

USEFUL PROBLEMS IN SURVEYING WITH THE THEODOLITE.

[*The simpler Problems in frequent use will be found in my Elementary Text Book.*]

PROBLEM I.

It is often desirable to be able to produce a given line BC, which is inaccessible and invisible at the required point of production E.



Take any stations, A and D, whence B and C are visible, and at A and D take the several angles required (see Chap. V *on the determining of the length of a new base line, by angles taken from an old line,*) and determine BC, CD, and the angle BCD: through D, draw DE, at any angle with DC.

Now, because the line DC and the angle ECD have been computed, and CDE has been assumed, the triangle DEC comes under the first case of triangles, and the angle CED becomes determinable, as well as the required length of DE; measure this computed length to E; and at E lay off EF, making the angle DEF equal to the supplement of the computed angle CED.

Example.—Being desirous of continuing an inaccessible line on the opposite side of a river, I measured a base AD, of 12 chains 50 links, and took the several angles made at each end of it, between this new base and two points that were visible on the given line, viz., at A, 50°, 37°; and at B, 85°,

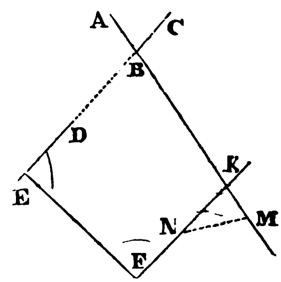
and 126°. I also measured, from the end of the new base line, nearest the river, another line, making an angle of 52°, with the new base. What must be the length of this line, and what the angle between this line and the production of the given line?

Answer: DE=13·17 chs. and the angle DEF=103° 48′ 20″.

PROBLEM II.

To measure the angle made between two inaccessible lines, as the angle of a fort, or the salient angle of a bastion.

Let B be the salient angle of the bastion ABC, whose faces are AB and BC. It is required to measure the angle ABC.



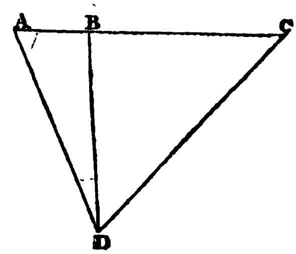
Take any point D, out of musket shot, and produce DE in a line with the face CB. At E, erect EF perpendicular to EC, about equal to ED, and also the perpendicular FK, such that AB, the other face, and K, shall be in the same straight line; produce the direction BK, to any point M. The angle FKM shall be equal to the angle ABC. Because DEF and KFE are two right angles, EB and FK are parallel, and, therefore, the outward angle FKM is equal to the inward opposite angle EBM, which is equal to the opposite angle ABC. Measure, therefore, the angle FKM, by the theodolite or the chain, and you obtain the measurement of the required angle.

PROBLEM III.

It sometimes happens in determining the position of two objects (visible, but inaccessible from each other), that both together are also inaccessible from any new station

The only plan that can, in such cases, be adopted, is to select that station, at which the angle between them can be

taken by the theodolite, and whence the most favourable lines, to clear the obstructions, can be drawn on either side, as *new bases,* to determine the lengths of the immeasurable sides, that include the angle taken.



First.— Let B and C be the given objects, whose distance BC is required. Let D be the station, whence B and C can be seen, and the angle BDC measured; but BD and DC cannot be measured.

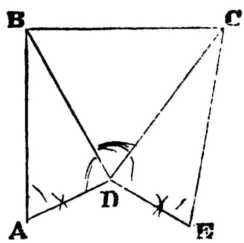
Now, if the stations B and C are of such a kind, and so situated, that, though AB cannot be measured, *the line of direction of CB can be produced to any point A, whence D can be seen;* by taking the angles at A and D, and measuring AD, you obtain the triangle ABD, and, therefore, BD and the angle ABD, which being the outward angle to the triangle BCD, is equal to the angles BDC and BCD. BDC being known, BCD, therefore, becomes known, and BD is known; hence, the distances required become determinable, under the first of the three cases of trigonometry.

Example:—Wanting to know the distance between two forts, on each side of the entrance to a harbour, I measured a base line, 35 chs. 20 links, along the beach, from a point on the beach in the produced range of the flags of the fort, to another point, which was nearly opposite to the centre of the entrance, and found, at this second point, the angles made between this line and the two flags of the fort, to be 8° 29′ 40″, and 45° 11′ 20″. The angle made at the first point between the range of the flags and the second point, viz., the angle DAB being 30° 17′ 38″, what was the length of BC?

Answer: 8 miles 3½ furlongs.

PROBLEM IV.

When CB cannot be produced.



It becomes necessary to consider DB and BC, as two new unknown distances required.

Select any points A and E, visible from D, and such, that B is visible from A, and C visible from E.

At D, take the angles ADB, BDC, and CDE; measure DA and DE; and, at A, take the angle BAD, and at E, the angle CED.

The triangles ABD and CDE, have one side and two angles given, and come under the first of the three cases.

Hence, BD and DC are determinable, and the triangle BDC, having now the two sides BD and DC, and the included angle known, comes under the second case.

Example.—On another occasion, wishing to obtain the distance between two forts, similarly situated, at the mouth of a river, I found that, in consequence of the high ground in the rear, the line of direction could not be produced.

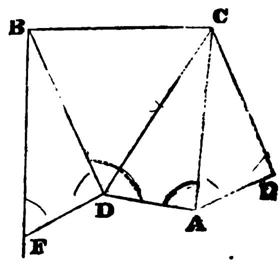
I was, therefore, under the necessity of adopting another plan. Placed my theodolite at the head of the harbour, and took the several angles, made between the flags of the fort and two new stations, whence these flags were also visible, viz., the angle ADB, 58°; BDC, 42°; and CDE, 72°; measured to these stations, DA, 32 chs.; DE, 35 chs.; and at A, and E, found the angles to be 83° and 86°.

Answer, 1433 yds.

PROBLEM V.

When no point can be found, whence B and C are both visible.

In this case some other arrangement is necessary.



Let B and C be unknown stations, as before, so placed that only one of them, B, is visible at D. Select any station A, whence D and C are visible; take the angle BDA; measure DA; and take the angle DAC. These data, if AC were known, would give us the length of DC, and the included angle BDC, as BDC would be the difference between the measured angle BDA, and the computed angle CDA; then, by obtaining the length of BD, the triangle BDC would have two sides, and the included angle known as before.

Find, therefore, the lengths of BD *and* AC, in the manner explained in the last example, considering them as two new unknown lines, by measuring DF and AE, and taking the angles BDF, BFD, and the angles CEA and CAE.

These data give you DB and CA.

In the triangle CAD, you have also DA, and the included angle DAC, therefore DC is determinable, and the angle CDA; but the whole angle BDA was taken, therefore the angle BDC, the included angle of the first triangle, becomes known by computation, which, by the position of the objects, could not be taken by the theodolite.

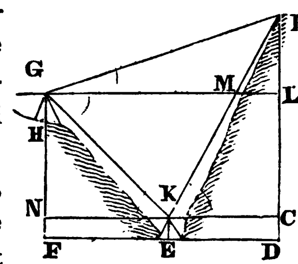
EXAMPLE.—On a third occasion, with a similar object, could not, from a number of buildings, find any point at the head of the bay, whence both flags were visible. Took a station D, at the head of the bay, and found the angles, between the flag B and two new stations F and A (whence the flags were visible); the angle FDB, 90°; the angle BDA, 125° 40′; measured as

follows viz. FD, 10 chains, and DA, 12 chains; again at F, found the angle DFB to be 54°; and at A, the angle DAC, to the other flag, 79°; and the angle CAE, to another station E, 50°; also AE, 8 chains 20 links; and then found the angle AEC to be 93° 30′. What is the distance BC?

Ans. 17 chs. 62 links.

PROBLEM VI.

To ascertain the height of a hill above the level of the country, where the ground is so broken that a horizontal line cannot be measured, and the only angles, that can be taken, are from a point in the rising ground of another hill, which does not admit of measuring the horizontal distance from the object, and from the bottom of the first hill.



Let EBD be the hill, whose vertical height is DB. Let E and H be the two stations, for taking the vertical angles, in the same vertical plane with the height BD.

At E, take the vertical angle BKC, and from E, measure the distance HE along the slope of the hill: at H, take the angle of elevation BGL, and the angle of depression LGK, setting a flag KE, at the point E, equal to the height GH. The heights required are DB, and FG.

Now, in the triangle BKC, because the angle BKC is known, if KC or KB were known also, the triangle would be determinable.

Because GL is parallel to NC, the angle MKC is equal to the angle GMK, which, being the outward angle, is equal to the angles BGM and GBM; that is, the vertical angle GBM is equal to the angle of elevation at K, minus the angle of elevation at G. The angle GBM thus being known the triangle GBK has the side GK, and two angles known, and is therefore determinable. In the triangle KCB you have the hypothenuse KB and the angle BKC known, to determine the height CB; to this must be added CD, the height of the theodolite, to obtain the height of BD.

The difference between the heights of the two stations, B and H, is found by subtracting NH from CB, their respective heights above the common *datum* line NC.

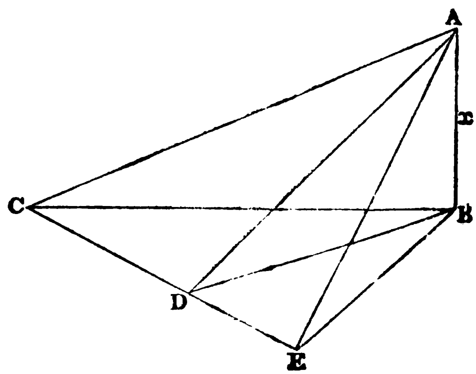
EXAMPLE.—Given at G, the angle of elevation BGL, 12° 15″ and the angle of depression LGK, 42°29′; the hypothenusal distance GK, 8·25 chains; and, at K, the angle of elevation CKB, 64° 15′. Required the difference of height of the stations G and B. Ans. 202 feet.

PROBLEM VII.

To determine the height of an inaccessible object by a sextant.

Select three points *in a straight line,* whence a distinct view of the object can be obtained, measure their distances from each other, and take the angles of elevation at the several points.

Let AB = the height = x; and the angles ACB, ADB, and AEB, or the angles of elevation = β, γ, δ, respectively, and CD = a, and DE = b; also let the angle CDB = ϕ.



$$\text{Now the cos. } \phi \text{ or angle CDB.} = \frac{a^2+DB^2-CB^2}{2\,a.DB.}$$

$$\left.\begin{array}{l}\text{and—cos. } \phi \\ \text{or the angle BDE}\end{array}\right\} = \frac{b^2+DB^2-BE^2}{2\,b.DB}$$

$$\therefore b(a^2+DB^2-CB^2) = -a(b^2+DB^2-BE^2)$$

$$\text{but } BD^2 = \cot^2\gamma.x^2$$

$$CB^2 = \cot^2\beta.\,^2$$

$$BE^2 = \cot^2\delta.x^2$$

$$\therefore \overline{b(a^2+x^2(\cot.^2\gamma-\cot.^2\beta)}=-a(b^2+x^2(\cot.^2\gamma-\cot.^2\delta)$$

$$x^2(\cot.^2\gamma.\, b-\cot.^2\beta.\, b+\cot.^2\gamma.\, a-\cot.^2\delta.\, a)=-(a\,b^2+a^2\,b)$$

$$x^2(b\cot.^2\beta+a.\cot.^2\delta-(a+b)\cot.^2\gamma)=(a+b)\,ab$$

$$x=\sqrt{\frac{(a+b)\,ab}{b.\cot.^2\beta+a\cot.\delta-(a+b)\cot.^2\gamma}}$$

Where the height equals the root of the product into the sum, divided by the sum of each distance into cot.^{2} of its remote angle *minus* the sum of the distances into the cot.^{2} of the middle angle.

When $a=b$

$$x=a\sqrt{\frac{2}{\cot.^2\beta+\cot.^2\delta-2\cot.^2\gamma.}}$$

Example.—Took three stations in the same straight line, at some distance from an object, whose height was required, and, by means of a pocket sextant, took the angles of elevation 12° 45′, 14° 45′, and 18° 7′. The distances between the stations were 12 chains, and 15 chains. What is the height of the object? Ans. 249 feet.

CHAP. IV.

SURVEYING BY THE THEODOLITE.

Having given the theories of solution of most of the practical cases, that can occur in the measurement of inaccessible heights and distances, by means of the Theodolite, we will now proceed to explain the measurement of a road and field, by this instrument, and the method of keeping the field notes.

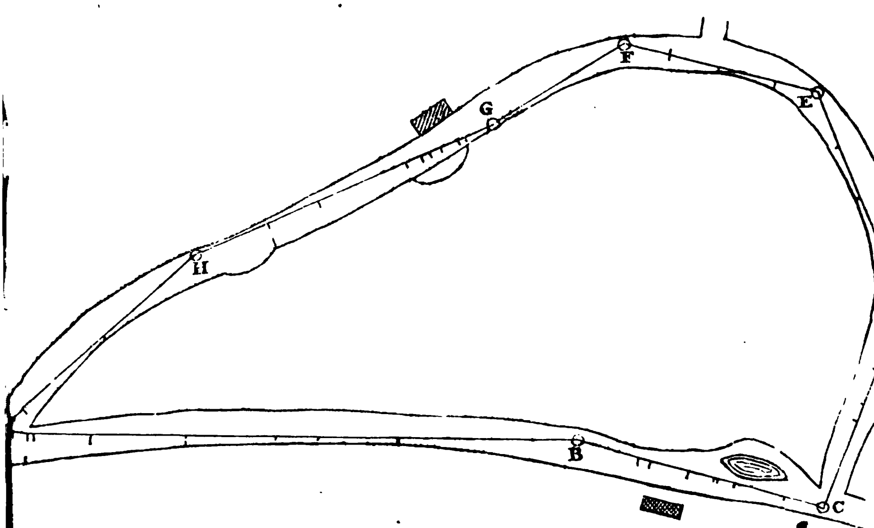
In the survey of a field, by the theodolite, as well as in the survey of roads, as explained in Chap. II., there are likewise two methods adopted: first, by selecting one of the *sides* as the base, and using the back angle; and secondly, by two

stations, by making the largest diagonal line the base of a number of triangles, and computing the position of the several corners, considered as vertices of triangles, whose angles at the base are determined by the theodolite, at the two stations.

The annexed field notes refer to both methods.

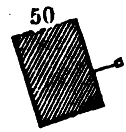
Example 1.

SURVEY OF A ROAD BY A BACK ANGLE.



Between △ H	43° 40′	and △ B
Check angle at △ A		
	8·50	to ▲ 40
to corner 65	7·30	60
30	6·00	90
5	3·90	95
	0·00	15
Between △ G	160°15′	and △ A
At △ H		

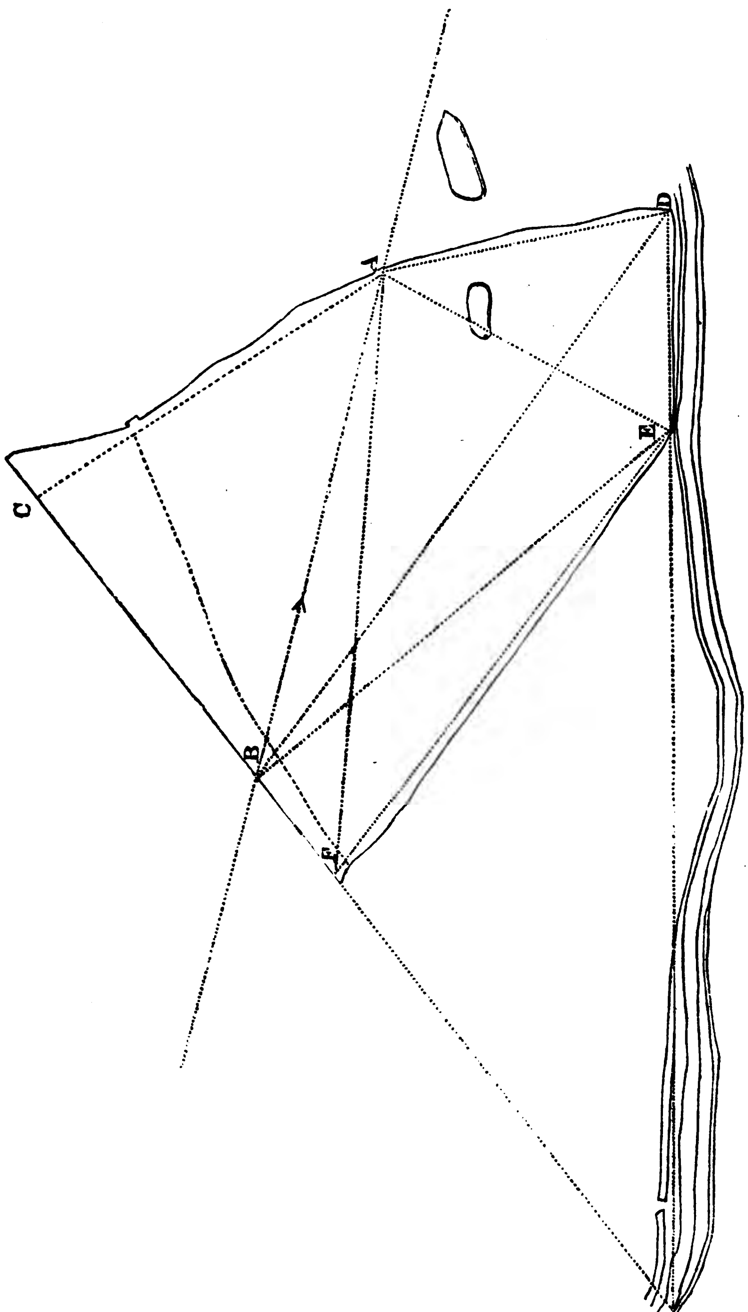
100	10·25	to H
88	7·70	
90	6·00	15
50	3·00	44
	2·20	55
25	1·00	80
Between △ F	189° 0′	and △ H
At △ G		
10	4·80	to G
12	2·50	95
	0·00	10
Between △ E	133°15′	and △ G
At △ F		
80	6·25	to F
40	4·00	70
15	3·15	90
12	1·35	75
1·00	0·00	15
Between △ D	126° 0′	and △ F
At △ E		
90	7·20	to E
25	5·20	80 to G. P.
7	3·50	85
45	1·00	60
	0·00	22
Between △ C	136°20′	and △ E
At △ D		
84	6·97	to D
75	6·30	35
64	4·60	60
	3·40	55
55	2·85	50
	0·70	40
25	0·60	
Between △ B	96° 0′	and △ D
At △ C		



4

	8 00	to C
50 + 60	6·70	36
50 + 70 + 20	5·10	6
60 + 50	4·50	0 + 50
65	3·60	
	3·35	70 + 50
25	2·00	
	0·20	85
Between △ A	195°30′	and △ C
At △ B		

to gate 44	18·30	to B
45	18·20	27
75	15·00	30
84	12·72	12
95	10·00	10
80	6·00	25
50	3·00	
to corner 23	1·15	
	1·00	70 to mile stone
	0·40	100 to corner of bend
At △ A		


A
B
C
D
E
F
G

EXAMPLE 2.

(BY TWO STATIONS).

	14·42	to △ G
road—	14·00	— ×
H—	13·73	— ×
to tree blazed 1	13·72	
D—	2·86	— ×
	2·64	△
	1·04	to stile
from △ B		
	23·41	to △ G
E		
8+30+10	21·63	12+D
	21·06	21 to G. P.
	20·90	20 to G. P.
E	19·00	6
D+28+30+20	18·00	0+D
	17·00	6
road+0	16·00	
road+D—	15·40	— ×
hedge—	15·35	— ×
hedge—	15·10	— ×
94	10·00	
38	8·00	
28	7·00	
D—	5·02	— ×
to G. P. and 25	4·33	
D+22	1·00	
D+21 path—	0·33	— ×
	324° 00	and △ G
between △ B	304°38′	and top of St. Paul's.
at △ D		
	308°23	and △ C
	129°10	and △ F
	127°55′	and △ (G) in Maiden Lane.
	36°17′	and △ E
between △ A	21°10′	and △ D
at △ B		

	350°48′	and △ F
	324°32′	Ball of St. Paul's
	284°51′	and △ E
	242°22′	and △ D
between △ B	42°40	and △ C
at △ A		

	17·37	
D—	11·17	— ×
	9·03	△ A
path—	1·27	— ×
D—	0·00	— ×
from △ B		

The notes, in the first example, are those of a survey of a road *by the back angle.* In the second example, they are those of a field surveyed *by two stations.*

The student will observe carried out, in these notes, the principle before recommended, of taking the angles always from left to right.

For instance, at station A, between station B, and station D, the angle is 242 degrees 22 minutes; that is, at the station 9·03, the direction of D (AD), is, measuring from left to right, not only 180 degrees, but 62 deg. more, extending into the third quadrant of the circle. The angle, reading the other way, from right to left, might have been taken, or the instrument might have been set to D. In the former case, the work would only have to be done in the field, which is now left to be done at home: in the latter, every station, making from left to right a greater angle than 180 degrees, must be made a new station; the entering of the field notes would be more complicated, and errors without number would result.

It is oftentimes requisite to take several angles at the same station, by considering the base line as fixed and constant, and determining the relative directions of the other points to this; the arrangement of the notes becomes simple and clear. As, for instance, *at* △ A (see page 90), denotes that all the following angles (*reading upwards*) are taken *at that* station, with

the theodolite unmoved. And *between* △B, shows that the station, at which the angles are taken, is on the line between which they are measured; and the following lines, in the margin to the left, being left vacant, proves that all the angles following are taken at the same station, △A, and measured from the same line.

I have preferred the phraseology of (" between—and ") to that of (" from—to "): there is less ambiguity about it; an angle is made *between* two lines, *at* a certain point.

The angles taken at this point are all of them, with the exception of the first, in the third or fourth quadrant. There is no possibility of error in this arrangement, and there is certainly nothing unmathematical in admitting an angle greater than 180 degrees.

In taking several angles at a station, when it is only desired to measure along the thus ascertained direction of a new line, this angle should be taken last, and placed immediately before the chain distances. Thus, at △D, the angle between △B, and △G, is placed last 324° 00′, because the following distances, 0·33, 1·00, &c., are distances on the line DG (to △G).

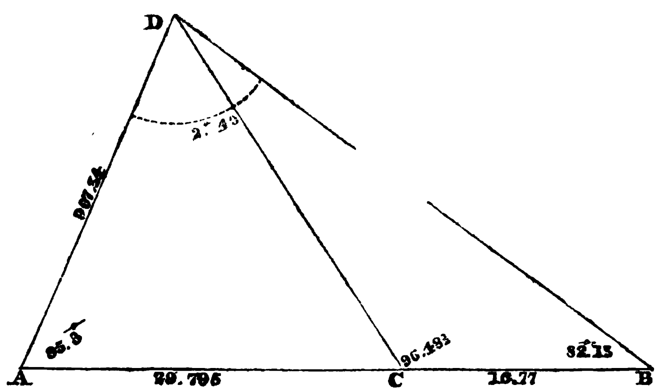
The field notes No. 2, (page 89) are those of the survey by the theodolite of a field, the plan of which is given at page 88.

I hope the student will understand, that I have not introduced the example of surveying a field (by the theodolite), as conveying an opinion, that it is indifferent, whether a field is surveyed by the theodolite or chain, far from it. I have merely given this, as an example of how it is to be done, on a larger scale. A theodolite would be quite out of place, in so small a survey as this.

CHAP. V.

FIELD NOTES.

Of a survey, actually made at Blackheath, explaining the method of keeping the field book, and the practical arrangement and computation of the angles.



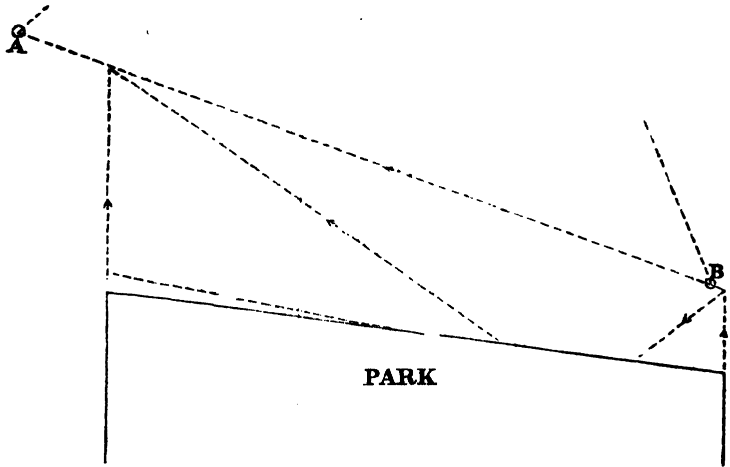
Total length of AB = 46·56 chs.

These Notes begin at page 95, from south-west corner of park, &c., and read upwards.

Between A on base line at 16·77 on base line	83° 11′	and Knockholt Beeches

The base here was so disproportionate to the unknown sides, that it became necessary to take a check angle to prevent inaccuracy.

vertical angle	0° 22′ 0″	to top of Spire
348°30′ 337°0′	348° 30′	and Charlton Church.
vertical angle	0° 33′	to top of tree
162°29′ 324°58′	162 29′	small bush right of large Tree on Forest Hill.
vertical angle	0° 33′	to top of Spire
108°20′ 216°38′ 324°57′	108° 19′	and Lee Church.
vertical angle	0° 30′ 0″	to top of Beeches
82°12′ 164°26′ 246°39′	82° 13′	and Knockholt Beeches △ D
vertical angle	1° 30′	to top of Spire
53°2′ 106°5′	53° 2′ 20″	and Blackheath New Church.
vertical angle	1° 15′	to top of Tower
21°29′ 42°58′ 64°28′	21° 29′ 20″	and pole at top of Sevendroog Castle.
between base line at △ B		
345°33′ 123°37′	345° 33′	and bush on right of large tree on Forest Hill.
vertical angle	0° 36′	top of Spire of Church.
325°41′ 83°54′ 202°6′	325° 41′ 20″	and Lee Church.
vertical angle	2° 8′ 0″	to top of Spire
284°46′ 209°31′	284° 45′ 40″	and Blackheath New Church.
vertical angle	0° 34′	to top of Beeches.
264°57′ 169°54′	264° 57′	and Knockholt Beeches △ D
vertical angle	1° 42′	to top of Turrets
207°29′ 54°54′	207° 29	and Sevendroog Castle.
between base line AB at △ A		



Scale of 12 Chains to the inch.

	46·56	to △ A
	40·71	to 1443 on 1443
	16·77	△
from △ B		
the distances were		

Correcting for the $2\frac{1}{2}$ inches.

This chain had been previously carefully measured at the standard chain length at Somerset House, and was $2\frac{1}{2}$ inches too short.

	46·70½	to △ A
	40·83	to 1443 on 1443
	16·82	△
from △ B		

The chain, which was used the first day, having been mislaid, another chain was used for the second measurement.

End of first day's work.

from 1400 on 3952	30·46	to 5 82 on 47·31
from 585 on 585	7·40	to 600 on 3952

	47·51	to 585 on 585
	46·51	△ B
	38·98	△ by side of road
	5·82	to 14·43 on 1443
from △ A by side of Shooter's		

	14·43	△
	1·35	to 39·52
from S. E. corner of park, ranging with easterly wall.		
	39·52	△
side of —	38·57	—road
to wall 37	30·00	
3	20·00	
	1400	△
to park gate 0	13·98	
	600	△
0	1·96	
to wall 4	1·27	
from 65 on 585		along wall of park
△	5·85	
	0·65	△ park wall on left
to wall 0	0·62	

from S. W. corner of Greenwich Park, at Blackheath, near the Princess Augusta's.

These Notes were taken in order to obtain the relative position of the assumed base AB (which was a loose line upon Blackheath Common), with the fixed line of the southerly wall of the park, which is laid down upon the ordnance map.

This was done for the purpose of testing our computed distance of the Beeches, with that given by scale from the ordnance map.

To find the value of the check-angle, 83° 11′ (page 92), *by computation.*

The base AB being given, 46·56, and the angles at the base 95° 3′, and 82° 13′, the other two sides were found to be 967 chains 34 links, and 972 chains 60 links.

Now, in the triangle ADC (page 92), we have the sides AD, AC, and the included angle at A; hence, by the second case of trigonometry, the other angles were computed, and that, at

the base, ACD, found to be 83° 11′ 12″, the supplement of which is 96° 48′ 48″; the measure of the angle DCB being only 12 seconds less than the observed angle 96° 49′.

Testing of the field work.

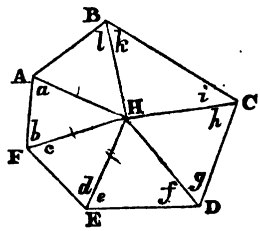
The length DB, being the distance from the Knockholt Beeches to △ B, was found to be 972 chains 60 links. To this was added the distance of B, from the S W. corner of the park, viz., $\sqrt{5{\cdot}85^2+100^2}$, or 5 chains 93 links, and the total distance, obtained thus, was 978 chains 53 links, or 12 miles, 1 furlong, and 8 chains 53 links.

The distance on the ordnance maps is somewhat less than 12 *miles* 2 *furlongs.*

CHAP. VI.

TRIGONOMETRICAL PROBLEMS.

Before proceeding to the solution of these Problems it must first be demonstrated, that, if any polygon be taken, and lines be drawn from a point, either within or without it, to the several angles of this polygon, the continued product of the sines of one set of alternate angles, made by these lines, and the sides of the polygon, will be equal to the product of the sines of the other set.



Let H be the point whence the lines HA, HB, HC, &c., are drawn to the angles of the Polygon.

Now, in the triangle AHF, $\frac{FH}{AH}=\frac{\sin. a}{\sin. b}$

In the same manner $\frac{EH}{FH}=\frac{\sin. c}{\sin. d}$; $\frac{DH}{EH}=\frac{\sin. e}{\sin. f}$, &c.

$$\therefore \frac{FH}{AH}\times\frac{EH}{FH}\times\frac{DH}{EH}\text{ \&c.}\times\frac{AH}{BH}=\frac{\sin. a}{\sin. b}\times\frac{\sin. c}{\sin. d}\times\frac{\sin. e}{\sin. f}\text{ \&c.}$$

$$\text{and }\therefore\ \frac{\sin. a.\ \sin. c.\ \sin. e.\ \text{\&c.}}{\sin. b.\ \sin. d.\ \sin. f.\ \text{\&c.}}=1 \text{ or}$$

$$\sin. a.\ \sin. c.\ \sin. e.\ \sin. g.=\sin. b.\ \sin. d.\ \sin. f.\ \text{\&c.}$$

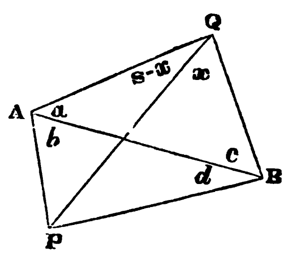
The same result would obtain, when H is *without* the Polygon.

PROBLEM I.

Given the position of two known points, to determine the length of a new base line, by means of angles taken from its extremities with the given base.

(Resolved analytically.)

Let PQ be a line determined in position; having selected two stations A and B, it is required to find their distance from each other, this distance cannot be measured, but the angles *a*, *b*, *c*, *d*, can be taken by a theodolite.



First let P fall outside the triangle AQB.

In this case AQ, QB, AB are the sides of the figure, and AP, QP, BP are the lines drawn to the angles.

Let s=AQB=(AQP+PQB,) or, 180—$(a+c)$ and x=PQB

$$\therefore (\sin. b. \sin. \overline{s-x}. \sin. (c+d) = \text{one set};$$

$$\sin. (a+b) \sin. x. \sin. d = \text{the other.}$$

$$\text{but } \sin. b. \sin. \overline{c+d}. \sin. \overline{s-x} = \sin. d. \sin. \overline{a+b}. \sin x.$$

$$\sin. b. \sin. \overline{c+d}(\sin. s. \cos. x - \sin. x. \cos. s) =$$

$$\sin. d. \sin. \overline{a+b} \sin. x;$$

[*dividing by sin. x.*] $\sin. b \sin. (c+d)(\sin. s \cot. x - \cos. s) =$

$$\sin. d. \sin. (a+b);$$

$$\sin\ b \sin. (c+d). \sin. s \cot. x - \sin. b \sin. (c+d). \cos. s =$$

$$\sin. d \sin. \overline{a+b};$$

$$\therefore \sin. b \sin. (c+d) \sin. s \cot. x =$$

$$\sin. b \sin. (c+d) \cos. s + \sin. d. \sin. \overline{a+b}.$$

getting rid of the coefficient of the cot x; we have

$$\cot. x = \cot. s + \sin. d. \sin. \overline{a+b}. \operatorname{cosec}. b. \operatorname{cosec}. \overline{c+d}. \operatorname{cosec}. s.$$

This will give the value of x.

Now sin. s: AB : : sin. a: QB;

$$\therefore AB = \frac{\sin. s}{\sin. a}. QB = \operatorname{cosec}. a. \sin. s. QB;$$

$$\text{and } \sin. (x+c+d): QB :: \sin. (c+d): PQ;$$

$$\therefore BQ = \frac{\sin. (x+c+d)}{\sin. (c+d)}. PQ.$$

and, therefore, by substituting this value of QB, we have

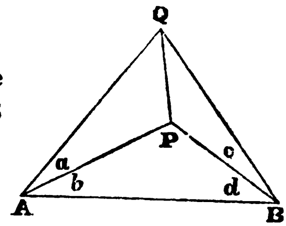
$$AB = \operatorname{cosec}. a. \sin. s. \operatorname{cosec}. (c+d). \sin. (x+c+d). PQ;$$

The sin. $(x+c+d)$ requiring the value of x, before it can be resolved, this value is given above in the formula of

cot. x=cot. s+sin.d.sin. $(a+b.)$ cosec. b. cosec. $(c+d)$ cosec. s.

Now, $(a+b)$ and $(c+d)$ being the angles opposite PQ, are known.

PROBLEM II.



Next let the point P fall within the triangle. It is required to find AB as before.

Let $s = \angle AQB = 180 - $(the angles QAB + QBA)
$= 180 - (a+b+c+d)$
And let $\angle$ PQB $= x$.

then sin. b. sin. $\overline{s-x}$. sin. $c =$ sin. d. sin. x. sin. a.

resolve sin. $\overline{s-x}$ and we have

sin. b. sin. c. (sin. s. cos. x—sin. x. cos. s.)$=$sin. d. sin. x. sin. a.

sin. b. sin. c. (sin. s. cot. x—cos. s)$=$sin. d. sin. a.

sin. b. sin. c. sin. s. cot. $x=$sin. d. sin. $a+$sin. b. sin. c. cos. s.

$$\therefore \text{cot. } x = \frac{\text{sin. } d. \text{ sin. } a.}{\text{sin. } b. \text{ sin. } c. \text{ sin. } s.} + \frac{\text{cos. } s.}{\text{sin. } s.}$$

and because $\frac{\text{cos.}}{\text{sin.}} = $cot. and $\frac{1}{\text{sin}} = $cosec.

$\therefore$ cot. $x=$cot. $s+$sin. d. sin. a. cosec. b. cosec. c. cosec. s.

Again because sin. s : AB : : sin. $(a+b)$: BQ.

$\therefore$ AB$=$sin. s. cosec. $(a+b)$.BQ

and because sin. c : PQ : : sin. $(x+c)$: BQ

$\therefore$ BQ$=$sin. $(x+c)$ cosec. c.PQ

and therefore, by substituting the value of BQ, we have

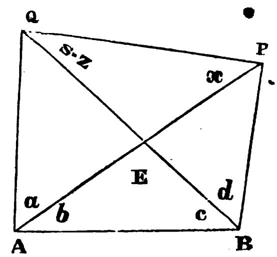
AB$=$cosec. $(a+b)$ sin. s. cosec. c. sin. $(x+c)$ PQ.

And, whether P fall within or without the triangle ABQ the required distance AB will always be

AB$=$PQ (cosec. BAQ. sin. AQB. cosec. QBP. sin. QPB.)

The value of x being always wanted to obtain the value of the *sin.* QPB.

PROBLEM III.



Next let P fall withont the triangle and on the same side of AB as Q is; and let the angles a, b, c, d be taken by the theodolite as before. In this case let E be the point whence the lines are drawn to the angles.

Now let $b+c=s=$QEA$=$EPQ$+$EQP and let EP Qx, therefore PQB$=s-x$

Now,

$$\sin. a. \sin. c. \sin. \text{APB}. \sin. (s-x)=$$
$$\sin. b. \sin. d. \sin. x. \sin. \text{AQB};$$

but $\sin. \text{APB}=\sin.$ of supplemental $\angle\ (b+c+d)$.

and $\sin. \text{AQB}=\sin.$ supplemental $\angle\ (a+b+c)$.

$$\therefore \sin. a. \sin. c. \sin. (b+c+d). \sin. (s-x)=$$
$$\sin. b. \sin. d. \sin. x. \sin. (a+b+c).$$

or, $\sin. b. \sin. c. \sin. (b+c+d). (\sin. s. \cos. x-\sin. x. \cos. s.)=$
$$\sin. b. \sin. d. \sin. x. \sin. (a+b+c).$$

(dividing by $\sin. x$) $\sin. a. \sin. c. \sin. (b+c+d). (\sin. s. \cot. x-\cos. s)$
$$=\sin. b. \sin. d. \sin. (a+b+c).$$

$$\text{or } \sin. a. \sin. c. \sin. (b+c+d). \sin. s. \cot. x=$$
$$\sin. b. \sin. d. \sin. (a+b+d)+\sin. a. \sin. c. \sin. (b+c+d). \cos. s.$$

hence, by transposition and getting rid of, as before, the coefficient of $\cot. x$, we have,

$$\cot. x=\cot. s+\sin. b. \sin. d. \sin. (a+b+c). \text{cosec}. a.$$
$$\times \text{cosec}. c. \text{cosec}. (b+c+d). \text{cosec}. s.$$

whence x, and $s-x$ are known.

Then as $\sin. (a+b+c) : \text{AB} :: \sin. (a+b) : \text{BQ}$

$$\therefore \text{AB}=\sin. (a+b+c). \text{cosec}. (a+b). \text{BQ}.$$

again as $\sin. (d+s-x) : \text{BQ} :: \sin. d : \text{PQ}$

$$\therefore \text{BQ}=\sin. (d+s-x). \text{cosec}. \text{PQ}.$$

$$\text{AB}=\sin. (a+b+c). \text{cosec}. (a+b). \sin. (d+s-x). \text{cosec}. (d). \text{PQ}.$$

$=$cosec. $(a+b)$. sin. $(d+s-x)$. cosec. d. sin. $(a+b+c)$. PQ.

AB$=$PQ(cosec. QAB. sin. QPB. cosec. QBP. sin. AQB).

Which is the same result as the two preceding cases, where the relative positions of the given and required bases materially differ from the present.

Hence, if the distance between two objects be correctly known by previous surveys, and a BASE LINE be required, the actual measurement of which cannot be accurately depended upon, at the same time that perfect accuracy is indispensable in determining it, as it is to become the base of other triangulations, by selecting two stations, from each of which the two given objects and the other station can be seen, whatever may be the relative position of the given and required bases, the length of the *new base* can be always determined.

CHAP. VII.

PROBLEM I.

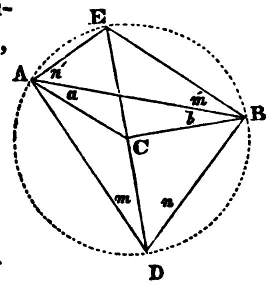
Given the position of three known points, to ascertain the position of a new station, whence those points can be seen.

(Resolved geometrically.)

HAVING previously determined the position of three different places, A, B, and C, not in the same straight line, and being desirous of connecting with them a new survey, which is at some distance, but from which the three objects can be seen, it is required to determine the position of any point D, within the new survey, by means of angles taken at that point between the three given points.

Let A and B be the extreme stations; join AB, AD, and BD. Now the other given point, C, can either fall on the line AB, or within or without the triangle ABD.

1st. *Let it fall within;* about the triangle ABD describe a circle, join DC, and produce to E; join AE and EB.



Let the ∠ ADE$=m$, and the angle BDE$=n$, (in all the three cases) then the angles ABE, BAE, will be also respectively equal to m and n, being upon the same segments of a circle, AE, and BE.

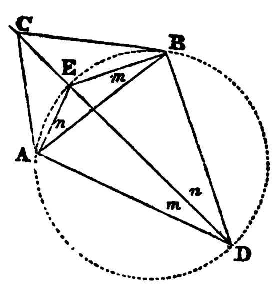
Now, therefore, there are in the triangle AEB, the base AB, and the angles BAE and ABE, known.

AE and EB are thus determined.

Again, the sides AC, and AE, being known, and the included angle, the angle AEC can be obtained. This is also an angle of the triangle DAE, whose other angle (m) at D has been taken, and therefore the angle DAE can be determined; again, because the angle CAE $(a+n)$ is known, their supplemental ∠ DAC becomes known; and there are, in the triangle DAC, the angles ADC, and DAC, and the side AC, given to determine the sides AD and DC; and in the triangle DCB, you have now DC and BC, and the angle CDB, (n), known.

PROBLEM II.

Now let C fall without the triangle ABD.



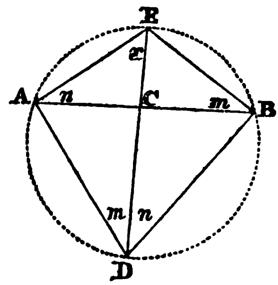
Describe a circle as before; join AE, and BE; then the angle BAE$=n$, and ABE$=m$; and the angle CAE$=a-n$; and the angle EBC$=b-m$.

Determine as before the sides AE, and EB, and, having the sides AB and BC known, you have two sides AC and AE, and their included angle, whence to obtain the angle ACD.

The after process is the same.

PROBLEM III.

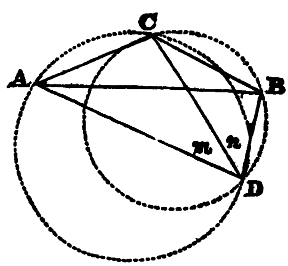
Lastly, let A, B, and C, be in the same straight line.



Let ∠ ADC=m, and CDB=n, as before, and describe round ABD a circle; produce DC to E, and join AE and EB. Determine AE and EB, as well as the angle E, as before; then, because AC also is given, therefore the two sides CA, AE, and the included angle CAE, are known, and therefore, the angle AEC; and in the triangle AED, the angle DAE, the supplement of the other known angles is known also; whence, in the triangle ADC, there are two angles and a side known, and also in DBC. The required sides AD, CD, BD, are thence found.

The above are examples of solution, when the student is not acquainted with analytical trigonometry.

There is an easy method of constructing the figure, and determining upon paper the position of the required station.



Let ABC be the given triangle, and D the given station, whence the angles ADC, and CDB are taken.

To find the point D.

Upon AC describe a segment of a circle, having an angle (m) equal to ADC, and upon CB the segment of a circle, having an angle (n) equal to CDB, the point of intersection of the circles shall be the station D required.

These cases may be proved analytically, if the student should prefer it, in the same way as the Problems in Chapter VI.

In the first case, where AB is given, and two angles a and b.

Make s=the difference between the known angles and 180°; and s and $s—x$, are the unknown angles required.

Equate the product of the sines of the alternate angles, as in Problem II, page 99, and resolve the sin. $(s—x)$ into its equivalent values of the simple sines.

Arrange the known and unknown values on their proper sides of the equation.

Divide by the co-efficient of cot. x, and you obtain the value of one of the unknown angles. $S—x=$ the value of the other. The sides are obtained by the formulæ, for the relative values of the sides, and the sines of their opposite angles.

In the second case, where AC and CB are given, and the included angle, ACB.

Let $s=$ 360 degrees—$(\text{ACB}+m+n)$; let the angle CBD$=x$, then the angle CAD will be equal to $S—x$; ED is the common base of the two triangles.

Equate the value of ED in the two triangles, thus—

$$\text{As sin. } m : \text{AC} :: \text{sin. } \overline{s—x} : \text{DC}$$
$$\text{and as sin. } n : \text{BC} :: \text{sin. } x : \text{DC}$$
$$\therefore \frac{\text{AC. sin. } \overline{S—x}}{\text{sin. } m} = \frac{\text{BC. sin. } x}{\text{sin. } n}$$

Or, AC. sin. n. sin. $(s—x)$=BC. sin. x. sin. m.

Then, by resolving sin. $\overline{s—x}$, we shall have

AC. sin. n. (sin. s. cos. x—cos. s. sin. x)=BC. sin. x. sin. m.

AC. sin. n. (sin. s. cot. x—cos. s)=BC. sin. m.

AC. sin. n. sin. s. cot. x=BC. sin. m. +AC. sin. n. cos. s.

$$\text{and } \therefore \text{ cot. } x = \frac{\text{BC}}{\text{AC}}\text{. sin. } m\text{. cosec. } n\text{. cosec. } s + \text{cot. } s.$$

CHAP. VIII.

TRIGONOMETRICAL SURVEY.

The following field notes are given for examples to the student. A base line was taken on Hampstead Heath, and angles were taken between it and the Churches of Highgate, Hampstead, and St. Paul's. There was also an angle taken to a Stone Monument, at some distance on the Heath, on the ascent of a hill, whence a second angle could be taken to St. Paul's, which was not visible from both stations on the *base line.*

The distance between St. Paul's and Highgate was subsequently calculated, and this distance being assumed as an *old line,* a new line was taken at Streatham, whence angles at each end were taken to this old line, for the purpose of ascertaining the exact direction of the new line, preparatory to determining the relative position of the two surveys at Streatham and Hampstead.

HAMPSTEAD HEATH FIELD NOTES.

∠ elevation	0° 19′	ball of St. Paul's △ F
	96° 2′	and St. Paul's △ F
between △ A	11° 36′ 35″	and Highgate Church △ E
at △ C		
angles of { elevation	0° 18′ 0″	of St. Paul's △ F
angles of { elevation	3° 15′ 5″	of Highgate Church △ E
angles of { depression	3° 15′ 0″	of bottom of hill on B. L.
	306° 3′ 40″	and Stone △ C
	293° 49′	and Hampstead Church △ D
	181° 51′ 40″	and St. Paul's △ F
between △ A	74° 47′	and Highgate △ E
at △ B		
	278° 11′	and Highgate Church △ E
	94° 29′	and Hampstead Church △ D
between △ B	81° 8′ 0′	and Stone △ C
at △ A		
from △ A	24·74 chs.	to △ B twice measured

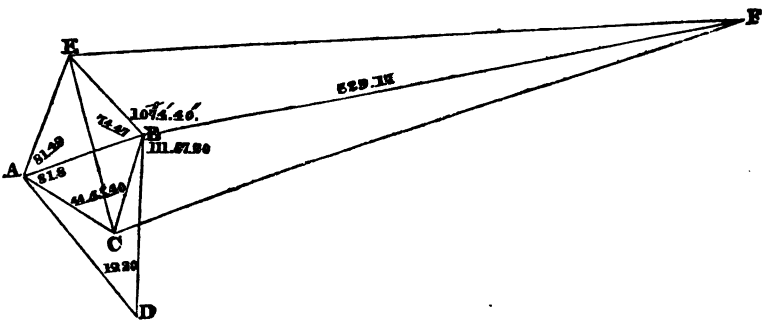
BASE LINE.

Explanations, &c.

Let AB be the base line, 24 chains 74 links long; E, Highgate Church; D, Hampstead; F, St. Pauls; and C, the Stone Monument by the roadside.

At station A the following angles were taken—to the Stone, 81° 8′; to Hampstead Church, 94° 29′; computing, always, from the base line towards the right. The first angle is the angle BAC; the second is the angle BAD; the angle CAD is equal to the difference of these two, or 13° 21′; the other angle is 278° 11′, or an angle in the fourth quadrant; the angle BAE is the difference between this and 360° or 81° 49′.

At station B, the first angle taken, is to Highgate Church, ABE, 74° 47′; the second angle, to St. Paul's, 181° 51′ 40″, being the angle ABE, 74° 47′ + the angle EBF, which is therefore 107° 4′ 40″. The next, to Hampstead Church, 293° 49′, making the angle ABD the difference betweeen this and 360°, or 66° 11′. The next angle is that to the Stone, which is 306° 3′ 40″, making the angle ABC, 53° 56′ 20″.



The angles of elevation were taken, in case of their being wanted.

And, lastly, at the Stone C, the angles were taken to Highgate Church and St. Paul's, 11° 36′ 35″, and 96° 2′.

The distances required were EF, BF, and CF.

CALCULATIONS OF ANGLES TAKEN AT HAMPSTEAD.

These calculations were made for the purpose of determining the distance from St. Paul's to Highgate Church. These being the only two objects that were visible both at Hampstead and Streatham.

Arrange, systematically, the work to be done, by dividing it into triangles, and apportioning the proper lines to be determined in each.

Thus in the triangle AEB, *find* AE and BE.
In the triangle ACB, *find* AC and BC.
In the triangle CAE *find* CE.
In the triangle CBE *find* CE.
Their *equality* proves the work.
In the triangle CBF, *find* CF and BF.
In the triangle EBF, *find* EF^1.
In the triangle ECF, *find* EF^2.
EF^1 and EF^2 must *agree*.

To find BE in the triangle AEB.

As sine $\angle$ E	$= 23° 24'$	$= 9{\cdot}5989523$
is to AB	$= 24{\cdot}74$ chs.	$= 1{\cdot}3933997$
so is sine $\angle$ EAB	$= 81° 49'$	$= 9{\cdot}9955552$
		$11{\cdot}3889549$
		$9{\cdot}5989523$
to BE	$= 61{\cdot}66$ chs.	$= 1{\cdot}7900026$

To find AE, in the triangle AEB.

As sine $\angle$ E : AB : : sine $\angle$ B : AE
AE $= 60{\cdot}11$ chs.

To find CB, in the triangle ACB.

As sine $\angle$ C : AB : : sine $\angle$ A : BC.
BC $= 34{\cdot}61$ chs.

Again, to find AC in the triangle ACB.

As sine $\angle$ C : AB : : sin. $\angle$ B : AC.
AC $= 28{\cdot}32$ chs.

In the triangle CAE, to find the unknown angles.

$$\text{As AC+AE : AC—AE : : tan. } \frac{\angle E + \angle C}{2}$$

: tan. $\frac{1}{2}$ difference $= 3° 5' 4''$

The angles being known to find E C.

As cos. $\frac{\angle E \backsim \angle C}{2}$: cos. $\frac{\angle E + \angle C}{2}$: : AC+AE : EC

EC = 87·58 chs.

In the triangle ECB, to find the unknown angles, and thence the side EC.

As CB + BE : CB—BE : : tan $\frac{\angle C + \angle E}{2}$: tan. $\frac{\angle C - \angle E}{2}$

tan. ½ difference = 7° 40′ 49″.

And as cos. $\frac{\angle C - \angle E}{2}$: cos. $\frac{\angle C + \angle E}{2}$: : CB + BE : EC

EC = 87·58 chs. *as before.*

In the triangle CBF, to find CF and BF.

As sine $\angle$ F : BC : : sine $\angle$ C : BF

BF = 329·17 chs.

And as sine $\angle$ F : BC : : sine $\angle$ B : CF.

CF = 349·80 chs.

In the triangle EBF, to find the unknown angles, and EF.

As BE + BF : BF—BE : : tan. $\frac{\angle E + \angle F}{2}$: tan. $\frac{\angle E - \angle F}{2}$ = 26° 49′ 42″.

$\angle$ BEF = 63° 17′ 22″.

$\angle$ BFG = 9° 37′ 58″.

As cos. $\frac{\angle E - \angle F}{2}$: cos. $\frac{\angle E + \angle F}{2}$: : BE+BF : EF

EF = 352·24 chs.

In the triangle ECF, to find EF.

As sine $\angle$ F : EC : : sine $\angle$ C : EF.

EF = 352·23 chs.

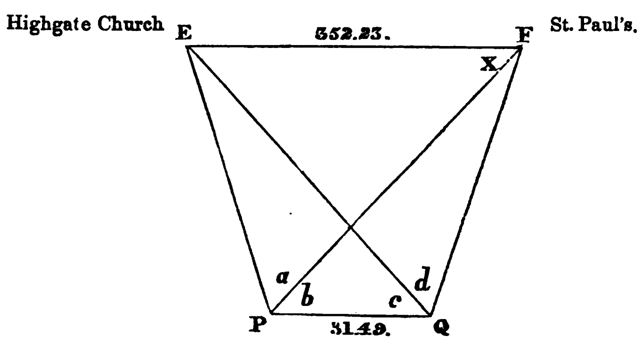
The same within one link as before. EF is the distance required—viz., from St. Paul's to Highgate Church.

STREATHAM COMMON FIELD NOTES.

Between △ P	86° 50′ 0″	and St. Paul's △ F
	69° 8′ 27″	and Highgate Church △ E
at △ Q		
between	108° 27′ 0″	and △ Q
Highgate Church △ E	19° 48′ 40″	and St. Paul's △ F
at △ P		
	32·10	△ Q
fence—	4·81	— ×
side of Reservoir 5	4·42	
side of Reservoir 5	0·61	△ P near New Church, opposite Crown and Sceptre.

These notes were taken to determine the length and the relative position with the Hampstead survey of the base line at Streatham.

As it was doubtful, however, whether, in the observation taken at station Q, the angle PQE was taken to the proper object, from the extreme fogginess of the morning, which prevented the spire of Highgate Church from being distinguished from many others in the neighbourhood, though that observation, if to the right object, was, in itself, correctly and carefully taken, it was checked by assuming the distance PQ (taking its measured distance), as known, and *calculating* what the angle should be, between the proper object and the given base line.



PQ being the Base Line at Streatham, this base measured on the ground 3 furlongs, 1 chain and 49 links, commencing (see notes) at 0·61 and ending at 32·10 chs.

First find PF in the triangle PQF.

As sine $\angle$ PFQ : PQ :: sine $\angle$ PQF : PF

PF$=$398·29 chains.

To find the angle PEF in the triangle PFE.

As EF : sin. $\angle$ FPE :: PF : sine $\angle$ PEF

the angle PEF$=$22° 32′ 4″

To find PE in the same triangle.

As sin. $\angle$ FPE : EF :: sine $\angle$ EFP : PE

· PE $=$ 700·04 chains.

To find the angle PQE in the triangle PQE.

$$\text{As PE} + \text{PQ} : \text{PE} - \text{PQ} :: \tan. \frac{\angle \text{PEQ} + \angle \text{PQE}}{2} :$$

$$\tan. \frac{\angle \text{PEQ} - \angle \text{PQE}}{2} = \tan. \tfrac{1}{2} \text{ difference} = 33° \; 21' \; 57''$$

which makes the required angle PQE$=$69° 8′ 27″

The same as that observed by the theodolite.

The other angle PEQ will be	$=$	2°	24′	33″
which added to PQE	$=$	69°	8′	27″
gives their sum	$=$	71°	33′	0″
Making their supplement	$=$	108°	27′	0″

which is also the observed angle EPQ.

The angles have therefore been taken to the right object at Hampstead. The intended calculations can now be proceeded with.

The following calculations were made for the purpose of verifying the correctness of the several angles taken at the two stations P and Q, by assuming the base PQ as unknown; and after computing its length by means of angles taken from each end of it to a known base EF, according to the problem given (at page 100), comparing the computed with the measured length of PQ, and thereby determining its correctness.

Example.—Having the old base at Hampstead, EF, given, to find the new base PQ at Streatham being a practical example of the trigonometrical problem given at page (100).

Let $s=b+c$, where b is the angle FPQ, 88° 38′ 20″, and c

is the angle PQE, 69° 8′ 27″; $b+c$ being together equal to the unknown angles QEF and EFP.

Let EFP$=x$, then QEF will be equal to $s-x$.

In calculations of this kind, the tables with 7 places of decimals should be used, they are therefore purposely referred to in this example.

Now, by Problem III., Chap. VI., let $s=b+c$.

cot. x = cot. s + sin. b. sin. d. sin. $(a+b+c)$. cosec. a cosec. c. cosec. $(b+c+d)$. cosec.. s.

sin. b =	88° 38′ 20″	=	9·9998774
sin. d =	17° 41′ 33″	=	9·4827426
sin $(a+b+c)$ =	177° 35′ 27″	=	8·6236120
cosec. a =	19° 48′ 40″	=	10·4699023
cosec. c =	69° 8′ 27″	=	10·0294406
cosec. $(b+c+d)$ =	175° 28′ 20″	=	11·1026903
cosec. s =	157° 46′ 47″	=	10·4223198
to rad. (10 index)		=	70·1305849
		—	70
(to rad. 1,)	1·3507831	=	0·1305849

again cot. s=cot. 157° 46′ 47″ and being in the second quadrant =— cot. 22° 13′ 13″.

— (log. cot. 22° 13′ 13″) =— (10·388892) to rad. 10.

subtract 10

(to rad. 1)—(2·4483819)=—0·388892

1·3507838 (see above)

—2·4483819

—(42° 30′ 48″=natural cot. x=—1·0975971 because

rad. cot. 42° 31′=1·0977020	5971=given angle	
rad. cot. 42° 30′=1·0970609	0609=42° 30′	
6411	5362	
	60	
6411	311720	48

but cot.—(42° 30′ 48″)=its supplement 137° 39′ 12″=x

and PQ=cosec. EPF. sin. PEQ. cosec. PQE. sin. EFP(x)

viz. log. EF. 352·23 chs. =	2·5468246
log. cosec. 19° 48′ 40″ =	10·4699023
log. sin. 2° 21′ 33″ =	8·6336120
log. cosec. 69° 8′ 27″ =	10·0294405
log. sin. 137° 39′ 12″ =	9·8284115
the length of PQ=log. 31·4912 chs.=	1·4981909

which is the same as the measured length of the base PQ.

CHAP. IX.

TO REDUCE ANGLES TO THE CENTRE OF STATIONS.

In large trigonometrical surveys, as those objects, which from their position, are more commanding, and are furthest visible, are generally so situated, as to prevent the theodolite being placed immediately in the centre, it has been found necessary to calculate expeditiously the angle of *reduction to the centre*, or the difference between the angle, as taken from a station (S) near the centre of this object, and the angle from the centre.

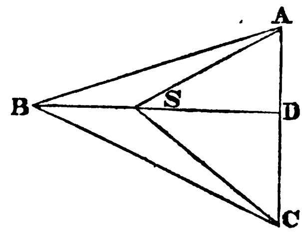
Suppose, for instance, you have determined the position of a church and tower, and taken angles to the vane of the church, or the pole of the tower, but on wishing to base upon the line connecting them a further triangulation, you find you cannot place your instrument at the centre of either, you are therefore compelled to take your angles as near to it as possible. This angle, thus taken, will fall either within the first triangle, or without, or upon one of its sides.

If it falls within, it will be greater than the true angle; if without the triangle, but within the circumscribing circle, it is greater; if upon the circumference, equal; if without it, less.

The amount of this excess or deficiency we are now to ascertain.

Let B be the true station, and S the station taken.

1st. Let S fall within the triangle ABC; the observed angle will be ASC.



In addition to this angle ASC, it will be necessary to take the angle at S, made between the true point B, and each of the two points A or C and to measure BS.

To find its excess, or, to reduce it to ABC.

Produce BS to D.

Now $\angle ASD = \angle ABS + \angle SAB$,

and $\angle CSD = \angle CBS + \angle SCB$,

$\therefore \angle ASC = \angle ABC + \angle BAS + \angle BCS$;

but BC and AB, and BS are known distances, and the angle BSC is known also, being the observed angle.

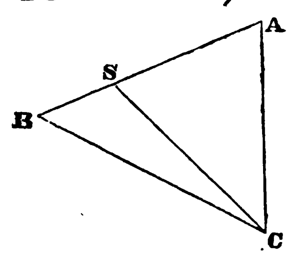
$$\text{Now, } BC : \sin. \angle BSC :: BS : \sin. \angle BCS,$$

$$\text{therefore } \angle BCS = \frac{BS}{BC} \cdot \sin. \angle BSC,$$

$$\text{and, therefore } \angle SAB = \frac{BS}{BA} \sin. BSA,$$

$$\text{whence } \angle ABC = \angle ASC - BS\left(\frac{\sin. BSC}{BC} + \frac{\sin. BSA}{BA}\right)$$

2nd. Let S fall upon the side AB,



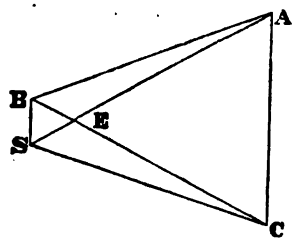
then $\angle ABC = \angle ASC$

$-BS\left(\frac{\sin. BSC}{BC} + \frac{\sin 180°}{BA}\right)$

but $\frac{\sin. 180°}{BA} = 0$

$\therefore \angle ABC = \angle ASC - BS \frac{\sin. BSC}{BC}$

3rd. Let S fall *without* the triangle ABC, and let E be the point of intersection of AS and BC. Now the $\angle$ AEC $= \angle$ ABC $+ \angle$ BAS $= \angle$ ASC $+ \angle$ BCS, therefore



$\angle ABC = \angle ASC + \angle BSC - \angle BAS$;

but $BCS = \frac{BS}{BC} \sin.\ (BSA + ASC)$

and $\angle BAS = \frac{BS}{AB} \sin.\ BSA.$

$$\therefore \angle ABC = \angle ASC + BS\left(\frac{\sin.\ (BSA+ASC)}{BC} - \frac{\sin.\ BSA.}{AB}\right)$$

In the case where BC and AB are infinite, each of the expressions, where they occur, will vanish; and $\angle$ ABC becomes equal to $\angle$ ASC, which is the case when the objects are heavenly bodies.

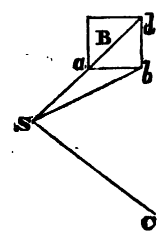
This obtains also, when the station falls upon the circumference of the circumscribing circle.

Measurement of the line BS.

It is not always possible, however, to measure BS, or take the angle CSB.

As it is desirable that the instrument should be placed close to the centre of the station, the vane of the steeple, or the flag pole of the tower, may be invisible from it.

When this is the case, if the centre be the vane of a church, as in the diagram, when the tower is square, select S in a line with the diagonal. Measure Sa and ab, and take the angle CSa = CSB.

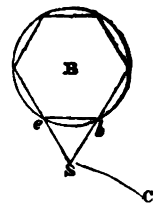


Now, $SB = Sa + \frac{ad}{2}$ and $ad = ab\sqrt{2}$

for $ad^2 = ab^2 + bd^2 = 2\ ab^2 \therefore ad = ab\sqrt{2}$

$$\therefore SB = Sa + \frac{ab}{2}\sqrt{2};$$

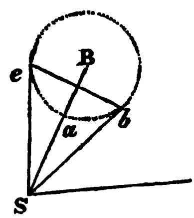
Again, let the tower be a hexagon.



Produce outward two of the sides to S, where place the instrument; (ebS) is an equilateral triangle, and $SB = e\,b\sqrt{3}$

The interior angle of a hexagon is 120°, the angle at e 120°, and Be bisects it, and the $\angle$ B eb, is, therefore, 60°, but the angle S eb is also 60°, being the supplement to the interior angle 120°, and therefore BS is bisected by eb, but each of its bisections $= \tan. 60° \times \frac{eb}{2}$. the whole BS $= \tan. 60° \times eb$, and, as $\tan. 60° = \sqrt{3}$ $\therefore$ B S $= eb. \sqrt{3}$ and $\angle$ CSB $= \angle$ C S $b + \frac{bSe}{2}$.

If the tower be circular,



$\angle CSB = \angle CSb + \frac{bSe}{2}$

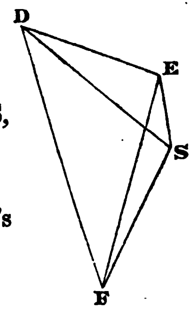
and SB $= bS. \sec. \angle \frac{bSe}{2}$. or if the circumference of the tower can be taken equals S a + radius.

PRACTICAL EXAMPLE OF CALCULATIONS ON THE REDUCTION OF ANGLES TO THE CENTRE.

EXAMPLE 1.—Determining the angle made at the centre of the spire of Highgate Church, between the spire of Hampstead Church and St. Paul's, from angles taken in the Church-yard, at Highgate, between the three churches of Hampstead, Highgate, and St. Paul's.

FIELD NOTES.

from △ S	2·56	to Highgate Spire △ B
	156° 16	and Highgate Church △ E
between △ F St. Paul's at △ S	85° 25′	and Hampstead Church △ D



Let x = the unknown angle DEF.
Now, by the preceding formula,
$\angle$ DEF + $\angle$ EDS = $\angle$ EFS + $\angle$ DSF.
$\therefore$ $\angle$ DEF or $x = \angle$ DSF + $\angle$ EFS $-\angle$ EDS,

but $\sin. \angle \text{EFS} = \text{ES}. (\frac{\sin. \angle \text{ESF}}{\text{EF}})$

where EF is the distance from St. Paul's to Highgate Church;

and the $\sin. \angle \text{EDS} = \text{ES} (\frac{\sin. \angle \text{ESD}}{\text{DE}})$.

where DE is the distance from Hampstead to Highgate Church.

Hence the values of the angles EFS, and EDS can be obtained; and, by adding their difference to the observed angle DSF, you obtain the value of the corrected angle DEF, from the centre of the previous point of observation.

Before these calculations are proceeded with it will be necessary to determine *the distances EF and DE,* which are at present unknown, from the data given in a preceding chapter.

In the triangle ADB (see Diagram p. 106), *to find the distance DB.*

As sin. $\angle$ ADB : AB : : sin. $\angle$ DAB : DB

(The angle ADB is the supplement of the observed angles DAB and DBA).

DB = 74·50 chs.

In the triangle EDB, to find ED.

Having calculated the length of DB, we have two sides and the included angle, BE having already obtained (See. p. 107).

$$\therefore \text{As DB+BE : DB} \backsim \text{BE} :: \tan. \frac{x+y}{2} : \tan. \frac{x-y}{2}$$

$$\tan. \frac{x-y}{2} = 1^\circ\ 30'\ 18''$$

$$\begin{array}{rrl} \text{now because} & 19^\circ\ 41' & = \tfrac{1}{2} \text{ sum} \\ \text{and} & 1^\circ\ 30'\ 18'' & = \tfrac{1}{2} \text{ diff.} \\ \hline \therefore & 21^\circ\ 11'\ 18'' & = \text{greater angle DEB.} \\ \text{and} & 18^\circ\ 10'\ 42'' & = \text{smaller angle EDB.} \end{array}$$

Again, in the same triangle, to find ED,

$$\text{As sin.} \angle \text{EDB} : \text{EB} :: \text{sin.} \angle \text{EBD} : \text{ED}$$

$$\text{ED} = 125{\cdot}36 \text{ chs.}$$

Now substitute their proper values in the two equations, viz.,

$$\text{sin.} \angle \text{EFS} = \text{ES} \left(\frac{\text{sin.} \angle \text{ESF}}{\text{EF}} \right).$$

and $\text{sin.} \angle \text{EDS} = \text{ES.} \left(\frac{\text{sin.} \angle \text{ESD}}{\text{DE}} \right)$, thus:-

$$\text{log. sin.} \angle \text{EFS} = \text{log. ES} + \text{log. sin.} \angle \text{ESF} - \text{log. EF},$$

and $\text{log. sin.} \angle \text{EDS} = \text{log. ES} + \text{log. sin.} \angle \text{ESD} - \text{log. DE}$,

log. ES, 2·56 chs.	= 0·4082400
+ log. sin. $\angle$ ESF, (156° 16′)	= 9·6030166
	10·0112566
— log. EF,	— 2·5468246
log. sin. $\angle$ *EFS* (0° 10′ 1″)	= 7·4644320
again log. ES 2·56	= 0·4082400
+ log. sin. $\angle$ ESD (70° 51′)	= 9·9755394
	10·3837794
— log. DE, 125·36 chains	= 2·0981630
log. sin. EDS (1° 6′ 22″)	= 8·2856159

but $\angle \text{DEF}$ or $x = \angle \text{DSF} + (\angle \text{EFS} - \angle \text{EDS})$

$\angle \text{EFS} = 0° \; 10' \; 1''$

subtract $\angle \text{EDS} = 1° \; 6' \; 22''$

their difference $= -56' \; 21''$

$\therefore \angle \text{DEF} = 85° \; 25'' + (-56' \; 51'') = (84° \; 28' \; 39') = x$

which is the angle at the spire of Highgate Church, between Hampstead Church and St. Paul's.

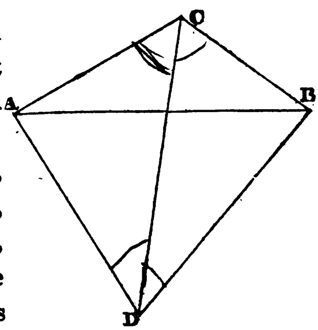
Example 2.—Having measured the distances between two objects AB 2 miles, 6 furlongs, and taken the angles at the base line, to a third object, viz., 59° 25′ 40″, and 78° 24′ 20″. It was required to *observe* the remaining angle at the third station, as a check upon the other two angles; planted the theodolite at a distance of 3 chains from it, and found one of

the angles 80°. What must the other be, so that the previous observation should be correct? Ans. 42° 3′ 25″.

CHAP. X.

ON TRIANGULATION.

In extensive surveys, carried on by a continued system of triangulation, the most important part is the proper selection of a base line, proportionate to the intended extent of the survey. This base line should be measured on a nearly level surface of country. In hilly countries the plane of a valley must be selected for that purpose.



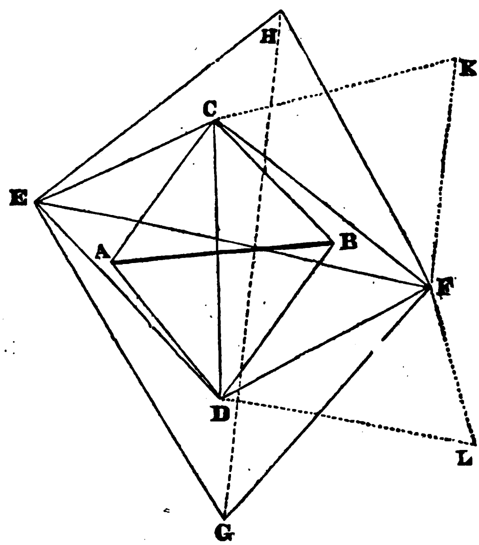
Let AB be the base, measured in a valley, and CD, two prominent objects, on opposite hills, which are visible from A and B.

Take at C, the angles ACD, BCD; and at D, the angles ADC, BDC; and the stations, C and D, are determined, by the formulæ given in Chap. VI., Prob. I. DC thus becomes a new base line, of longer extent and equal accuracy with AB; or, by taking at A and B, when B is visible from A, the angles between the base, and C and D respectively; C and D can be more directly and more accurately determined, as in either triangle its length will be the same. The three angles would, in all cases, be taken, if practicable, as from their sum,

which should be equal to 180 degrees, the accuracy of the work can be tested.

The sides of the triangle should be, as nearly equilateral as possible, or the angle at the new stations should not differ, materially, from 90 degrees. All the sides should be calculated and plotted from their determined lengths, and not protracted from their angles, as the smallest error of an angle would be of injurious effect, in determining the position of the new stations.

The accompanying diagram will exhibit the method of extending a system of triangulation, and of obtaining, between inaccessible stations, a base line, commensurate with the extent of the survey.



Let GH be the base line required, and inaccessible; AB, the only favourable base line that can be measured.

By determining from AB the position of C and D, CD becomes a new base; from CD determine E and F, EF again becomes a new base for the stations G and H, by which means the line GH, by observing the angles EHF and EGF, and

comparing their calculated with their observed values, becomes a base, as much to be depended upon as the first AB.

It is not, of course, necessary that the triangulation should be carried on in the regular manner, exhibited in the figure, as it might be branched off in any direction that may be required; for after having, from CD, determined E and F, CF might be taken as the new base, as correctly as DC, and the triangulation extended towards K; or, DF being taken as the base, it might be extended towards L; and by a similar process, in any other direction whatever.

Having carried the triangulation in the direction, and to the extent required, it becomes desirable, for the sake of testing the accuracy of the work, to make one of the lines, of the last triangles, *a base of verification*, by selecting for it a level position, along the slope of a hill, or in the bottom of a valley. This base, as it can be computed trigonometrically, being compared with its actual length by measurement, is a test, not only of its own accuracy, but of all the various triangles that subserve to its determination.

As the triangulation goes on, the sides increase in length, and the angles taken are between objects of some miles distant. It becomes then imperative to call in the aid of science to make the objects distinct. For this purpose, various contrivances have been, at different times, adopted, such as plane mirrors, disks of tin, plane convex lenses, parabolic reflectors, to receive the rays of artificial light, that were thrown upon them, by means of Argand lights, balls of burning lime, &c. These latter were introduced by Captain Drummond, who managed to send through a flame of alcohol, a powerful stream of oxygen gas, upon the lime, which was placed in the focus of a parabolic reflector.

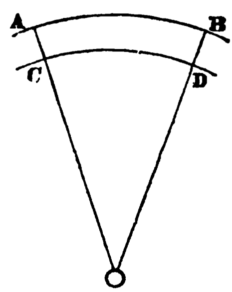
By these means, angles have been taken to objects from 40 to 60 miles off.

CHAP. XI.

REDUCTION OF A BASE LINE, TO THE LEVEL OF THE SEA.

(*It is only in very large trigonometrical surveys that this reduction is necessary.*)

Let CD be the length of the line at the level of the sea $= x$, AB, (L) the measured line above it. Let AC be the height of the measured line above the level of the sea $= h$, and CO be the radius of the earth $=$ R. It is required to find the length of CD.



Because the arcs of different circles, subtending the same angle, are proportional to their radii, we have,

$$AO : CO :: AB : CD, \text{ or}$$
$$R+h : R :: L : x$$
$$\therefore\ x = \frac{R}{R+h}L.\text{; and } \therefore\ L - x = L - \frac{R}{R+h}.L = \frac{h}{R+h}.L$$

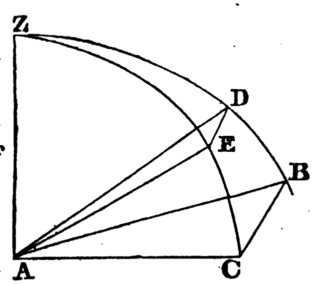
Or the *excess* of any line AB, measured above the level of the sea, is equal to the length of the line, into its height above that level, divided by its distance from the centre of the earth. Now the radius of the earth is 21,008,000 feet, therefore log. L in feet + log. h in feet—log. (21,008,000+h) in feet = log. excess of measured base above the true base at the level of the sea. This base is the arc CD. Where the triangulation is of some extent—as the distances thus obtained must be considered arcs of the circumference of the earth, the lengths of which would vary according to their radii or distances from the earth's centre,—these radii have to be brought down to the

mean radius or level of the sea, and the distances reduced accordingly. The heights of the measured lines above that level are determined by the mountain barometer.

CHAP. XII.

OBLIQUE ANGLES.

When angles have been taken by the Sextant they are taken in an oblique plane—before they can be used in calculation they must be reduced to the horizontal plane.



Let DAE, the angle taken at the point A, between the objects D and E, be in a plane, inclined to the horizon. Let BAC be the horizontal plane, and Z the zenith of the observer at A, then AZEC, AZDB will be portions of planes, of large circles, passing through the radius AZ, and the horizontal angle will be BAC.

Let CAE be the angle of elevation of the station E, and DAB that of D, of which EC and DB are the measures; ZE and ZD are the zenith distances, or measures of the complements of these angles of elevation; and DE is the measure of the observed angle DAE, as given by the Sextant, CB being its true measure. The angle BAC is the measure of the angle Z, or the plane angle made between the two planes. In the triangle ZDE, there are ZE, ZD, and DE given, hence the sine $\frac{1}{2}$ BAC or $\frac{1}{2}$DZE $= \sqrt{\frac{\text{sin. } \frac{1}{2}\text{ (S}-\text{ZE) sin. } \frac{1}{2}\text{ (S}-\text{ZD)}}{\text{sin. ZE sin. ZD}}}$

S, being the sum of the sides of the triangle ZED; or sin. $\frac{1}{2}$ Z

$$=\sqrt{\frac{\sin.\frac{1}{2}(DAE+BAD-CAE).\sin.\frac{1}{2}(DAE+CAE-BAD)}{\text{cosine } BAD.\ \text{cosine } CAE}}$$

If the angle BAD = angle CAE, or the objects be of the same altitude, then,

$$\sin.\tfrac{1}{2}Z=\sqrt{\frac{\sin.^2\frac{1}{2}\ DAE}{\cos.^2\ CAE}}=\frac{\sin.\frac{1}{2}\ DAE}{\cos.\ CAE}$$

In the first case, when the angles of elevation differ slightly, and when each of them is but small, not exceeding 2° or 3°, as the cosines of angles vary slowly, the following formula may be safely adopted, viz., $\sin.\frac{1}{2}Z=\frac{\sin.\frac{1}{2}DAE}{\cos.\frac{1}{2}(H+h)}$, where H and h are the respective angles of elevation, which becomes a convenient logarithmic formula, viz.,

$$\log.\sin.\tfrac{1}{2}Z=10+\log.\sin.\tfrac{1}{2}DAE-\log.\cos.\tfrac{1}{2}(H+h).$$

Spherical Excess.

The angles taken between any three points on the surface of the earth, by a theodolite, are, strictly speaking, spherical angles, and their sum must exceed 180°; and the lines bounding them, are not the chords, as they should be, but the tangents to the earth.

This excess is inappreciable in common cases, but in the larger triangles it becomes necessary to allow for it, and to *diminish* each of the angles of the observed triangle, by one third of the spherical excess.

To calculate this excess.

Divide the area of the triangle in feet, by the radius of the earth in seconds, and the quotient is the excess, viz.,

$$\text{excess}=\frac{\text{Area in feet}}{\text{Rad. in seconds}}$$

because the radius = as above, 21,008,000 feet, and one second of space = 101·43 feet, then (101·43 feet)^2 = the area in a square second, and radius = 206, 264 seconds of space

$\therefore$ excess in seconds $= \dfrac{\text{area in square feet}}{(101\cdot43)^2 \times 206264}$ or,

log. excess $=$ log. area $-$ 9·3267737.

When two sides and the included angle are given.

$$\text{cot. } \tfrac{1}{2} \text{ excess} = \frac{\text{cot. } \frac{1}{2} a - \text{cot. } \frac{1}{2} b + \text{cot. C.}}{\text{cos. C.}}$$

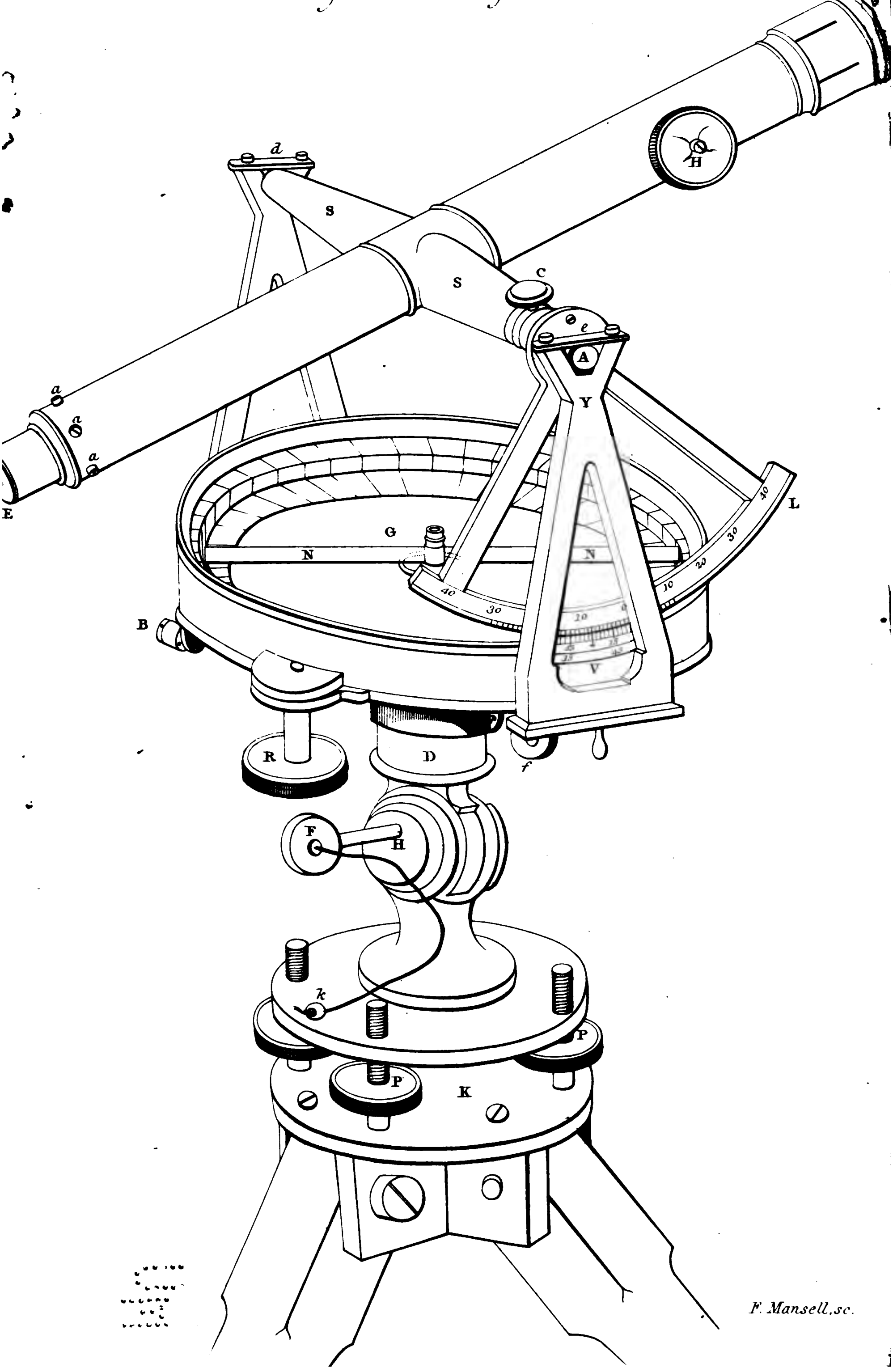
When the three sides are given.

$$\text{tan. } \tfrac{1}{4} \text{ excess} = \left(\frac{\text{tan. } a+b+c}{4}\right) . \left(\frac{\text{tan. } a+b-c}{4}\right) . \left(\frac{\text{tan. } a-b+c}{4}\right) . \left(\frac{\text{tan. } b+c-a}{4}\right)$$

To allow for this excess.

In any triangle take the three angles, find their sum. Calculate from these angles the spherical excess, by the above rule; the sum of the angles taken should amount to 180° + this spherical excess; if not, the difference must be divided among the three observed angles, so as to make them, when thus corrected, equal to 180° + the excess; then subtract one third this excess from each of the angles, and their sum will be reduced to 180°, the correct measure of a plane triangle.

Streatfield's Circumferentor.



F. Mansell, sc.

LAND SURVEYING.

Part the Third.

CHAP. I.

THE CIRCUMFERENTOR.

THE Circumferentor is the mariner's compass differently divided, and furnished with sights, standing upon one or three legs, and capable of a horizontal motion, by means of the usual parallel plates, or a ball and socket. In new countries, where expedition is required, the Circumferentor is generally used with a ball and socket, and with one leg or staff, strongly shoed with iron. After taking the angle at any station, the head part is taken off, and is carried by the surveyor under his arm, the staff is seized in his hand, and the next station is proceeded to. The mould of the woods is soft, and easily penetrated; and as, for the purpose of blazing the trees (which, in American phraseology, means taking a slice off the bark of a tree, just cutting into the wood) the general direction alone is wanted, which may be blazed within 10 yards on either side of the line, there is no delay arising from the placing of flags and driving of stakes, the axemen, being generally in advance or close upon the heels of the surveyor, and the chain men overtaking him, before he has taken his next sight.

All the LINE trees, as trees are termed, which stand directly in the line, in addition to being *blazed*, are *witnessed* with three notches, and being marked where the line strikes them, become permanent boundary marks—of so permanent and distinctive a character, that the very year almost of their being blazed is distinguishable by an experienced eye. The scar seldom grows over entirely: never without a seam, which, if carefully cut, will, by the number of rings, that every year has added to it, show the exact year of the incision.

The three legged staff, however, is very useful; especially in old countries, or in the surveying of roads in frosty weather, and under various other circumstances, when the single staff cannot penetrate, or when walls intervene, upon which the short legs of the staff can be advantageously placed.

Streatfield's Circumferentor.

This instrument consists of a compass box (G); divided into degrees, and, by means of a vernier, subdivided into three minutes.

It stands upon three legs, and, by means of a pair of parallel plates (K*k*), is capable of a truly horizontal position, which is determined by a level, placed, so as not to interfere with the reading, under the compass box (the end of it alone can be seen in the accompanying plate at B).

This compass box (G), has an absolute horizontal motion round its centering at D, and is fastened by the clamp screw (a side of which is visible) at *p*.

When this is clamped, by detaching the pin *f*, which passes through the two plates of the compass box, the brass one, which, with the vernier attached, works round the inner one, on which are divided the degrees, is capable of a relative motion, and thus partakes of the character of the theodolite; this motion is communicated to it by a rack and pinion at R.

There is, however, one thing wanting in this instrument,

which is indispensable in a new country. There is no means of clamping the vernier, except when it is at zero of the dividing-plate. In running lines, of a given direction, through the woods of a new country, where the bearings are often odd minutes, it would be impossible, at every fresh station, to be altering the vernier.

In the circumferentors that are used in America, they have placed the vernier outside, and by that means have contrived to clamp it in any position; there is also a tangent-screw for fine adjustment, so that being properly adjusted for the odd minutes, or the variation of any kind, and clamped, the bearing of the full number of degrees need alone be referred to throughout the whole line. None but a practical man can be aware of the immense advantages resulting from this arrangement.

YY are two frames, or supports, capable of being taken on and off, on the Y's of which rests the arm (S*s*) of a telescope, so contrived, as to move in a truly vertical position, when the instrument is horizontal. The telescope has its usual adjustments for the object and eye-glasses, and for the line of collimation.

To one end of the cross piece of the telescope is attached a graduated arc of a circle, with a vernier fastened to the supports of the Y, for taking vertical angles. This instrument has also, in addition to the telescope (not shown in the plate), the usual pair of sights, whose position would be, in the meridian line of the instrument, in the same plane as the telescope moves in.

The introduction of the telescope was forced upon me, from the constant inconvenience I experienced in the back woods of America (where I was engaged in government surveys), from the almost uselessness of the common sights, in surveying up and down hills, across steep but narrow vallies; all these difficulties were remedied by this contrivance, the advantages which I have tested practically in the woods. The telescope is capable of a vertical motion either way of 45 degrees, and

moving, truly vertically, enables you to carry a line of a given direction either down or up a hill, and from its magnifying power to secure, at the same time, a check position at the top of the ascent beyond; so that, after having descended and ascended, you may prove your correctness.

The instrument, also, has other contrivances; the pin F, when taken out, allows the whole instrument to be turned upon its side, and the spirit level at B, being then in a horizontal position, the instrument is made capable of a vertical motion, reading off to three minutes, by means of the vernier.

This may sometimes become a fair substitute for a sextant on a clear night; though I should myself, in all cases, prefer each instrument being kept to its especial purposes, as, the more simple an instrument is, the more accurate.

Division of the Circumferentor.

The line of sights is made the north and south end of the *instrument*, and from each of these the circular rim is graduated toward the east and west points, from 0° to 90°, *the west being on the right and the east on the left, looking northwards.*

When the needle is released, and is allowed to play freely, it points towards the magnetic north. The north of the instrument points to the object whose bearing is required, the angle made between these two must necessarily give the relative position of the line of the object, and the magnetic north and south line; and the bearing of the object, by reading off the number of degrees to which the needle points on the graduated circular rim, is thus obtained.

As the needle points to the north, should the object bear to the west, the line of sights would be on the left of the needle, and the north end of the needle would be on the right of the line of sights; if, therefore, on the right of the instrumental north, looking northwards, were marked east, the needle, which should at once, with the number of degrees, read off

the bearing, would give east, but the bearing is west. But if, as was shown above, it was marked west, the needle would read west, that is, north so many degrees west. By this arrangement the needle gives at once the degrees and bearing.

As the accuracy of the bearing depends, in a considerable degree, upon the goodness of the needle, great care should be observed in using it, and in marking whether it continues vibrating, or soon settles.

In the latter case there is some radical defect, arising either from the diminution of magnetic power in the needle, or from the wearing away of the centre on which it plays; this must be corrected immediately.

A really good needle is actually wearisome in its tardiness to settle.

Definitions.

Meridian lines are due north or south lines, and strictly considered, are arcs of large circles of the earth meeting at the poles; these arcs, however, are subtended by so small an angle, and are so infinitely small in comparison of the whole circle, as to admit of their being assumed as parallel.

The distance of a line is its horizontal measurement, as a tangent to the earth, not following the surface of the ground, and is usually calculated in chains and links.

The angle of bearing of any line, is the angle of bearing made between that line and a meridian line running through the point, where the instrument is placed; and is measured always from north or south, eastward or westward.

The *reverse bearing* of a line, is merely the bearing taken in a contrary direction, but measured by the same angle.

Difference of latitude, or *northing* and *southing,* is the distance that the end of the line is further north or south than the beginning.

The difference of longitude, or departure, is the distance, that the end of the line is east or west from the beginning.

The meridian distance of any station, is the distance of that station from the meridian line passing through the first or any other assumed point, and is equal to the difference between the sums of the eastings and westings from that point; and is east or west, as the eastings or westings predominate.

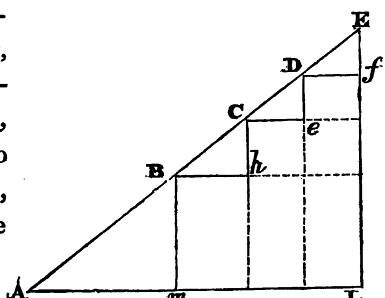
THE TRAVERSE TABLE.

Is a table giving the latitudes and departures to any distance and bearing, to the extent of the number of minutes to which it may be calculated.

The tables, which are appended to the present work, are calculated to every three minutes of the quadrant, and to every number from 0 to 10, from 10 to 100, &c. They have been prepared upon the following principle:

The whole northing or southing of a line, of any bearing, is equal to the sum of the northings and southings of any number of lines of the same bearing, when the sum of the several distances is equal to the one distance of the whole line.

Let AE be a line, having any bearing whatever, through E draw the meridian line EL; and, from A, draw AL perpendicular to EL. AL is the departure, and EL is the latitude of the line AE.



Now, divide AE into AB, BC, CD, DE, and through B, C, D, draw the several meridian lines, B*m*, C*h*, D*e*, and B*h*, C*e*, D*f*, at right angles to them.

B*m*, C*h*, D*e*, E*f*, are the latitudes of the several parts of the line AE, and are together equal to EL, that is, the sum of the northings of the parts is equal to the one northing of the whole.

And, in the same way, Am, Bh, Ce, Df, the sum of the eastings, are together equal to AL, the one easting of the whole line AE.

Again, if AE be a multiple of AB, (by similar triangles), AL is the same multiple of Am, and EL is the same multiple of Bm, being but sines and cosines of similar arcs to different radii.

If Bm, therefore, be the northing of a line, whose distance is AB, or one chain, then the northing of a line AE, which is ten times AB, or 10 chains, is LE, or ten times Bm.

This being premised, the understanding of the tables of latitudes and departures, or traverse table, becomes very simple.

EXAMPLE.—Let it be required to find the latitude and departure of a line bearing north 4° 3′ east, and 15 chains 25 links long.

Look in the margin of the pages for the degrees, and down the column on the left hand for the minutes, then on the same line, colaterally which the three minutes, will be found the latitude and departure for any number of links or chains.

Take the latitude first.—For the 15 chains, look out in the tables for the latitude of 1·00 chains	=	·998	
multiply by		10	
and you have			9·98
the latitude of 5·00 chains	=		4·99
For the 25 links, look out in the latter for the latitude of 2·00 chains	=	1·99	
divide by		10	
(because each 10 links are one tenth of a chain)			
and you have			0·19
the latitude of 5·00 chains	=	4·99	
divide by		100	
(each link being one hundredth of the chain)	=		0·05
Add them together, and you obtain, as the one			——
northing of the whole line;			15·21

Or, by supposing the number at the top of the page not to be confined to 1 chain, 2 chains, &c., for it is not necessary that

they should be so limited, but to be 1 chain or 10 chains, 2 or 20, 3 or 30 chains, &c.; or 10 links or 1 link, 20 or 2 links, 30 or 3 links, &c., a more expeditious method is used, in obtaining the same results.

To find the latitude of 15·25 *chains to the angle of bearing of* 4 *degrees* 3 *minutes.*

Look in the tables as before, and for one chain you find 0·998, that is some decimal less than 1; instead of 1 chain, let this be 10 chains, then the northing becomes 9·98, or some decimal less than 10 chains; therefore the latitude.

of 10 chains	=	9·98
of 5 chains	=	4·98
		14·96

Of 20 links, by looking in the tables under the head of 2, you find 1·99, that is, as before, some decimal less than 2. This may be 20 as well as 2, and 20 chains or 20 links indifferently. The latitude, therefore, of 15 chs. = 14·960

of 20 links	=	0·199
of 5 links	=	0·50
the total of which is as before.		15·21

which is the latitude for 15 chains 25 links, at a bearing of 4 degrees and 3 minutes.

This is north latitude, because the given bearing is north.

To find the Eastings or Westings

Look in the same place as before, under the head of degrees, and you will find that, at a bearing of 4 degrees, 3 minutes, the departure of 1 chain = ·071

and (by the first method) by multiplying by		10
the departure of chains	=	0·710
of 5 chains	=	0·353
of 2 chains = 0·141, dividing by 10	=	0·014
of 5 chains = 0·353, dividing by 100	=	0·003
total, east departure,		1·080

which is the required departure of the whole line.

Examples.—Required the difference of latitude and departure of a line, which bears south 16 degrees 30 minutes east, 3 chains 47 links. *Ans.* 3·33 south lat.; 0·98 east dep.

Given a line, bearing north 13 degrees 30 minutes west, and 6 chains 10 links long, to find the latitude and departure.

Ans. 5·93 north lat.; 1·42 west dep.

What are the latitude and the departure of a line bearing north 41 degrees 9 minutes east, 4 chains 47 links?

Ans. 3·36 north lat.; 2·95 east dep.

A line bears north 22 degrees 45 minutes west, 27 chains 62 links, required its latitude and departure.

Ans. 25·47 north lat.; 10·68 west dep.

These tables are found to be extremely useful; they enable you quickly to test the accuracy of the survey of a large extent of country; for, as the given distances and bearings are all capable of being resolved into their respective latitudes and departures, and as you cannot, from any point, go northwards without (to return to that point) coming back the same distance southward, nor any distance eastward, without remeasuring westward the same distance back, it follows, that, in going completely round a tract of country, the sum of the northings must be equal to the sum of the southings, and the sum of the eastings to that of the westings.

To find the bearing of a line.

Plant the instrument over one end of it, level it, set the needle free, then turn the sights towards the other end, taking care to look through the *south* sight. The angle denoted by the *north* end of the needle, when it has ceased playing, will be the angle of bearing. In case of its being requisite to look through the *north* sight, still read off by the *north* end of the needle, but insert the *reverse* bearing; as it seldom happens that both ends of the needle read alike, it is not safe to read by them indifferently.

Should the needle not point to a full degree, set it to the next lower; this will remove the sights from the object: clamp the instrument; release the upper plate, and by means of the rack and pinion, as in the Theodolite, bring the sights to cover; this small angle of minutes will be given on the vernier.—There will be no occasion to repeat the mode of doing this, as it has been fully explained in the theodolite.

The vernier of the circumferentor and the vertical one of the theodolite are upon the same principle.

Having taken the angle, the two plates must again be brought to the proper position, by making the broad arrow of the vernier coincide with the 360°, where the connecting pin keeps them, till they are again required to be separated. The needle must be fixed, and the whole instrument clamped. It might not, perhaps, be out of place to observe here, that the kind of surveying required in a new country, as Australia or New Zealend, is directly the reverse of that which must be in use here. Here a representation is, in most cases, wanted on paper, of actual boundaries of properties in the field: there, various lines, or certain positions, are to be marked down upon the ground. Here, the course of the hedge of a field is required: there, lines have to be run of given courses. The circumferentor, as constructed in this country, answers exceedingly well for the nature of the country it is generally used for here; but, when wanted to run a particular course, the objection, I have before mentioned, holds specially against it.

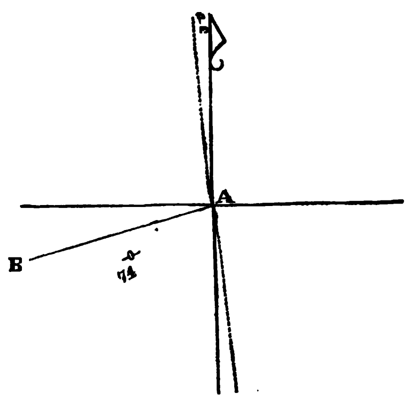
Having measured the distance to the next station, before taking the forward angle of the second line, take carefully the reverse bearing of the first, this will verify the last forward bearing, as the two should be the same; it also prevents, in a great measure, the probability of the needle being acted upon, without detection, by any magnetic substance in the neighbourhood; and I would especially recommend the young beginner invariably to adopt it. It is anything but a loss of time, and no survey can be depended upon without it.

Variation.

Before proceeding, it were better here to observe, that the magnetic meridian may not be the true meridian. The pole-star is not exactly at the pole; the needle seldom points either to the one or the other; two needles seldom point exactly the same way; and the same needle seldom points exactly the same way two years together. There are, therefore, two or three kinds of variation. The variation of the needle, strictly so termed, is the angle made between the magnetic north and the true north, and the variation is called east or west, according as the magnetic north is on the east or west side of the true north.

The measure of this angle, or the variation, is different in different countries, at the same time; and in the same country at different times.

Given AB, S. 74° W. variation 3° East; required the true bearing.



AB bears south 74 degrees west; draw any line AB (see diagram). At A lay off an angle of 74 degrees, this will give you the meridian line, which reversed will give the magnetic north. This needle is however 3 degrees east of the true north;

the true north, is therefore, 3 degrees west of it, lay off an angle of 3 degrees to the west and produce it southwards.

This line being on the left of the magnetic meridian in the northern hemisphere, will be on the right in the southern, and the angle of true bearing will consequently be the angle of magnetic bearing, plus the variation, viz. $74+3=77$; that is, S. 77° W.

2. Given AB, N. 85° 17′ W.; variation 24° W., required the true bearing, Ans. South 70° 43′ W.

In the same manner may be determined the angle between any two lines when their bearings are given. In the first example, if the variation be 3 degrees east, the bearing of the true north by that needle will be N. 3° W., and by the same needle the bearing of the other line is S. 74° W., the angle between them is required. This is a special case, but the general principle is the same in all.

The best practical method therefore, in every case, is to draw a north and south line, and from the same point, lay off the two lines in their given directions in relation to it; their position will at once shew the angle between them.

CHAP. II.

MODE OF SURVEYING WITH THE CIRCUMFERENTOR.

This instrument is very useful in obtaining approximately the shape and contents of any large irregular block of land; in the surveying of woods and lakes; in running boundary lines through forests in new countries, where the property is not sufficiently valuable to pay for any great accuracy.

This instrument, also, might, in good hands be of invaluable

assistance upon a railway survey for parliamentary deposit. It is very expeditious, and though never perfectly correct, is seldom far wrong.

Place flags at each of the corners of the block you are required to survey, and take the several bearings of each, from A to B, from B to C, &c., and measure AB, BC, CD, taking the necessary offsets upon each.

Observe, it is not necessary for any purpose of this kind, to know the magnetic variation, as whatever be the variation, all the lines of the survey will still lie in their relative positions to each other, and the area will not be changed.

For fear, however, of any local attraction, or any accidental mis-reading of an angle, it will be at all times indispensable when you get to B, to take the backward or reverse reading to A, and compare it with the forward bearing. The same must be done with every line.

It must be mentioned here, that the usual check upon the interior angles of any rectilineal figure will not hold good, as the sum of the interior angles of any figure obtained from the bearings of its sides, whether those angles have been taken correctly or not, will always amount to 4 less than twice as many angles as the figure has sides. Some other check, therefore, must be adopted. The principle referred to in the remarks at the end of the chapter on the traverse table will now be found useful.

Resolve the latitudes and departures into their northings and southings, and eastings and westings, by the TRAVERSE TABLE, then add the northings together, and the southings, and see if they agree, do the same with the eastings and westings. Should these be found to agree within a certain limit, the field work is correct, and you may proceed to plot the figure or calculate the area by the traverse table.

This allowable error should not exceed one link to every five chains of the sum total of distances. Beyond this a resurvey becomes necessary.

EXAMPLE 1.—Given the following bearings and distances, viz., No. 1. S. 16° 30′ E., 3 chains 47 links; 2d. S. 45° E., 4 chains 50 links; 3d. S. 26° E., 5 chains 51 links; 4th. S. 31° 30′ E., 7 chains 34 links; 5th. S. 5° 20′ E., 10 chains 55 links; 6th. S. 15° E., 5 chains 9 links; 7th. N. 18° W., 4 chains 3 links; 8th. N. 13° 30′ E., 4 chains 70 links; 9th. N. 45° 30′ W., 6 chains 50 links; 10th. N. 64° 45′ W., 7 chains 34 links; and 11th. N. 4° 19′ W., 17 chains 11 links to the place of beginning, to find their northings and southings, and eastings and westings, and never determine their correctness.

Stations.	Bearings.	Distances.	Latitude. N.	Latitude. S.	Departure. E.	Departure. W.
		Chains.				
1	S. 16° 30′ E.	3·47		3·33	0·99	
2	S. 45° 0 E.	4·50		3·18	3·18.	
3	S. 26° 0 E.	5·51		4·95	2·41	
4	S. 31° 30′ E.	7·34		6·26	3·83	
5	S. 5° 20′ E.	10·55		10·51	0·98	
6	S. 15° 0′ E	5·09		4·92	1·32	
7	N. 18° 0 W.	4·03	3·83			1·25
8	N. 13° 30′ E.	4·70	4·57		1·10	
9	N. 45° 30′ W.	6·50	4·56			4.63
10	N. 64° 45′ W.	7·34	3·13			6.64
11	N. 4° 19′ W.	17·11	17·06			1·29
		76·14	33·15	33·15	13·81	13·81

The northings being the same as the southings; the eastings the same as the westings.

It is seldom, however, that they agree so exactly, as has been given in this example. As the sum total of the distances amounts to 76 chains 14 links, there would have been an allowable difference of 15 links (*i. e.* of one link in every five chains), between the northings and southings, and the eastings and westings.

In example 2 (*see below*) where a difference is found between both northings and southings, and eastings and westings, the error, if it does not exceed one link in every five chains of the sum total of distances, must be apportioned among each of the distances, by the following proportion; viz., as the sum total of the distances is the whole error, so is each distance to its correction.

This must be done for the latitudes and departures, and must be placed in a column appropriated to each, called the north and south correction, and the east and west correction; the correction, thus determined, must be placed, collaterally with the distance to which it refers, without distinguishing as to north, or south, east or west.

In making this proportion, the links of the distances need not be taken into account, and frequently, as an approximation is sufficient, the apportionment may be made without reference to calculation.

Having found the several corrections for each of the latitudes and departures, add them together severally, and see whether their total agrees with the whole error; then draw four other columns, heading them, corrected northings, southings, eastings, westings, and proceed to allot the corrections.

If the error be an excess of northings, substract each correction from its collateral northing, or add it to the collateral southing; if an excess of easting, add to the westing, and subtract from the easting; the respective sums of their corrected latitudes and departures will now be found exactly to agree.

Example. 2.—Given the following bearings and distances; viz., No. 1. N. 64° 20′ E., 18 chains 30 links; 2d. N. 68° 42′ E., 8 chains 29 links; 3d. N. 3° 45′ W., 4 chains 99 links; 4th. N. 76° E., 14 chains 50 links; 5th. S. 28° 15′ E., 7 chains 40 links; 6th. S. 72° W., 24 chains 33 links; and 7th. S. 73° 54′ W., 18 chains 90 links to the place of beginning. It is required to find their northings and southings, eastings and westings, and apportion the corrections for any excess of the same, if found to exist, among, them severally, according to the rule as given above.

EXAMPLE 2.

Stations.	Bearings.	Distances.	Latitude.		Departure.		Corr.	Corr.	Corrected Latitude.		Corrected Departure.	
			N.	S.	E.	W.	For Lat.	For Dep.	N.	S.	E.	W.
1	N. 64° 20′ E.	18·30	7·92		16·50		3	4	7·89		16·46	
2	N. 68° 42′ E.	8·29	3·01		7·73		1	2	3·00		7·71	
3	N. 3° 45′ W.	4·99	4·98			0·32	1	1	4·97			0·33
4	N. 76° E.	14·50	3·51		14·07		2	3	3·49		14·04	
5	S. 28° 15′ E.	7·40		6·52	3·50		1	1		6·53	3·49	
6	S. 72° W.	24·33		7·52		23·13	4	5		7·56		23·18
7	S. 73° 54′ W.	18·90		5·23		18·16	3	3		5·26		18·19
		96·71	19·42	19·27	41·80	41·61	15	19	19·35	19·35	41·7	41·70
			19·27		41·61							

Deficiency ·15 S. ·19 W.

$\frac{15}{97} \times 10 = \frac{150}{97} = 1\frac{1}{2}$ link to every 10 chains for correction for latitude.

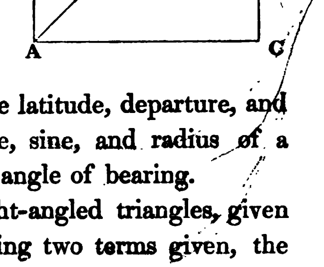
And $\frac{19}{97} \times 10 = \frac{190}{97} = 2$ links to every 10 chains for correction for departure.

CHAP. III.

ERRORS OR OMISSIONS OF SURVEY.

As it is necessary, for the correction of any errors, or supplying any omissions of survey, when any two of the following data, viz., bearing, distance, difference of latitude, and of departure are given, to understand the method of determining the other two; we will first briefly say a few words on the subject.

Any two of the above data being given, to determine the other two.



Let AB be the distance of the given line; DAB or ABC is the angle of bearing; then AD or CB is the difference of latitude, and AC or DB is the departure; but BC is the cosine of the angle DAB, which is the angle of bearing; and AC or DB, is the sine also, of the same angle of bearing; therefore, the latitude, departure, and distance, are respectively the cosine, sine, and radius of a circle, whose arc is measured by the angle of bearing.

By the rules for the cases of right-angled triangles, given in any work on Trigonometry, having two terms given, the rest can be found.

Example 1.—A line bears N. 45° E., 31 chains 40 links; required its difference of latitude and departure.

As sin. 90° or rad. : 31·41 chs. :: sin. 45° : 22·20 chs.

∴ 22·20 chs = diff. of latitude, but sine 45° = cos. 45°,
∴ departure = latitude = 22·20 chains.

Example 2.—The difference of latitude of a line was 4 chains 40 links N.; the departure was 25° W.; required the distance of the line.

Ans. 25·57 chains.

Example 3.—A line bears N. 72° W.; its departure is 24 chains 58 links: what is its length?

Ans. 25·84 chains.

Example 4.—A line is 38 chains 45 links in length, and its departure is 14 chains W.; what is its angle of bearing, the same being between the north and west?

As 38·45 chains : rad. : : 14 chains : sin. ∠ of bearing.
∴ sin. ∠ of bearing = 21° 20′. Ans. N. 21° 20′ W.

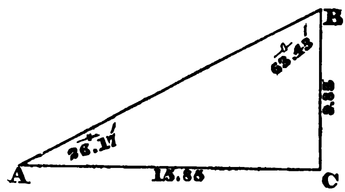
FIRST OMISSION.

When the length and bearing of any one line are omitted.

Rule.—Add up the northings and southings, and subtract them for the northing or southing of the unknown line; do the same with the eastings or westings to obtain its departure; then, by the preceding note, having the departure and latitude of a line given, the distance and bearing can be determined.

Stations.	Bearings.	Distances.	North Latitude.	South Latitude.	East Departure.	West Latitude.
		Chains.				
1	S. 49° 30′ E.	7·55		4·91	5·74	
2	S. 73° E.	8·82		2·58	8·43	
3	N. 6° 15′ E.	15·41	15·32		1·68	
4	S. W.					
			15·32	7·49	15·85	
			7·49			
Deficiency of Southing.			7·83		15·85	Deficiency of Westing.

Now, AC, the westing or base of the right-angled triangle, being given, 15 chains 85 links; and BC the southing or perpendicular, being 7 chains 83 links; the angle ABC, or the bearing of the line AB, and its distance, which are required, can be determined by the rules of the cases of right-angled triangles.



Thus: take AC as radius, and describe the circle CD; then CB becomes the tangent to angle BAC, and AB is the secant of the same angle; then say—

As 15·85 : radius : : 7·83 : tan. $\angle$ BAC$=$26° 17′.

Also, as radius : 15·85 : : sec. $\angle$ A : AB$=$17·65 chains.

but the angle of bearing$=$90°—$\angle$ BAC$=$90°—26° 17′$=$63° 45′.

Ans. S. 63° 45′ W.

Examples.

1.—Given the following bearings and distances of an old waggon road, through a new settlement, running from one concession road to another, viz.,—1st. N. 78° E., 2·20 chains; 2nd. N. 45° 30′ E., 14·80 chains; 3rd. N. 16° W., 21·75 chains; 4th. N. 68° 20′ E., 13·90 chains; 5th. N. 10° W., 15·60 chains; 6th. N. 70° E., 8·96 chains. It is required to lay out a new straight road, connecting the two ends of the present road. What will be its bearing and distance?

Ans. S. 24° 38′ W., 60·82 chains.

2.—Given the boundaries of a tract of land, as follows, viz.:—1st. S. 16° 30′ E., 3·47 chains; 2nd. S. 17° E., 3·02 chains; 3rd. S. 26° E., 5·51 chains; 4th. S. 31° 30′ E., 7·34 chains; 5th. S. 5° 20′ E., 10·55 chains; 6th. S. 15° E., 5·09 chains; 7th. S. 8° W., 4·03 chains; 8th. S. 3° 30′ E., 4·70 chains; 9th. S. 45° 30′ W., 6·50 chains; 10th. S. 64° 45′ W., 7·34 chains; 11th. to the place of beginning. Required the bearing and distance of this eleventh line.

Ans. N. 1° 21′ E., 49 chains 18 links.

SECOND OMISSION.

When the distance of two sides cannot be obtained.

RULE 1.—Find, by the preceding chapter, the *length* and *bearing* of the closing line, connecting the known sides of the survey. This line and the two unknown sides will form a triangle, having its base known, as well as the bearings of all its sides; with these data compute the several angles of the triangle, by the previous rule (page 136), and you have the three angles and one side given; whence the two required sides are determinable by the first case.

EXAMPLE 1. The boundaries of a tract of land were taken as follows: 1st. S. 16° 30′ E.; 2nd. N. 24° W.; 3d. S. 26° E., 5·60 chains; 4th. S. 31° 30′ E., 7·80 chains; 5th. N. 5° 20′ W., 8·50 chains; 6th. S. 15° E., 9·60 chains; 7th. N. 18° W., 4·85 chains; 8th. N. 30° W., 8·60 chains; 9th. N. 45° 30′ W., 6·48 chains; 10th. N. 64° 45′ W., 7·34 chains; 11th. N. 12° 30′ E., 20·16 chains, to the place of beginning. Required the distances of the two first stations, which, from local obstructions, could not be measured.

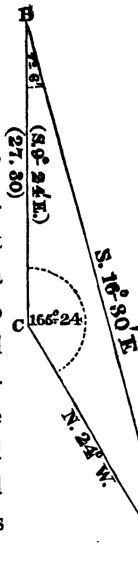
Now the first two lines are the unknown lines: let the following lines therefore be considered a portion of a block of land, the last line of which has been left out, which line it is required by the preceeding rule to discover. Resolve these lines into their respective latitudes and departures, and add them together, the northings will be found to amount to 47·89 and the southings to 20·95; making a deficiency of southings of 26·94: in the same way, there will be found a deficiency of 4·46 of eastings. The closing line consequently will bear south-east, so many degrees. Find the distance and departure of this line from these required data by the rule given in Chapter III. viz., south 9° 24′ east, 27 chains 30 links.

EXAMPLE THE FIRST.

Stations.	Bearings.	Distances.	Latitude. N.	Latitude. S.	Departure. E.	Departure. W.
		Chains.				
1	S. 16° 30′ E.	(52·72)		(50·55)	(14·97)	
2	N. 24° W.	(25·85)	(23·61)			(10·51)
3	S. 26° E.	5·60		5·03	2·45	
4	S. 31° 30′ E.	7·80		6·65	4·08	
5	N. 5° 20′ W.	8·50	8·46			0·79
6	S. 15° E.	9·60		9·27	2·49	
7	N. 18° W.	4·85	4·62			1·50
8	N. 30° W.	8·60	7·45			4·30
9	N. 45° 30′ W.	6·48	4·54			4·62
10	N. 64° 45′ W.	7·34	3·13			6·63
11	N. 12° 30′ E.	20·16	19·69		4·36	
			47·89		13·38	17·84
			20·95			13·38
		Deficiency	26·94	S.	Deficiency	4·46 E.

Ans. The Angle of Bearing of closing line is S. 9° 24′ E., and distance 27·30 chs., from which result:—
first distance = 52·72 chs.
and 2nd do. = 25·85 chs.

The conditions of the question will now be BC=27 chains 30 links, and its bearing S. 9 24′ E.; AB, S. 16° 30′ E., making the angle ABC, therefore 7° 6′; and AC bearing N. 24° W., and AB, S. 16° 30′ E., or N. 16° 30′ W., making the angle BAC, 7° 30′; and therefore, the remaining angle at C, 165° 24′; that is, the three angles and one of the sides of a triangle will be given to find the other two sides. The first distance from this triangle will be 52·72, and the second 25·85. Find the latitude and departure of these, and verify the work by the usual checks. The latitudes will be found to amount each to 71·50 chains, and the departures each to 28·35 chains, thus proving the work.



By means of a changed bearing.

Rule 2. Suppose the whole tract of land to revolve, until it becomes in such a position, as to have one of the unknown sides in a direct meridian line. This will not change the relative position of the sides, as, whatever angle of variation be taken for this line, the same will be taken for all; but, as in the first instance, there were but two lines, whose departures were unknown, so now, in consequence of one, from having been made due north and south, having no departure, that of the only one that remains unknown, is the measure of the difference of the sums of the eastings or westings.

The changing of the bearing is performed in the same way as correcting for the variation, viz.,—by bringing all the given angles to a *new* meridian, which makes so many degrees east or west, with the first meridian. (*See page* 135.)

Arrange, therefore, the bearings and distances, as in the following columns; change the bearings of each, so as to make the bearing of one of the unknown sides a meridian line. Place these changed bearings in their proper column; find the latitude and departure of these bearings, to their several distances. Find the sum of the eastings and the sum of the westings; their difference will be the departure of the second unknown side, in terms of the deficiency.

Calculate the latitude and distance of this second side, from its bearing and departure, which are known, and place them also in their proper column. (*See page* 141.)

Then add the northings together, and the southings together, and their difference will be the distance of the side, which was made a meridian.

Example.—Solve the preceding example, given in Rule 1, page 144, by using a changed bearing.

Stat.	Bearings.	Changed.	Dist.	N.	S.	E.	W.
1	S. 16° 30′ E.	South.	(52·72)		(52·72)		
2	N. 24° 0′ W.	N. 7° 30′ W.	(25·85)	(25·63)			(3·37)
3	S. 26° 0′ E.	S. 9° 30′ E.	5·60		5·52	0·92	
4	S. 31° 30′ E.	S. 15° 0′ E.	7·80		7·54	2·02	
5	N. 5° 20′ W.	N. 11° 10′ E.	8·50	8·34		1·65	
9	S. 15° 0′ E.	S. 1° 30′ W.	9·60		9·60		0·25
7	N. 18° 0′ W.	N. 1° 30′ W.	4·85	4·85			0·13
8	N. 30° 0′ W.	N. 13° 30′ W.	8·60	8·36			2·00
9	N. 45° 30′ W.	N. 29° 0′ W.	6·48	5·67			3·14
10	N. 64° 45′ W.	N. 48° 15′ W.	7·34	4·89			5·48
11	N. 12° 30′ E.	N. 29° 0′ E.	20·16	17·64		9·78	
				49·75	22·66	14·37	11·00
			Deficiency	of S.	52·72	of W.	3·37
				(75·38)	75·38		14·37

Make the first line a meridian line, by turning the southerly bearing S. 16° 30 E., 16° 30′ more to the west. Now, as the reverse of S. 16° 30′ E., is N. 16° 30′ W., so is the reverse of south, north; therefore, the turning a southerly bearing westward, will have the effect of turning a northerly one eastward,

All southerly bearings must therefore be brought 16° 30′ more westerly, and all northerly bearings 16° 30′ more easterly. The changed bearings will of course make no change in the distances.

Find the latitudes and departures, according to these changed bearings. The departure of the first one being due south, will of course be zero. They will therefore all have departures but the second.

The difference of the sums of the eastings and westings of the other lines, will be the departure of this second line, which will be found to be west 3·37 chains.

Now, as this second line has its bearing and its departure given, the latitude and therefore the distance, can be ascertained by the Rule in Chapter III. This distance will be 25·85 chs., and its departure 25·63 chains. This disposes of one line out of the two, the other has its departure known, being zero, its distance, therefore, is equal to its latitude, which is, by hypothesis, due south. The difference between the north and south latitudes will be the distance required. The sum of the northings is 75·38 chains, that of the southings 22·66 chains, leaving a deficiency of southing of 52·72 chains, which is the distance of the first line.

The two distances therefore are 52·72 chains, and 25·85 chains; the same results as before.

Example. 2.—The following bearings and distances were taken, viz.: at station 1., the bearing of 2nd station was, S. 71° 24′ W.; at station 3, the same point bore S. 46° 18′ E., the distances could not be obtained; at station 3, N. 15° 42′ E., 6·20 chains; at station 4, N. 52° 18′ E., 6·75 chains; at station 5, S. 78° 48′ E., 5·96 chains; at station 6, S. 5° 51′ E., 4·84 chs.; at station 7, S. 49° 15′ W., 4·75 chains; at station 8, S. 4° 57′ E., 3·98 chains, to the place of beginning.

What are the distances of the first and second lines?

Ans.: First distance 5·48 chains. Second distance 6·80 chains.

THIRD OMISSION.

When the bearings of any two sides of a tract of survey have been incorreetly taken.

ARRANGE the given bearings and distances in their proper columns, and find, as before, the difference of the northings and southings, and of the eastings and westings, for the latitude and departure of a closing line. The line, and the distances of the two unknown sides, form a triangle, with sufficient data to determine its inward angles; and thence, as the bearing of the closing line is known, the required bearings of the two other sides are obtained.

CHAP. IV.

TO FIND THE AREA.

When the bearings and distances of a tract of land are given.

Arrange the several bearings and distances as before, and find the respective latitudes and departures corresponding to each.

Determine, as in the preceding problems, the several corrections to each, for latitude and departure, and allow for these corrections, placing the amended latitudes and departures in their proper columns.

Now make five more columns, and head them—East Double Departure; West Double Departure; Multipliers; North Areas; and South Areas.

Take the sum or difference of every two consecutive departures (the first and last departures are as much consecutive

as any other two adjacent departures are, as they join each other upon the ground,—if this be borne in mind by the student, there will be no difficulty in the matter); adding them, if of the same kind, and subtracting them, if of different; and place this sum, or excess, in the column to which it belongs; of west in the column of W. D. D.; of east, in that of E. D. D.

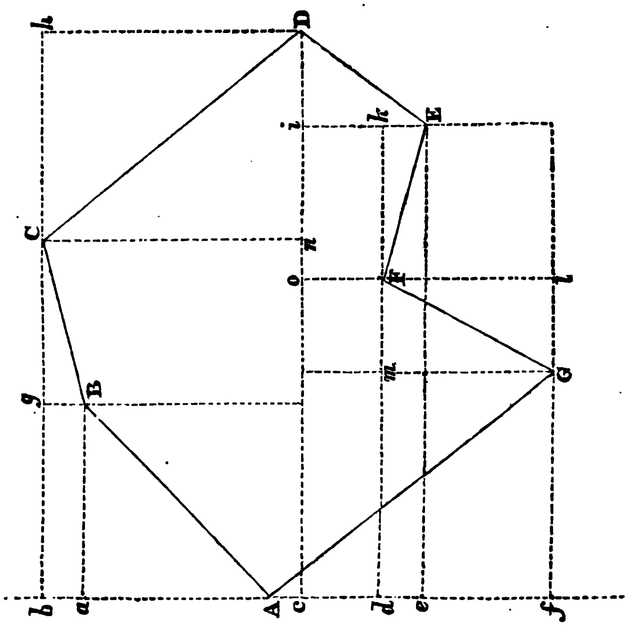
Commence at any station whatever of the survey, and assume any line, either close to, or at any distance from it, as a meridian or multiplying line, so called, because the meridian distances of all the stations, or perpendiculars, let fall from the several corners of the figure upon this line, become the several multipliers, which multiplied into their corresponding latitude, give the north and south areas. If this line is close to the figure, the first multiplier will be the first departure, place this, therefore, in the column of multipliers collateral with its corresponding departure, terming it either east or west, according as that departure is east or west. The sum of this assumed multiplier, and that of the double departure, corresponding to the next side, if they are of the same kind, or their difference, if of opposite kinds, in terms of the greater, will be the next multiplier; proceed with the second multiplier, and the following double departure, until multipliers have been found for all the sides. The last multiplier thus obtained, if the work be correct, will be found the same as the last departure.

Multiply each of the corrected differences of latitude by its collateral multiplier, and place the product in the north or south area columns, observing, that whenever the multipliers are of the same name as the assumed, those products are to be placed in the column of areas, which is of the same name as the latitude; if of different names, in the contrary column.

Half the difference, between the sum of the north and south areas will be the area of the survey.

Demonstration.

Let ABCDEFG be a tract of country whose area is required;



through A draw the meridian or multiplying line *fb*, and through the several points, B, C, D, E, F, and G, draw the meridian distances, B*a*, C*b*, D*c*, E*e*, F*d*, and G*f*, perpendicular to this meridian, and also *g*B, C*n*, D*h*, and C, parallel to it; *a*B will be the first multiplier.

Now the area of the figure, *b*CDEFG*f*, is equal to the figure *b*CB AG*f*+the required area of the tract. And the difference of latitude and departure of the several sides will be found as in the following table.

Thus A*a* is the northing of AB, and *a*B is its easting; *ab* is the northing of BC, and *g*C its easting, and so on.

Also, according to the rule, *a*B+*g*C or *b*C, is the E. D. D. of BC, as they are of the same kind, and *f*G—*a*B is the W.D.D. of *a*B.

Having obtained the double departures for the several sides, assume the multiplier *a*B, and find, by the rule, the several other multipliers *a*B+*b*C, &c., observing carefully, whether these multipliers are east or west. And place the areas of the product of the multipliers into their collateral northings or southings, in the columns of north or south areas, according to the proper relations between these multipliers and latitudes as given in the rule. These areas are double the required areas —being each of them double the areas of the several trapezoids into which the figure has been divided.

Half the difference, therefore, of the north and south areas will be the required area.

	N.	S.	E.	W.	E. D. D.	W. D. D.	Multipliers.		N. Areas.	S. Areas.
AB	A*a*		*a*B			*f*G—*a*B	*a* B	E	2 A*a*B	
BC	*ab*		*g*C		*b*C		*a*B+*b*C	E	2 *ab*CB	
CD		*bc*	C*h*		*gh*		*b*C+*c*D	E		2 *b*CD*c*
DE		*ce*		D*i*	*ni*		*c*D+*ci* (*e*E)	E		2 *c*DE*e*
EF	*ed*			*k*F		D*o*	*e*E+*d*F	E	2 *d*FE*e*	
FG		*df*		F*m*		*km*	*d*F+*dm*(*f*G)	E		2 *d*FG*f*
GA	A*f*			G*f*		*fl*	*dm*	E	2 AG*f*	

Now "*dm*" the multiplier last found equals G*f* or the last departure.

It will be perceived that all the multipliers, with the exception of the first and last, which are single, being the perpendiculars *a*B, *f*G of the triangles A*a*B, A*f*G, are double, being the sides of the trapezoids respectively, of which the other portion of the figure is made up—viz., the trapezoids B*ab*C; D*cb*C; and &c.

EXAMPLE 1.—Given the bearings and distances of a tract of country, to find the area, viz.:—1st. S. 16° 30′ E., 3·47 chains; 2nd. S. 17° E., 3.02 chains; 3rd. S. 26° E., 5·51 chains; 4th. S. 31° 30′ E., 7·34 chains; 5th. S. 5° 20′ E., 10·55 chains; 6th. S. 15° E., 5·09 chains; 7th. S. 8° W., 4·03 chains; 8th. S. 3° 30′ E., 4·70 chains; 9th. S. 45° 30′ W., 6·50 chains; 10th. S. 64° 45′ W., 7·34 chains; 11th. S. 60° 15′ W., 4·98 chains; 12th. N. 22° 45′ W., 27·62 chains; 13th. N. 67° E., 2·40 chains; 14th. N. 13° 30′ W., 6·10 chains; 15th. S. 62° 30′ W., 4·16 chains; 16th. N. 27° 30′ W., 8·24 chains; 17th. N. 41° 10′ E., 4·47 chains; 18th. N. 4° W., 16·40 chains; 19th. N. 84° E., 4·03 chains; 20th. S. 69° 30′ E., 18 21 chains —to the place of beginning.

Station.	Bearings.	Dist.	N. Lat.	S. Lat.	E.Long.	W.Long	Corrections of Latitude.	Cor. N.	Cor. S.	Cor. E.	Cor. W.	E. D. D.	W. D. D.	Multipliers.	N. Areas.	S. Areas.
1	S. 16° 30′ E.	3·47		3·33	0·98		—		3·33	0·98		18·04		0·98 E.		3·2634
2	S. 17° E.	3·02		2·89	0·89		—		2·89	0·89		1·87	—	2·85 E.		8·2365
3	S. 26° E.	5·51		4·95	2·41		1		4·94	2·41		3·30	—	6·15 E.		30·3810
4	S. 31° 30′ E.	7.34		6·26	3·84		1		6·25	3·84		6·25	—	12·40 E.		77·5000
5	S. 5° 20′ E.	10.55		10·51	0·97		2		10·49	0·97		4·81	—	17·21 E.		180·5329
6	S. 15° E.	5·09		4·92	1·31		1		4·91	1·31		2·28	—	19·49 E.		95·6939
7	S. 8° W.	4·03		3·99	—	0·56	—		3·99	—	0·56	0·75	—	20·24 E.		80·7576
8	S. 3° 30′ E.	4·70		4·67	0·28	—	—		4·69	0·28	—	—	0·28	19·96 E.		93·6124
9	S. 45°30′W.	6·50		4·56	—	4·64	1		4·55	—	4·64	—	4·36	15·60 E		70·9800
10	S. 64°45′W.	7·34		3·14	—	6·64	1		3·13	—	6·64	—	11·28	4·32 E.		13·5216
11	S. 60°15′W.	4·98		2·47	—	4·33	1		2·46	—	4·33	—	10·97	6·65 W.	16·3590	
12	N. 22°45′W.	27·62	25·47	—	—	10·68	5	25·52	—	—	10·68	—	15·01	21·66 W.		552·7632
13	N. 67° E.	2·40	0·94	—	—	—	—	0·94	—	2·21	—	—	8·47	30·13 W.		28·3222
14	N. 13°30′W.	6·10	5·93	—	—	1·42	1	5·94	—	—	1·42	0·79	—	29·34 W.		174·2796
15	S. 62°30′W.	4·16	—	1·92	—	3·69	—	—	1·92	—	3·69	—	5·11	34·45 W.	66·1440	
16	N. 27°30′W.	8·24	7·31	—	—	3·80	1	7·32	—	—	3.80	—	7·49	41·94 W.		307·0008
17	N. 41°10′E.	4·47	3·36	—	2·95	—	—	3·56	—	2·95	—	—	0·85	42·79 W.		152·3324
18	N. 4° W.	16·40	16·36	—	—	1·15	3	16·39	—	—	1·55	1·80	—	40·99 W.		671·8261
19	N. 84° E.	4·03	0·41	—	4·01	—	—	0·41	—	4·01	—	2·86	—	38·13 W.		15·6333
20	S. 69°30′E.	18·21	—	6·37	17·06	—	4	—	6·33	17·06	—	21·07	—	17·06 W.	107·9898	
		154·16	59·78	60·00	36·91	36·91	22	59·88	59·88	36·91	36·91				190·4928	2556·6389
				59·78												190·4928
				·22	deficiency of northings.										2	2366·1461
		5|154												A. R. P.	10	1183·0730
		31 links of allowable error.												Ans. = 118 1 9		118·3073

EXAMPLE 2.—Given the bearings and distances as follows, viz.,—S. 16° 30′ E., 3 chains 47 links; S. 45° E., 4 chains 50 links; S. 26° E., 5 chains 51 links; S. 31° 30 E., 7 chains 34 links; S. 5° 20′ E., 10 chains 55 links; S. 15° E., 5 chains 9 links; N. 18° W., 4 chains 3 links; N. 13° 30′ E., 4 chains 70 links; N. 45° 30′ W., 6 chains 50 links; N. 64° 45′ W., 7 chains 34 links; and N. 4° 18 W., 17 chains 11 links, to the place of beginning, to find the area.

A. R. P.
Answer, 9. 0. 3.

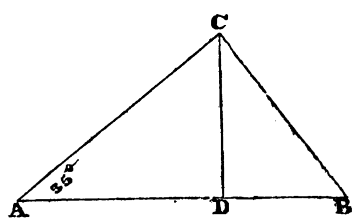
CHAP. V.

DIVISION OF LAND.

PROBLEM I.

To lay out a given area in the form of a triangle, when the base, and the angle at the base, are given.

Let ABC = the given triangle, having the base AB, and the angle BAC, given; and let the area of the triangle=A; it is required to find, first, AC, and thence AD, and DC.



By the nature of triangles,

$$\text{Area} = \frac{AB}{2} \cdot DC = AC.\ \frac{AB}{2R}.\ \sin.\ \angle\ BAC.$$

$$\therefore \frac{AC}{2} = \frac{\text{Area}\ .\ R}{AB\ .\ \sin.\angle\ A}$$

$$\text{Again } DC = AC \sin.\ \angle\ A$$

$$\text{and } AD = AC \cos.\ \angle\ A$$

EXAMPLE 1.—There are 46 acres of land to be laid out in form of a triangle, whose base is 15 chains, and the angle 35 degrees; required the lengths of the other two sides.

Here AB = 15 chs., and the angle CAD = 35°,

$$\text{now } \frac{AC}{2} = \frac{\text{Area . R}}{\text{AB. sin.} \angle A}$$

log. area (460 square chains)		=	2·66276
+ log. rad.		=	10·
			12·66276
log. AB (15 chs.)	= 1·17609		
+ log. sin. ∠ A (35°)	= 9·75859		
deduct	10·93468	=	10·93468

$$\frac{AC}{2} = \text{log. } 53{\cdot}47 \text{ chains} = 1{\cdot}72808$$

$$\underline{\quad 2 \quad}$$

$$AC = 106{\cdot}94 \text{ chs.}$$

EXAMPLE 2.—Having the same data, to find the length of BC, in the previous example, and also DC; and thence to determine, whether, with these lengths, the triangle will contain the given area.

PROBLEM II.

From a given triangle, to cut off any given area, by a line drawn from the vertex to the base.

Triangles, of equal altitudes, are proportional to their bases (see Euclid. lib. vi., prop. i.); therefore, making A, the given area of the triangle; a, the part to be cut off; and x, the required portion of the base, we have

$$x = \frac{a \text{ base}}{A}$$

EXAMPLE. 1.—There is a Gore of land between two townships, whose area is 425 acres, and base, 85 chains. It is required to cut off 400 acres by a line drawn from the vertex.

Example 2.—From a Gore of land, having a base of 40 chs., containing 125 acres, to cut off 50, by a line from the vertex, required the base.

Example 3.—From a triangle, with a base of 74·54 chs., containing 35 acres, to cut off 860 square yards, required the base.

CHAP. VI.

PRACTICAL EXAMPLES.

The following questions are of a more practical nature, and have been fully worked out, so that the reader may make himself acquainted with the means of ensuring that practical accuracy, which is indispensable for success, in a profession to which so much responsobility is attached, and where the consequences of inaccuracy are so serious and lasting as in the survey of a new country. In Canada where I was engaged for some time, one third (I believe I am speaking within compass) of the common law cases were either disputed surveys, or originated in the bad feeling engendered from encroachments, or supposed encroachments upon each other's property.

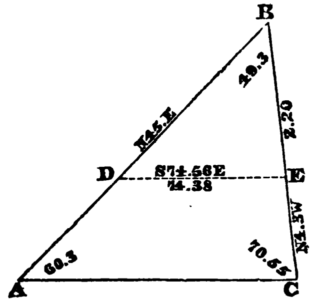
PROBLEM I.

The bearings and distances of two sides of a triangle being given, to cut off a given area at the vertex, by a line parallel to the base.

Ex. 1st. There is a Gore of land between two townships, the boundary line of one being N. 45° E., and that of the other N. 4° 3′ W.; the length of the first line, 240 chs., that of the second 220 chains.

It is required to cut off, at the vertex end, 300 acres, by a line parallel to the base of the Gore.

Because the one line bears N. 45° E., and the other N. 4° 3′ W., the angle between them is 49° 3′.



$$\frac{240 \times 220 \times \sin. \angle 49^{\circ} 3'}{2\ R} = \text{area.}$$

log. $\frac{240}{2}$ = 120 chains	=	2·07918125
log. 220 chains	=	2·34242268
log. sin. 49° 3′	=	9·87810900
		14·29971293
Divide by radius =	−	10
1994 acres = 19940·00 sq. chains	=	4·29971293

Now, as similar figures are proportional to the squares of their homologous sides, the whole area is to the given area as square of one of the given sides is to the square of that portion of the side, that is to be cut off: that is,

As 19940 sq. ch. : 3000 sq. ch. : : $240^2 : x^2$ = 93·09 chs.

where x = length to be cut off.

The other side BE will by the same process be found to be 85·33 chs.

Having obtained the lengths of BD and BE, the two sides of the piece to be cut off, that will contain 300 acres, together with the included angle 49° 3′, we can calculate the angles at the base by the second case of trigonometry.

These angles will be found to be as follows, viz. : 70° 55′ = greater angle, and 60° 3′ = the less.

To prove the calculation.

$$\text{Area} = \frac{DB.BE. \sin. 49^{\circ} 3'}{2\ R} = \frac{85 \cdot 33 \times 93 \cdot 09 \times \sin. 49^{\circ} 3'}{2\ R} = 300 \text{acres}$$

Having verified the correctness of the calculated sides, we proceed to obtain the length and bearing of the base.

As sin. 64° 3′ : 85·83 chs. : : sin. 49° 3′ : DE = 74·38 chs.

Again ∵ AB bears N. 45° 0′
add 60° 3′

∴ 105° 3′ is the angle made with the north. Because it is greater than a right angle, subtract it from 180, and its supplement, 74 5′7, is the angle made with the south point of the needle, being, therefore, S. 74° 57′ E.

In order, therefore, to cut off 300 acres from the Gore ABC, measure from B to A, 93·08 chs., and at that point run a line, bearing S. 74° 57′ E, which will be parallel to the base of the Gore, and should be 74·38 chs. in length.

Example 2.—*The same data being given,* it is required to cut off 300 acres towards the township, whose boundary line bears N. 45° E, by a line drawn from the vertex to the base.

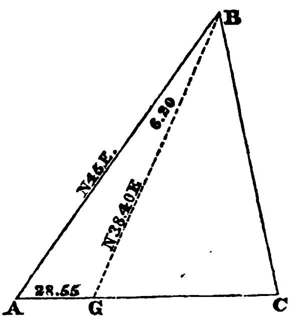
Using whatever calculations may have been made above, that may be useful to our purpose, first ascertain the base of the Gore.

As sin. 60° 3′ : 2·20 chs. :: sin. 49° 3′ : AC=191·70 chs.
191·17 chs. = the base of Gore.

Now, triangles of equal altitudes are to each other as their bases; therefore, the whole Gore is to the piece to be cut off, as the whole base is to the base of the piece cut off.

As 19940·00 chs. : 3000·00 chs. :: 191·77 chs. : AG.
AG = 28·85 chs.

Next find the angles of the triangle ABG, by the rule for the second case of trigonometry, so as to determine the length and bearing of BG.



These angles will be found as follows, viz.: $113° 36' =$ larger angle AGB, and $6° 20' =$ the smaller angle ABG.

BA is S. 45° W.

$45° - 6° 20' = 38° 40'$ W.

The bearing, therefore, is S. 38° 40' W.

To find the length of the line BG.

As sin. 6° 20' : 28·85 chs. = sin. 60° 3' : 226·65 chs. = BG.

To lay off the line BG, cutting off 300 acres on the side AB, proceed to the vertex B, and run BG S. 38° 40' W., till you come to the base, and measure the distance GA, which should be 28·85 chs.; if the actual distance be found to the same within a few links, the work is finished, but if not, it becomes necessary to correct the bearing of the line by the compass, and run it over again.

To find the angle of correction.

When the line run by the circumferentor has been slightly out of the right direction.

Say: As the whole distance run is to radius, so is the measure of the variation, to its angle.

Note.—The circumference of a circle $= 2\,\pi$. rad. This circumference is measured by 360 degrees, therefore π. rad. is measured by 180°, and $\frac{180}{\pi}$, which is 57° 3′ is the measure of that portion of a circumference which is equal to the length of the radius.

Example 1.—In the preceding problem let GA, instead of 28·85 chs. measure 29·85 chs.; that is, one chain too much to the east.

Then to find the angle of correction, because the whole distance run will be the hypothenuse of a right-angled triangle, of which the error will be the perpendicular, by making the former radius; and the latter, the sine to the angle of error, we shall have,

As $226 \cdot 63 : 57^{\circ}\ 3' :: 1 \cdot 00 : x = 15' =$ angle of correction.

Present bearing	S. $38^{\circ} 40'$ W.
Add	$15'$ W.
Corrected bearing	S. $38^{\circ}\ 55'$ W.

PROBLEM II.

The bearings of three sides of a triangle being given, to cut off a given area at the vertex, by a line parallel to the base.

RULE 1st.—These three lines form a triangle, whose angles are all known; make one side equal to unity, and calculate the proportionate lengths of the other sides. Making this side $= x$, and multiplying by the other two sides, all the sides are known in terms of x. Find the area in terms of x, according to the formula, area $= \sqrt{s.(s-a)(s-b)(s-c)}$, and make this equal to the given area. The solution gives the answer.

EXAMPLE 1.—Let one of the boundary lines of a Gore, between two townships, bear N. 38° W., and the other N. $16^{\circ}\ 30'$ E.; it is required to cut off 325 acres from the Gore end, by a line bearing N. $88^{\circ}\ 15'$ E.

RULE 2nd.—First, find the interior angles, and let AB $=$ 1·00 chains or unity.

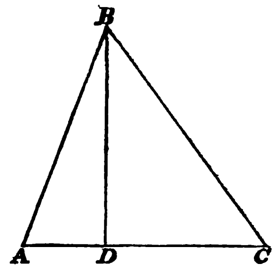
To find BC or AC.

As sin. $53^{\circ}\ 45'$: 1·00 ch. :: sin. $71^{\circ}\ 45'$; BC

BC $= 1 \cdot 18$ chs.

Or, As sin. $53^{\circ}\ 45'$: 1·00 ch. :: sin. $54^{\circ}\ 30'$: AC

AC $= 1 \cdot 01$ chs.



$$\begin{aligned} AB &= 1{\cdot}000 \\ \text{therefore} \quad AC &= 1{\cdot}010 \\ BC &= 1{\cdot}178 \end{aligned}$$

Let fall a perpendicular BD.

Now the area $= \dfrac{AC.AB}{2} \cdot \dfrac{\text{sin. } \angle\, CAB}{R} = 4794$ sq. chains.

then because similar figures are as the squares of their homologous sides.

Therefore 4794 sq. chs. : 3250 sq. chs. :: $1^2 : x^2$

where $x = AB = 82{\cdot}34$ chains.

which is the measure of the side taken as the unit side of the three.

To find AC and BC, the other two sides (by the first rule); multiply each of the sides of the unit triangle by this common unit of measurement, or, making $A =$ the actual area, and a, the unit area, and x, as before, the length of any side, in the unit triangle a, which was taken, we have, by similar triangles,

$$a : A :: x^2 : y^2.$$

i. e. $\dfrac{Ax^2}{a} = y^2$ where y is the actual length of the same side.

$\therefore x \sqrt{\dfrac{A}{a}} = y$ or log. $x + (\text{log.} \sqrt{\dfrac{A}{a}} = 1{\cdot}91560) = \text{log.}\, y$

where $\sqrt{\dfrac{A}{a}}$ or $1{\cdot}91560$ is the actual length of the unit side (AC).

Let x $(= 1{\cdot}010)$ then log. x	$= 0{\cdot}00412$
log. $\sqrt{\dfrac{A}{a}}$	$= 1{\cdot}91560$
$\therefore$ AC $= 83{\cdot}12$ chs.	$= 1{\cdot}91972$

Again,

Let $x = 1{\cdot}178$	$= 0{\cdot}07102$
log. unit of measurement	$= 1{\cdot}91559$
$\therefore$ BC = 96 chs. 96 lks.	$= 1{\cdot}98661$

$$\text{therefore} \quad \begin{aligned} AB &= 82{\cdot}34 \text{ chs.} \\ BC &= 96{\cdot}96 \text{ chs.} \\ AC &= 83{\cdot}12 \text{ chs.} \end{aligned}$$

EXAMPLE 2.—Given the bearing of a Gore of land, north 12 deg. east, and north 21 deg. west, and the base 156 chains, 50 links, bearing north 86 deg. east.

It is required to cut off 1000 acres from the frustrum end of the Gore.

First find the area of the given triangle, 2066 acres, 2 roods.

Deduct 1000 acres from it, and you have the portion to be cut off at the vertex. This now comes under the previous rule.

$$\text{Ans.} \begin{cases} AC = 77{\cdot}39 \text{ chs.} \\ DB = 77{\cdot}79 \text{ chs.} \\ \text{Height} = CE \text{ or } DF = 74{\cdot}39 \text{ chs.} \end{cases}$$

Practical Questions.

1. One side of a tract of land of which a survey is to be taken, passes through a pond. Two stations are therefore taken on one side of the pond. The bearings and distances from the first end of the side to the first station, from that to the second, and thence to the other end of the side are: 1st. S. 52° W. 10·70 chs.; 2nd. S. 7½° W. 13·92 chs.; and 3d. S. 34¼° E. 9 chs. Required the bearing and distance of the side.

Ans. S. 10° 33′ W. 28·31 chs.

2. Given the bearings and distances of the sides of a tract of land as follows; 1st. S. 40½° E. 31·80 chs.; 2nd. N. 54° E. 208 chs.; 3d. N. 29½° E. 2·21 chs.; 4th. N. 28¾° E. 35·35 chs.;

5th. N. 57° W. 21·10 chs.; and 6th. S. 47° W. 31·30 chs.; to place of beginning. Required the area of the tract.

	A.	R.	P.
Ans.	92	3	30

3. In taking a survey of a tract of land bounded by six straight sides, I was prevented going directly from the 3rd to the 4th corner by a pond of water. I therefore set up two stakes near the edge of the pond, and took the bearing and distances from the 3d corner to the first stake, from the first stake to the second, and from the second to the fourth corner, and noted them in my field book, as all belonging to the 3rd station of the survey. The field notes being as follows, the bearing and distance of the third side, and the area of the survey are required. 1st. N. 7·81 chs. 2nd. S. 76¼° E., 18·15 chs. 3rd. S. 52° W., 10·70 chs. S. 7½° W. 13·92 chs., and S. 33¼° E., 9·00 chs. 4th. N. 84¼° W. 27·12 chs. 5th. N. 4½° W. 22·00 chs. 6th. East, 16·58 chs.

Ans. 3d side, S. 10° 47′ W. 28·42 chs.; and area 80 acres, 0 rood, and 25 poles.

4. Given as follows: 1st side, N. 62° 15′ W. 2nd. N. 19 E. distance 18 chs.; 3d. S. 77° E. distance 15·25 chs.; 4th. S. 27° E.; to cut off 35 acres by a line, from the first side to the last, running N. 82° 30′ E. Required the length of the division line, and the distance on the first side.

Ans. Division line 22·98 chs.; distance on first side 5·14 chs.

5. Given as follows: 1st. side S. 78° W. 8 chs.; 2nd. N. 26½° W. 11·08 chs.; 3d. N. 38½° E., 12·82 chs.; 4th. S. 64° E., 10·86 chs.; 5th. S. 23¼° E.; to cut off 25 acres by a line running from the place of beginning, and falling on the 5th side; required its bearing and distance.

Ans. N. 45° 1′ E., distance 10·67 chs.

6. The boundaries of a trapezoidal field ABCD are given as follows: viz. AB, N. 80° W., 60 perches; BC, N. 39½° W., 45·5 perches; CD, S. 80° E., 89·4 perches; and DA, South, 30 perches; and it is required to divide it into two equal parts

by a line FE parallel to AB or CD. What will be the length of the division line FE, and the distance AF?

Ans. FE, 76·13 perches, and AF, 16·46 perches.

7. The boundaries of a field ABCD are given as follows: viz. AB, S. $10\frac{1}{2}$° W. 720 chs.; BC, S. 67° W. 12·47 chs.: CD, N. 23° W., 13·33 chs.; and DA, S. 89° E. 18 chs.; and it is to be divided into two parts ABEF and FECD, in the ratio of 3 to 4, by a line FE, running due South. Required the length of the division line FE, and the distance AF.

Ans. FE, 10·10 and AF, 8·12.

8. The boundaries of a tract of land are as follows: 1st. N. 14° W., 15·20 chs.; 2nd. N. $70\frac{1}{2}$° E., 20·43 chs.; 3d. S. 6° E., 22·79 chs.; 4th. N. $86\frac{1}{2}$° W. 18· chs. to the place of beginning; within the tract there is a spring, the bearing and distance of which, from the second corner, is S. 75° E., 7·90 chs. It is required to cut off 10 acres from the west side of this tract by a straight line running through the spring; what must be the distance of the division line from the first corner, measured on the fourth side. *Ans.* 4·64 chains.

CHAP. VII.

THE MERIDIAN LINE.

To determine the meridian line.

There are several methods by which this can be obtained, any one of which is sufficiently correct for the surveyor.

1st. *Approximately by the shadows of the sun,* when at equal altitudes before and after noon. This is a simple practical

contrivance for dispensing with an instrument in observing equal altitudes and azimuths. It is useful when White's Ephemeris or the Nautical Almanack are not at hand, and is sufficiently correct for railway purposes, that is, for running trial levels.

2nd. *By the needle, or any angular instrument.*

1. By the Azimuth of the Pole Star.
2. By the sun's amplitude at setting or rising.
3. By determining its transit at noon.
4. By equal altitudes and azimuths.

The method by the pole star, and that by determining the sun's transit at noon, are, perhaps, the only two, that can be used in the forests of a new country. A natural horizon, by which to determine the sun's amplitude at rising or setting, can seldom be obtained; and no dependence could be placed upon securing sufficient opening between the trees, to obtain equal altitudes and azimuths.

Again, the method by the sun's amplitude, is decidedly the best at sea, or on the sea shore, where a decided horizon can be obtained: and in the interior, where there are no woods, the method by equal altitudes and azimuths, from the power the surveyor has of taking several equal altitudes at the same point, and thus verifying his work, may perhaps be most beneficially adopted.

APPROXIMATELY.

By the sun's shadow.

Drive a stake down into the ground, having a pointed top. Plomb it carefully so that it be quite perpendicular; mark where the sun's shadow is thrown, and describe upon the ground, from the stake, as their common centre, two or three circles, decreasing in radii. Note where the shadow from the point of the stake, touches the circumference of the larger circle, before noon, and also after noon. Do this with the other circles: connect these points upon the circumferences with the centre, and bisect the

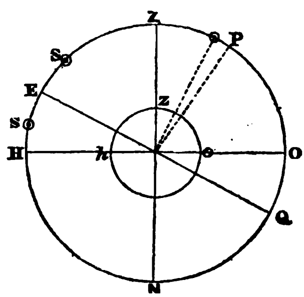
angles; these should have a common bisection. Should this not be the case; the mean may be safely taken, provided the difference is not great between them.

CHAP. VIII.

2ND. BY AN ANGULAR INSTRUMENT.

BEFORE entering upon this method, a few preliminary observations will be found useful.

It will also be necessary that the student should understand the method of *finding the* LATITUDE.



Let z be the place of the Observer, and *zoh* the earth: then Z will be the Zenith; EQ, the Equator; and HO, the horizon, at the earth's centre. ZE will be the *latitude*; and ZP the co-latitude or *polar distance.* H*s* will be the sun's altitude, E*s* the sun's declination, and *s*Z, the zenith distance.

Now E*s*, or E*s'*, which is the sun's declination, varies every day in the year; it is to be found in White's Ephemeris, or in the Nautical Almanack.

H*s*, the sun's altitude, must be measured, by an instrument,

and a natural or artificial horizon. The only natural horizon is the sea: on land, an artificial horizon of quicksilver, in which the sun is reflected, must necessarily be used. The angle observed by this latter method is double the angle of altitude, which will be north or south according to whether the observer is north or south of the sun.

The complement of this, Z*s*, will be the sun's zenith distance. E*s*, the sun's declination being added to this, if they are both north or south, or subtracted from it, if they differ, you obtain ZE or the latitude, in terms of the greater; the complement of which, ZP, is the polar distance.

To find the latitude.

Find the sun's centre by allowing for his semi-diameter, 16 minutes, deduct for refraction 1 minute. Deduct his true centre thus corrected from 90 degrees, which will give you the zenith distance. Turn out the sun's declination in the tables for the given day and subtract or add them, as the zenith distance and the declination may or may not be of the same name, and you will obtain the latitude.

EXAMPLE 1.

May 18th, 1842, the meridian altitude of the sun's lower limb was 47° 50′, the observer being north of the sun; required the latitude of the place of observation.

Obs. alt, ☉'s L. L.	47° 50′
☉'s semi-diameter	+ 16′
App. alt. sun's centre	48° 6′
Refraction	— 1′
True alt. sun's centre	48° 5′
	90° 0
Sun's zenith distance	41° 55′ N.
Sun's declination	19° 41 N.
Latitude	61° 26′ N.

EXAMPLE 2.

September 9th, 1842, the meridian altitude of the sun's lower limb was 62° 33′, the observer being north of the sun required the latitude of the place.

Obs. alt. sun's lower limb	62° 18′
Semi-diameter	+ 16′
App. alt. sun's centre	62° 34′
Refraction	— 1′
True alt. sun's centre	62° 33′
	90° 0′
Sun's zenith distance	27° 27′ N.
Sun's declination	5° 23′ N.
Latitude	32° 50′ N.

The *amplitude* of a heavenly body is the angle made between that body when on the horizon, at its rising or setting, and the east or west points of the compass.

The *azimuth* is the same angle, measured on the horizon, when the body has attained to some altitude above it; it is reckoned from the north or south points. Thus, the sun's amplitude is the angle made with the true east or west, and the pole star's azimuth, is that between the true pole and the pole star, taken at its greatest elongation, measured on the equator, and taken between two large circles; the one passing through the polar axis of the *Earth*, the other, through the pole star.

Having now ascertained how the latitude is to be obtained, we will return to the method of obtaining a meridian by an azimuth of the pole star.

1st. *By the Azimuth of the Pole Star.*

The α of Ursa Minor, or as it is more commonly termed Polaris, is about $1\frac{1}{2}^{\circ}$ from the true pole, and revolves round it in 23 hours 56 minutes. When it is at its greatest distance, east or west, it is said to be at its eastern, or western elongation.

The angle that measures this distance, that is, the angle made at the centre of the earth, between the true pole and the pole star, is called the polar azimuth; this, which should be taken at the time of its greatest elongation, depends upon the latitude of the place, and the distance of the star from the pole, This distance is called the polar distance, it is subject to a small annual diminution, called precession, which is 19·3 seconds annually. In the year 1830 this distance was $1^{\circ}\ 35'\ 50''$; by multiplying the number of years since by 19·3 seconds, and deducting the product, the actual polar distance can be obtained.

Now, the azimuth, or angle of variation of the pole star, from the true pole, can always be determined by the following proportion:—

As radius : sec. lat. :: sin. polar dist. : sin. azimuth.

To lay down a Meridian Line therefore,
find the time in the nautical ephemeris, when the star is about its greatest eastern or western elongation, and having calculated the polar distance for that day, make all the necessary arrangements for the observation about 20 minutes before this time.

Having set the telescope of the Theodolite to the star (a circumferentor with a telescope will answer the same purpose), watch the star carefully, which will be seen to move in one direction for some time, until it becomes stationary. Now, fix the telescope, so that the web covers the star, and keep it thus, until the star begins to retrograde. Then clamp it care-

fully in that place, and direct an assistant to take a stake, with a lighted candle upon it, and put it down in the same line, at some 8 or 10 chains distance; this is the line of elongation. Further operations had better be put off till the morning. Then, in order to obtain the meridian line, another line must be laid off upon the ground, making an angle with the line, equal to the line of the angle of azimuth, which was previously determined, to the east of the line, if the elongation was westerly, and to the west, if it was easterly. This meridian line had better at once be marked out with stakes at either end, so as to be a standard of reference.

2. *To find a meridian line, by the sun's amplitude at rising or setting.*

Except upon the sea shore, there must be some uncertainty about this method, for the result cannot of course be perfectly correct, unless the observation be taken when the sun is exactly on the meridian. There is, however, very little "departure" about that time.

The method of applying this is very simple; set your instrument to cut the sun's limb, either the right or left, at the time of its setting. If working with a theodolite, lay off the angle between this and the true west.

To find the true west, seek in the Nautical Almanack for the sun's declination for the given day, which being obtained, the true amplitude can be found thus:

As cos. lat. : R. :: sin. dec. : sin. amplitude.

A perpendicular to this will of course be a meridian line.

If using a Circumferentor, the difference between the magnetic and the true bearing of the sun at its setting, will give the variation of the compass.

EXAMPLE 1.

Given the latitude of Greenwich $51^\circ\ 28'\ 40''$ N., and the sun's declination on the 21st of June, 1842, $26^0\ 27'\ 41''$N.; required its amplitude.

As cos. $51^0\ 29'$	=	9·794308
is to radius	=	10·000000
So is sin. dec. $23^\circ\ 28'$	=	9·600118
		19·600118
To sin. amplitude $39^\circ\ 45'$	=	9·805810

Which is the amplitude for the east or west point of the horizon; and its compliment $50^\circ\ 15'$ shows how far from the north the sun rises or sets on the longest day at Greenwich.

3. *By determining the sun's transit at noon.*

The Circumferentor must be furnished with a telescope for this purpose, and whether to this, or to the Theodolite, a darkened glass must be used. About ten minutes before noon, set your instrument perfectly level, and cover the sun's centre with your web; follow its motion carefully with the telescope, keeping it always covered, until, after ascending, it remains stationary, and is beginning to descend ; clamp the telescope. Two points fixed with its line of collimation will give a meridian.

4. *By equal altitudes and azimuths.*

Measure carefully the angle of the sun's altitude, about nine o'clock in the morning, three hours before noon, and lay down the line upon the ground. Then, about $2\frac{3}{4}$ hours after

noon, keeping the instrument in the same place, and set to the same vertical angle, but with its horizontal movement free, watch the sun until it descends, so as to be again covered by the web, as in the morning; clamp the instrument, and lay down this direction also; half the angle between these two lines laid off, will give you the meridian line required.

I have entered only so far into the subject as to furnish the surveyor with a few simple practical means of laying down a meridian line; an approximation is all that he requires; but any approximation were better than the want of knowing how to distinguish between the true and magnetic meridian.

Anything were preferable to the shameful ignorance of many surveyors on the subject; especially now, when a good and easy method of determining the variation of the compass is invaluable to a surveyor on the preliminary railway surveys.

CHAP. IX.

ON LOCAL ATTRACTION.

THE needle being often, from various causes, diverted from its polarity, it becomes requisite, in running a line, to try the *backward* bearing at every station, and to see if it corresponds with the forward: it it does not, try the last forward station again, to see if any error may have been committed in taking it; should there be none, as the previous backward bearing, by

assumption, must correspond with the preceding for ward station, there can be no attraction there, and the attraction must be at the one where the backward bearing differs. Allow for the error or augle of attraction at the next station, and proceed until this error be compensated. During the continuance of this angle of error, there is local attraction in the neighbourhood.

At the first station, in starting, as there is no backward bearing to prove the non-existence of attraction, it is impossible to say whether the error be at the first or second station, should the backward bearing of the second station not correspond to the forward bearing of the first; by taking, however, a third station, and taking therefrom and thereto, the bearings of both the first and second stations, the error will be discoverable immediately, as the backward and forward bearings of (from 2 to 3 and from 1 to 3) cannot both agree. Where the disagreement is, there is the local attraction.

Attraction may, however, commence with the first station, and not be discovered until some station afterwards; should this be suspected, it becomes necessary to test the first bearing relatively, by a line making any angle with it, (*which angle has been measured by the chain,*) to such a distance, as may be considered beyond the sphere of the supposed attraction. There are many other *local* checks besides this; lines are seldom run in space unconnectedly in the woods, and, by measuring the distance from the next base line, its correctness may soon be determined.

CHAP. X.

This page contains a small collection of those conventional signs that are most in use; the method of representation is such as is generally adopted.

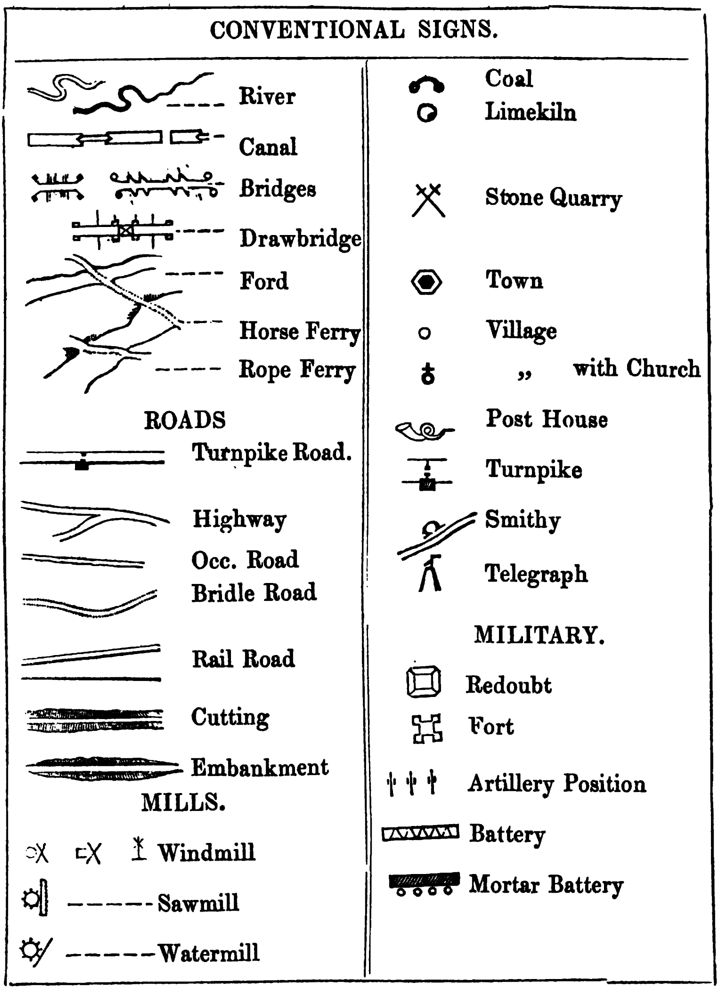
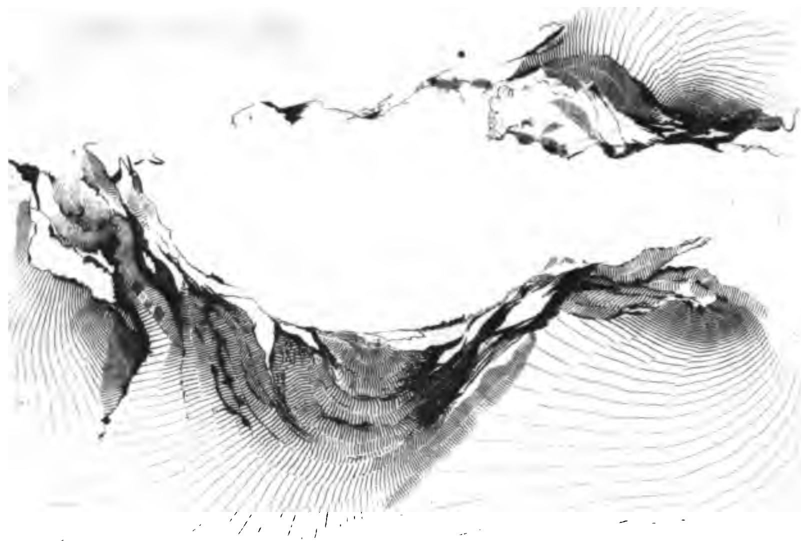


Plate 6.



F. Mansell sc.

CHAP. XI.

TOPOGRAPHICAL SURVEYING.

In surveying a tract of country, it is frequently not only necessary that a correct delineation should be obtained of the various outlines of hill and dale, of river and forest, but that some method should be adopted of conveying upon the same plan a tolerably correct representation of the relative heights of its different parts. The positions of the various boundaries in the plan are, or always should be, the horizontal spaces they would occupy on the earth's surface, totally independent of their height; this height must, therefore, be obtained in some other way; straight lines drawn from the summit, diverging to the bottom, more or less close, as the hill is steep or otherwise, are used in the one case; this is called the vertical method; in the other, the irregularities of the earth are represented by waving horizontal lines, approaching or receding in proportion to the steepness of the ground, or its graduated ascent. The plate, No. 6 is an example of the first method. There is, in this, a boldness of style, and a faithfulness of representation, which I am disposed to consider must give it the preference. This style of drawing is called military drawing, probably from the special demand for these topographical features of country for military purposes. By this, an officer is enabled, at a glance, to ascertain whether a commanding point, he is desirous of occupying, is accessible to cavalry, or unapproachable by infantry; and he is thereby enabled to decide, whether he can venture to attempt the dislodgement of an enemy from one hill, or can permanently occupy another, in spite of odds that may be sent against him.

This kind of drawing has its advantages even to the civil surveyor; if well done, the introduction of the features of the country is a great improvement to the plan, abstractedly; and, to the proprietor of an estate, the topographical details are often useful in assisting him in the laying of it out.

I need scarcely refer to the great assistance, that the engineers of the present day have derived from the faithfulness of the topographical delineations of this country in the ORDNANCE MAPS, in the selection of the best lines of routes for railways; every professional man must have experienced this himself.

LEVELLING.

Part the Fourth.

CHAP. I.

Levelling is the art of representing the inequalities of the earth's surface, and of determining the relative heights of any number of points above or below a line, equidistant, at every point, from the centre of the earth. This line is what is understood by the term—a level line; it is that line which is assumed by water when at rest.

The instrument, used for the purpose of levelling, is called a spirit level.

SPIRIT LEVELS.

Description of the usual kinds.

The spirit level is merely one portion of almost every other instrument, carried out to its greatest practical perfection. The bubble, which in most instruments forms only a subordinate part in the construction, is, in this, the chief; the only object of the instrument being to obtain a practical tangent to the earth's surface, or to place the line of collimation of the telescope in a

truly horizontal line. Hence it is termed **THE SPIRIT LEVEL**—*par excellence.*

THE Y LEVEL.

The Y level consists, like other instruments, of its parallel plates with their two pairs of conjugate screws, of its telescope, and its spirit level beneath. The telescope stands upon Y's, as in the case of the Theodolite; and has, also, like that instrument, its milled-head adjusting-screw for the object-glass; and the moveable eye piece for neutralizing the parallax. The cross wires, however, are not arranged the same way as in the Theodolite; one horizontal and two perpendicular wires being used instead. By this arrangement you are enabled to ascertain, whether the staff is perpendicular or not.

The bubble here, also, is furnished with its capstan-headed screws, for making it parallel to the axis of the telescope, vertically and laterally,

There is also a contrivance for raising or depressing one of the Y's, or supports, on which the telescope rests, so as to have the axis of the telescope always at right angles to the axis of the instrument.

Adjustments.

I need only repeat, that, for the line of collimation (the two first adjustments are similar to those of the Theodolite), the telescope must be turned round on its axis, that the intersection of its wires may always intersect one point, this adjustment will be found fully explained in the description of the Theodolite. The adjustment of the second liability to error, in the non-parallelism of the level and the telescope, is obtained as before, by reversing the telescope in its Y's. These two adjustments must be completed first. Afterwards

To make the axis of the telescope always at right angles to the axis of the instrument; in other words, to secure the line of collimation being perfectly level in any portion of a complete revolution of the instrument.

Set the telescope over any pair of conjugate screws, and make the bubble level; turn the instrument, till the telescope be over the conjugate pair: level it in this position; then turn it back to the first pair, and correct for any error that may have arisen from the last levelling, and continue till the bubble be central over the two pairs of conjugate screws; then turn the instrument one half revolution round, and, if the bubble still remain in the centre, the instrument is in adjustment; if not, the error can only be occasioned by the axis of the bubble, or, which is the same thing, the axis of the telescope not being truly perpendicular to the eentering of the instrument.

To correct this error, raise or depress the moveable (Y) support by the milled-headed screw beneath, until the bubble be brought half-way to its proper position, and correct, for the other half, by the parallel screws. By repeating the correction two or three times, the greatest accuracy will be obtained.

It is necessary to examine the adjustment every morning before starting, and it should be seen to at every observation, though it will scarcely require re-adjusting the same day. I should observe, that there is, or ought to be, a cap over this adjusting (Y) screw which should be carefully kept on.

The worst of this instrument is, that it soon gets out of adjustment even with the greatest care.

There are several kinds of levels—Troughton's, Gravatt's, the Y., &c.: all good of their kind, and each, perhaps, more fitted than the rest for some peculiar kind of levelling.

In Trial and Check-levels, I would recommend Gravatt's Dumpy, or Troughton's improved; being calculated, by their lightness and non-tendency of disarrangement, to get rapidly over the ground.

For the Main sections, at every two chains, I should prefer the Y level; and for the putting down the rails, the formation of roads, and all work where accuracy, and not expedition, are required, I should decidedly give it the preference.

There is one fault I have found with most levels, that the tube of the telescope is not long enough, to admit of reading off the staff within short distances; few reading within half a chain. Having had placed, for myself, in addition to the extending tube at the object end, another at the eye-glass to remedy this defect, I have been enabled to read within three yards; to this inner tube, of course, was attached the diaphragm of the cross wires and the lengthening eye-piece.

TROUGHTON'S IMPROVED.

This level does not stand so high as the Y, and is therefore more steady; at the same time, that being more simple, it is less likely to get out of adjustment.

The level in this instrument is not detached from the telescope, but is imbedded into it, being made by the instrument-maker as nearly parallel to the line of collimation, or optical axis of the instrument, as possible. It may as well be observed here, that this optical axis, is simply the line that the observer reads by, that is, the line running through the centres of the object and eye-glass, and the horizontal web.

This line, however, must not only approximate to, but must be adjusted perfectly parallel to the bubble.

There are three ways by which this can be done; *first*, by means of a natural level line, or surface of a sheet of water.

Drive two stakes down by the side of the water, about 6 chains apart, until the tops of them are just level with the water. Place the level over one, and measure carefully the height of the horizontal web in the telescope above it; say it is 5 feet. Set a staff up on the other stake, and read off from the

staff, where the horizontal web intersects it. If the line of collimation is correct, this will be 5 feet; should it read either more or less, the small screws at *c* and *d*, must be turned, the one set up, the other down, until the reading is exactly 5 feet, when the error will be adjusted.

This method, however, is not as good, as placing the instrument outside the staves, and altering the screws (the method will be seen below) till they both read alike.

Secondly. Where no water or natural level is near.

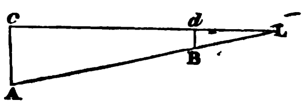
1st. by one staff.

Drive two pegs, about 6 chains a part as before, into the ground. Set the level over the one (*a*) measuring the height of the line of collimation as before, and read off the staff on the other (*b*). Let the height of the level (*a*) be 5 feet, and the reading (*b*) of the staff, 7 feet, that is, let (*a*) be 2 feet higher than (*b*).

Now reverse the arrangement, and let the level be placed at (*b*), and the staff at (*a*), and take the height of the level and the staff reading as before. If their difference be two, the collimation is correct; if not, first let the difference be less than two, the telescope therefore points too high, and as it pointed too high also, the previous time, and in the same ratio, and to the same distance (but which was not then known), this difference is double the error. Say now that the telescope stands 4 ft. 10 in., and the reading on the staff is 3 ft. 2 in.—the difference of which is 1 ft. 8 in., or 4 inches less than before—half this or 2 inches, deducted from the 3 ft. 2 in. as too high, or a reading of 3 ft. on the staff (the level and staff remaining untouched) will be the true line of collimation. Make it read 3 feet, and the instrument is in this respect adjusted.

Thirdly (with two staves). Drive two stakes down as before, say, about four chains apart; A and B; and consider the stake, on the left to be A, that on the right, B. Place the level outside the stakes; first, on the side of A, then on that of B, and at the distances of one chain from either; and levelling the

instrument, find in each case, the difference of heights of A and B. Now, if the differences are the same, the collimation is correct; if not, as the error is bothways, half the sum of their differences will be the real difference between them. Let for example, the first reading be 8·40 and 5·40; and the second, 4·20 and 6·70; making the first difference 3 feet, and the second 2·50; the true difference will be 2·75; for in the one case, it makes the difference too great by half; in the other by half too small; that is, ·25 feet of error each way. To adjust the level, therefore, keep it in the same place, and having carefully levelled it again, read off the nearer staff: say, that it reads 4·25, then the other should read 2·75, either greater or less; say, for example, greater; and therefore should read 7·00 feet Suppose it be found to read 6·75 feet. Raise the line of collimation, by means of the screws till it reads 7·00 feet; having done so, it will be found, however, on turning the level back to the first staff; that it will then read more than 4·25 feet.

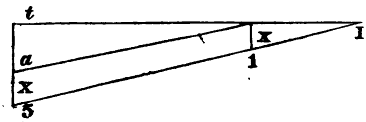


Thus let L be the Level, and A and B the staves. Let AB = 4 chs., and BL = 1 ch.

Also, let AC be the error of ·25 feet, then as BL is one fifth of AL, B*d* is one fifth of A*c*, and therefore equals ·05. Therefore, when the farther staff now reads 7·00, the nearer will read 4·30, making a difference between them of 2·70, instead of 2·75.

Turn the screws again, so as to make A*c* read ·06 higher, this will then make B*d* read ·01 higher, and therefore make a difference between them of ·05, which was required. The two readings will now be 7·06 and 4·31 having the required difference of 2·75.

The following, however, is the simplest and quickest method of at once obtaining the reading upon the staff, to which the horizontal line is to be set, thus;



L being the instrument and the staff being at 1 and 5 chs from it, as before: Let Lt be the line required; let (a) be the error to be corrected; and let x, x, be the unknown height to which the line of collimation is to be raised; then, by proportion,

$$1 : x :: 5 : x+a; \text{ or } 5x = x+a; \text{ and } x = \frac{a}{4}.$$

In this case x would be $= 0{\cdot}06$; the height to which the nearer staff is to be raised; add this to 0·25, and you get 0·31, the height to which the further staff is to be raised.

Having now got the line of collimation parallel to the bubble, the next adjustment is to make the line of collimation exactly perpendicular to the vertical axis; or, in other words, that the bubble be in the centre throughout a complete revolution.

Set the telescope over any two opposite screws and level; turn it one quarter round, that is, over the pair of conjugate screws, and level again; turn it back to its first position and level it again; and so on, until it is level in both positions; Now, turn it *one half* round; if the bubble is in the centre there is no error; if not, bring it half back by the plate screws; and for the other half, turn the small capstan headed screws, which support the telescope. These, in the plate, have a cap over them to protect them, but they are shewn more fully, and on a larger scale, at fig. 3, with the cap off. The plate *a, a,* is the end view of the plate *a, b,* fig. 1. (S) fig. 3, is the full view of the support or shoulder, of which the side view (S) is seen in fig. 1. On observing this fig 3, closely, it will be seen, that the two outer screws screw through the plate *a, a,* and have their ends, so as to press against the support (S), without screwing into it; their heads, it will be seen, are removed from the plate so as to admit of their pushing against the support (S), and removing it still further from the plate *a, a,* than is shewn in the diagram; by

these means, if the plate *a, a*, be not perfectly horizontal, the line of collimation, and the bubble, which reads it, can be made so exactly, The centre screw, on the contrary, *passes through* the plate *a, a*, without *touching* it, and screws into the support S; it is a larger and a stronger screw than the other two; its use is, when the two plates *a, a*, and S are placed at their proper distance, to screw them tight. The head of it should be always flush against the plate *a, a*, and the apex of the two other screws flush against the support S; by these means they will be kept at their proper distances. When the outer screws have to be tightened, of course, the the centre one must first be slackened.

GRAVATT'S DUMPY.

This is a very excellent instrument, very light, compact, and complete; from its simplicity, it requires fewer contrivances than the Y, and is therefore much lighter. The several parts are closer together, and by being more compact, it is more steady. It is admitted to be as complete as any other instrument yet made, though I am satisfied they are all capable of improvement.

The special, or rather *one* of the special advantages of this instrument, is, that from the increased diameter of the object glass, and the shortness of its focus, more light is admitted into the telescope. Another is, that in case of a close reading being required, the length of the rack, which runs nearly the whole length of the telescope, when the slide is elongated, secures a steady and parallel motion. To this instrument has also been attached a moveable mirror with a large joint to fit on the level above the telescope, so contrived as to enable the observer to note, just as he is taking his reading, whether the bubble has moved or not. This mirror is to be seen at (*m*), fig. 2.

With respect to the working parts of the instrument, the

dumpy principally differs from the "Improved Level," by having its spirit level tube not imbedded like the latter in the telescope, but separate from it. This level is generally furnished with another small level, placed at right angles to the larger one; but this might also, with advantage, be added to the "Improved." And the compass box, instead of being placed *above* the level, as in the other instrument, is placed in this *under* it. In this respect the "Improved" has certainly the advantage.

The first adjustment is to make the line of collimation correct, or the optical axis of the instrument parallel to the bubble.

Drive down three stakes 5 chains apart; say at A, B, and C, and placing the instrument half way between each pair, find the several correct heights of each. Let A = 100·00; B = 104·20; and C = 102·80 feet.

Now place the instrument close to A, and find the heights. If they are the same as before, the instrument is in adjustment. For example sake, however, let the heights be A, 100·00; B, 104·00; and C, 102·60; that is, A being the constant, the reading of B is ·20 lower than it should be, and so is that of C; but if AB is equal to BC, which it is, both being 5 chains in length; if the bubble points ·20 too low at B, it should be ·40 at C; the ·20, therefore, is owing to an error in the line of collimation. To correct this, use the *small screws* at the *eye-piece of the telescope*, and move the web until the reading at B, deducted from the present reading, becomes just half of the reading of C, deducted from its present reading. This will give you the adjustment for the line of collimation.

The next adjustment is that of getting the telescope level. This is done precisely in the same way as in the "Improved."

Besides these adjustments, there are also two or three imperfections in the telescope, which are common to all LEVELS. These are—1st. The occasional indistinctness of the web,

arising from the interference of light. 2. The uncertain position of the web; reading sometimes to a difference of 0·10, which arises from the largeness of the aperture at the eye-piece, and the closeness of this aperture to the eye-glass.

The first imperfection is frequently called parallax, but after having adjusted the telescope (and the principle of the telescope is the same in this as in the Theodolite), according to the rules for so doing given there, you will often find, that if the sun comes out after you have done it, the web, which before was quite distinct, becomes almost invisible.

The best remedy for this, is, to put on the cap at the object end, and to cover partly the object-glass with the shade, as seen in fig. 4. You will be able to see just as much as before, and more distinctly; as the interference of *the strong* light will be counteracted.

The second imperfection is, that if you move your eye up and down, you will read sometimes ·10 more one way than the other. This will be found to be the case the stronger the light is. The previous correction will oftentimes (but not always) remedy it in some degree, but the most effectual remedy would be this; the eye-piece slides in and out, so as to bring the web within the focus of it, to suit every eye (at least it should do so). I have, however, found that instead of making the tube in which the eye-piece slides, as short as itself, so as to suit very short-sighted persons, its own length, when elongated, being always sufficient to suit long-sighted ones, there is a quarter of an inch left between the eye-piece and the web, and with bad instruments sometimes more. The consequence is, that at times, you cannot see the web at all, and for this there is no remedy, except in moving the level which, at the moment, may be next to impracticable.

To this eye-piece is at present attached a cap, which screws on, having the aperture in it; this cap, and, the hole through which you look, is therefore, at a fixed distance. Now this distance is made too small, and the eye aperture too large.

It will be easily understood, that the web being fixed, the nearer the eye is to it, the greater will be the angle, under which any error in the position of the eye will be read, and also, the larger the aperture, the greater the error. It is not possible to keep the eye in one place, the immense distance of the staff from the web, in comparison of that of the web from the eye, multiplying as it does, shews the slightest change. It must be remembered, that this is also much affected by the state of the atmosphere. Taking all these things into consideration, I would suggest the following to instrument-makers as a remedy: viz.

To place one end of the eye piece as close as possible to the web, flush with it—it can if necessary be pulled out to its full length—upon this eye piece, to screw on the cap, as at present, but to make it twice as long as it is, and to extend the screw which works into it, further down the tube, so that the cap can be further elongated if necessary. The eye aperture should not be above one quarter of its present diameter.

Where this error to exist in the telescope of the theodolite, the instrument would be comparatively valueless. There, however, the eye-glass is fully twice the distance from the cross web, to what it is in the spirit level, owing probably to the levels generally reading inverted.

Breaking of webs, how to replace them.

In case of any accidental breaking of the cross webs in the country, where it is difficult to have them repaired, the surveyor had better become acquainted with the method of doing it himself.

The simplest plan, perhaps, if you can get a spider's web, is to suspend the spider in the air by his web, and wind the web round a fork or piece of card board. This can be done at any time and kept by you, until you want it. The inner tube to which the web is affixed can be easily taken out, and this web fastened to it by a little gum.

Where a spider's web is not to be obtained, a very fine skein of silk or a thin hair will do instead, until you have an opportunity of doing it properly.

Levelling Staves.

These are generally made from 12 to 16 feet high, divided into feet, and tenths of a foot, and again subdivided, for facility of computation, into hundredths.

The method of arranging this subdivision, constitutes the difference between the several staves in use.

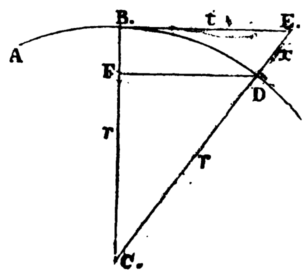
Some persons prefer one kind, some another; the reader must therefore choose for himself. There are Sopwith's, Gravatt's, Cowper's, Stephenson's, &c. The one I myself prefer is, that, which I believe goes by the name of Stephenson's: and was first used on the London and Birmingham railway. The mode of arrangement is this, the hundredths are obtained in the same way as in the common ivory protractor; the tenths of a foot through the whole length of the staff are bisected, making the two divisions "twentieths"; and these division lines extend the whole breath across the staff; the opposite ends of these lines are connected by diagonal lines, each one with its preceeding, viz.,—the left of No. 1 with the right of No. 2, the right of No. 2, with the left of No. 3, and so on. And five vertical lines are drawn along the whole of the staff, which thus divides each of these diagonal lines into five equal parts, each being the fifth part of the twentieth, or the hundredth of a foot.

The feet are distinguished by large red figures; the tenths by black, with a large full point at every ·5.

CHAP. II.

CORRECTION FOR CURVATURE.

As the lines obtained by reading off the staves, are only level lines for very short distances, being, in fact, tangents to the earth at the several points of observation, and not their corresponding arcs, which are the true levels, the nature and value of this error had better be at once explained, as well as the practical methods adopted to dispense with the correction altogether.



Let ABD be a portion of the earth's circumference, whose centre is C. Let BE be any level distance of the instrument (t), the true level line will be BD, and the error between the apparent and true level will be the versed sine BF.

When the distance BE is great, this versed sine must be calculated by trigonometry, but, for the usual distances of observation by the spirit level, DE, or sec. rad., can be safely taken instead.

Now CBE is a right-angled triangle, and therefore

$$\overline{r+x}^2 = r^2 + t^2$$
$$r^2 + 2rx + x^2 = r^2 + t^2$$
$$2rx + x^2 = t^2$$

throwing x^2 away, as indefinitely small, in relation to $2rx$, we have

$$2rx = t^2$$

and $$x = \frac{t^2}{2r}, \text{ or}$$

the error x = the square of the tangent, divided by twice the radius; and, as this divisor is a constant quantity, this error is proportionate to the squares of the distances.

The mean diameter of the earth is 7·916 miles; for one mile distance, therefore, we shall have $x = \frac{1}{7916}$ miles, or 8·004 inches; for two miles distance, four times that quantity; for three miles, nine times. Throwing away the $\frac{4}{1000}$ of an inch as immaterial, the error of one mile's distance is 8 inches or $\frac{2}{3}$ of a foot; for two miles, $\frac{2}{3}$·4 feet; for three miles, $\frac{2}{3}$·9 feet, &c.; or x in feet = $\frac{2}{3}$· (distance in miles)^2; which formula may be easily remembered. This value of x would not be sufficiently correct for many miles. Referring to the equation $\overline{r+x}^2 = r^2 + t^2$, the value would then be

$$x = \sqrt{r^2+t^2} - r.$$

Example of applying this correction.

Placed a spirit level at any point B (see fig. pag. 191), on the earth's surface, and found the point E, at 3 miles off, to be on an apparent level with the point B. What is the comparative height of the object E?

Now, BE is the apparent level, and BD the true level, B and D being points equidistant from the earth's centre. DE (x) is the height of E above B, which, in feet, equals $\frac{2}{3}$ the distance (DE, in miles)$8^2 = 3^2 \times \frac{2}{3} = 9 \times \frac{2}{3} = 6$ feet, the height of the object E, above B.

The apparent height, therefore, of every distant object, as observed by a spirit level from any point, is always *less* than the true height, by this value of (x), for the curvature of the earth.

CHAP. III.

REFRACTION.

There is also, in long observations, another correction necessary, arising from the effects of the density of the atmosphere, in refracting the rays from the object, which makes the apparent *greater* than the true height. The correction, therefore, must be subtracted from its apparent height.

Refraction increases the distance at which objects can be seen (cæteris paribus), in a proportion of 14 to 13, and *raises* the apparent height one-seventh of its correction for curvature.

Thus, in the last example, the object observed at a distance of 3 miles was apparently level with the instrument, but the correction for curvature being 6 feet, the height (independently of the subsequent correction for refraction) was 6 feet. Now $\frac{1}{7}$ of $\frac{6}{1} = \frac{6}{7}$ feet $= 10\frac{2}{7}$ inches, which is the correction for refraction, and therefore 5 feet $1\frac{5}{7}$ inches = the true height of the object, allowing for both corrections.

Examples.

1st.—The observed height, of three objects, above the point of observation, at a distance of 4, 6, and 8 miles (calculated from observations taken by the theodolite), were found to be respectively, 24, 25, and 28 feet. What are their true heights?

Ans. $33\frac{1}{7}$, $45\frac{4}{7}$ and $64\frac{4}{7}$ feet.

2nd.—Found the angle of elevation of the spire of a church, which was 420 chains 75 links off, to be 1° 10′. What is its real height above the point of observation?

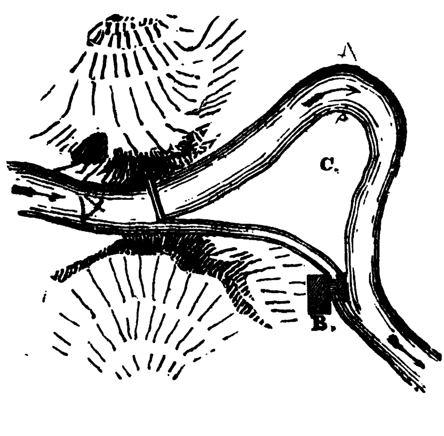
Ans. $581\frac{3}{4}$ feet.

CHAP. IV.

SIMPLE LEVELLING.

Simple levelling is merely the determining, by one observation, the relative heights of two given points.

Thus, supposing that it is required to ascertain, in the accompanying diagram, whether there was sufficient fall of water between that part of the river at A, and that at B, to turn a grist and sawmill at B. Place the level at C, between the two stations, and find the first or back reading 0·45 feet, and the forward 11·52 feet; let the back distance be 2 chains, and the forward distance 20 chains. Now, were there no correction for curvature or refraction, the point B would be 11·07 feet lower than A. As the forward distance, however, is so much greater than the back one, both these corrections must be taken into account.



The back reading 0·45 feet, as it is at the short distance of 2 chains, will require no correction for curvature or refraction.

On the other hand, the forward reading, being at a distance of 20 chains, must be corrected for both.

Now correction for curvature (x) equals in feet $\frac{2}{3}$ of the distance squared in miles; 20 chains equal $\frac{1}{4}$ mile; therefore $x = \frac{2}{3} \cdot \frac{1}{16}$ of a foot $= \frac{1}{24}$ foot, $=$·0416 feet for the correction for curvature; that for refraction is so small as not to be worth noticing.

This must be subtracted from the apparent difference of reading, and you obtain the true difference: thus,

the observed reading is	11·52 feet
subtract correction for curvature	·0416
there will result	11·4784 feet for

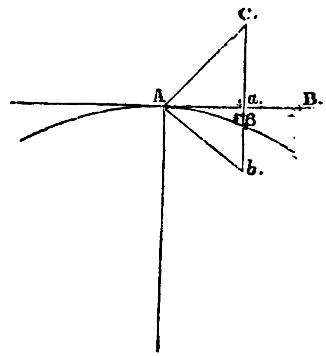
the true forward reading.

11·478	feet for the forward reading
0·45	feet the back reading

whence 11·028 feet is the true difference between them or the point B is 11·03 feet under the point A.

If the refraction, small as it is, be taken into account, then add, for refraction, $\frac{1}{7}$ of ·0416, or ·0059, and true height of the water at A, above that at B, is 11·034 feet.

As in this case, to obtain the true reading, the correction for the curvature has been subtracted, and that for refraction added, which is the reverse of the preceding rule at page 192, we had perhaps better investigate the subject a little more fully.



Let AB be a tangent line to the earth, at the point A, being the apparent level line; let Aa be the apparent level and Aβ be the true level, and β the corresponding point to a. Let C be any object *above* the apparent level line, and b any object *below* the same level line. The object above would be observed by the theodolite, those beneath (for short distances and for the common purpose of levelling) by the level staves. Let aC be the distance above AB being the positive height $(+h)$, and ab the distance beneath, which will then be $(-h)$. Now aC is the height above the line AB, and βaC, the true height, therefore $\beta a + a$C or $(+h+a\beta)$ is the true height of the point C, above A.

On the other hand, ab, the reading by the level staff, is the distance beneath the apparent level line AB, but the true line is Aβ, and $b\beta$ is the true difference of levels between A and b, and which is, therefore, equal to $(-h + b)$, or the difference between the two.

Again for the refraction.

As refraction raises the apparent above the true height of an object, its correction must always lower the apparent height (it is therefore negative—r).

If the object be above the apparent level, the error for refraction must be subtracted from the apparent height, thus $(h+\text{C}-r)$, where $+\text{C}$ is the correction for curvature, and r is that for refraction, when these values are positive.

But, if the object be below the apparent level, we have $(-h+\text{C}-r)$, that is, the curvature (which is the only positive value) must be subtracted, and the refraction must be added,

to the *negative* value of h, which still remains negative, or beneath the apparent level.

Generally, therefore, let the reader bear in mind, that the correction for the curvature raises the object, and that for refraction depresses it, reducing or increasing its distance from the apparent level, according as the object is above or below it, *i. e.*, has been observed by the Theodolite, or read off by the level staves.

CHAP. V.

COMPOUND LEVELLING.

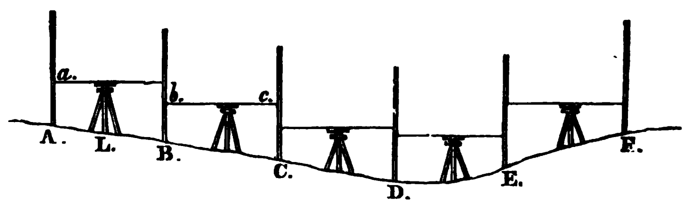
As the necessary calculation for curvature and refraction would be exceedingly tedious, in extensive operations, the following method renders them altogether unnecessary.

Set the level in the centre of the object, as nearly as the *eye can tell*, and these corrections for both objects become equal and opposite, and therefore neutralize each other. The apparent level, being always parallel to the chord, that connects two objects, equidistant from the place of observation, must, therefore, have the same versed sine.

To find the difference of levels between several points, or to trace a sectional line of the inequalities of the earth's surface.

Let ABCDE be the line to be traced. Set the level (L) between the object, and read off the height A*a* and that of B*b*, the difference between *Aa* and B*b* will be the number of feet that B is higher or lower than A*a*; if B*b* be greater than *Aa*,

the point B will be the lower (by this difference) than A*a*; for the height, read off by the level staff, is the number of feet that each point observed is ***beneath*** the level of the line of collimation of the telescope—hence, where there is a number of points beneath the same level line, the greater the reading of the staff, the lower this point must be.



Then, because, in the first observation, the height at B (read by the level staff) is greater than that at A, the point B is lower then the point A. Again, in the second example, where, it must be observed, another line of collimation is taken, because the height by the staff at C is greater than at B, the point C is lower than B. In the third observation, also, D is lower than C, and C being lower than B, and B than A, the ground falls thus far. At the next observation, however, because the height at D is greater than that at E, the point D is lower than E, and, therefore, E being higher than D, the ground rises to E, and as the reading at E is greater than at F, it goes on rising to F. The relative heights of the two ends of the line, at A and F, depend upon whether the ground falls, more or less, from A to D, than it rises from D to F.

Now, the difference between the reading at B and A, in the first observation, added to the difference of readings at C and B, in the second observation, plus the difference between D and C, in the third, as there is continued descent to the point B, will give the actual fall from A to D, or the number of feet, that the point D is lower than A. In the same way, the sum of the difference of readings of D and E, and of E and F, in their respective observations, will be the number of feet F is

higher than D; if, therefore, the fall from A to D, be greater, than the ascent from D to F, the difference will be the actual fall from A to F, or the number of feet that the point A is higher than the point F.

The following example of keeping the field book will, it is hoped, enable the learner to understand clearly the principle on which all levelling is conducted.

CHAP. VI.

THF FIELD BOOK.

With the means of obtaining the reduced levels.

Divide the page into the several columns, as in the example of the field notes of a portion of a line of railway between Hereford and Marden, commencing near Hereford, at a point which is carefully described, and can at any time be easily ascertained, being on string course (taken in the centre), of the Canal Bridge, on the road from Hereford to Mordiford close to where the line crosses.

This point was assumed to be 100 feet above an imaginary level, to which the heights, at the several points on the line, bear reference, which is called a datum line.

This point might have been considered zero, but as, whenever any other points were below this, their height would be negative, the constant changing, from positive to negative, would have been productive of considerable trouble and pro-

bable error, especially to persons not fully conversant with Algebraical calculation.

The height of the point of starting is put in the column of reduced levels, 100·00 feet, and the B. M. in the column of distance, shows that it is a Bench Mark. The instrument is now placed between this point and the next favourable station, (not exceeding in any case 6 to 8 chains from the level to the station) on this occasion the distance between the *two* stations was 1·50 chains, and the back reading was found to be 9·96 feet, and the forward reading 6·38.

These are placed in their respective columns. Between these two, however, there were two other (intermediate) observations: viz. 10·85, the centre of the road from Hereford to Mordiford, where the line is intended to begin, and 4·81, the surface of water in mill pond.

Now, in order to obtain the reduced levels (which are the actual heights of each point respectively above the assumed datum line, to which they all have reference), of these observations, as the backset reads less than the immediate, the ground is lower than the B. M. and the difference between the two readings (0·89) is placed in the column of "*Falls.*" Again, because the next intermediate reads less than this, the surface of the mill pond is higher than the road, and the difference 6·04 is put in the column of "*Rises.*" The foreset, however, reads more again, the next difference 1·57, therefore, is put in the "*Falls.*"

The height of the ground, where the instrument was placed, of course is not known. The instrument was then removed to between the second and third station, and the staff turned round, when a second reading of each was taken, and placed in their proper columns. The last reading being greater, the ground now falls and the difference (3·76) is placed in the column of falls and its distance (*always from the starting point*) is marked down.

The rises and falls are the differences at each point of its

own and the preceding height, and denote whether the ground has arisen or fallen between any two consecutive points. By adding, therefore, each rise to the preceding actual height, we obtain the actual height or *reduced* level of the point, and by deducting this difference where there is a fall, we also obtain the same result.

Thus, at a centre of the road, the intermediate backset being greater than the 0·89, is placed in the column of falls, which, subtracted from 100·00 the reduced level at the starting point, gives 99·11, the reduced level at this point.

The next observation which is an intermediate, is 604 higher than this; this 604 must therefore be added to the preceding reduced level 99·11, making the present reduced level 105·15; the next one, by subtracting 1·57 which is a fall, becomes 103·58.

It must be observed generally that there should be no *foreset* without its distance. All B. M's of every kind—*the surface of water,—in rivers and drains,—in fields,—on banks,*—and every height taken for the purposes of illustration, are generally made intermediate.

Having filled the page with the observed backsets, intermediates and foresets, add up the foresets and backsets, their difference should be equal to the difference between the sums of the rises and falls; it should be also equal to the difference between the assumed datum line and the last reduced height.

Having these three checks, there is no possibility of error when they all agree, and in *all cases every page must be so tested.*

Before calculating the reduced levels, as any error in the rises and falls must induce the same error in the reduced levels, observe first, whether the difference between the rises and falls is the same as that between the sums of the backsets and foresets; you can then safely proceed to the calculations of the reduced levels.

Observe, in placing the different readings, that, in the first line in the column of reduced levels, must be placed the

assumed height of the starting point above the datum line; collateral with this must be a full and clear description of the exact locality of the starting point. (Consider that you are describing the place to a perfect stranger, who has never been on the spot before.)

On the next lower line, the first back reading must be placed, and collaterally with it, *in the same line*, the next reading. If this reading is not intended to be the backward station to the next observation, place it in the intermediates. If it is, place it in the column of foresets, collaterally with the backset, in the same line. Observe, that every line in the reduced level must have a given height, and that to each of these heights, in the column of distances, there must be a distance given, or an observation of some kind.

The reason of placing the foreset in the same line as the backset is, that the height of each backset point is the height of the preceding foreset, and that it is the difference between the actual backset and foreset upon the same line, that gives the height of the latter.

In calculating the rises and falls, take the difference between the two readings in the same line, when the one is a back reading, and the other, intermediate or foreset, and place the difference in its proper place, in the same line. Then, if the next reading be an intermediate, it will be placed under the preceding intermediate; the difference between these two intermediates must be placed in the same line as the latter, and so on throughout any number of intermediates. After these intermediates must, of course, come a foreset point, before the instrument can be removed, which foreset becomes a backset point of observation at the next placing of the level. The difference must be taken between this foreset and the last preceding intermediate, and placed collaterally in the same line with the foreset. The better plan perhaps, is to consider each backset and foreset to form a link in the chain of levelling. These links are entirely distinct: in each link find the diffe-

rence between any observation and its preceding one, whether backset, intermediate or foreset, and place these differences in the proper columns.

This closes one portion of the line of section, and another is commenced that would be, as I before said, totally unconnected with the former, were it not that the last point in that becomes the first point in this.

Collaterally with the reduced levels, if these levels refer to bench marks, write B. M. in large characters in the column of distances, (or they may have a separate column for that purpose,) and the nature of it under the head of remarks. If the reduced levels refer to points taken out of the line, which is frequently the case in trial levels to avoid obstacles, write "out of line"; also, in the column of distances, carry the distances on always from the starting point to 80 chains, and, if an observation be taken at that point which should be done if possible, write 1 mile against it, and begin the chaining afresh. Continue to another 80 chains, marking this point 2 miles; begin here again also, and proceed as before.

CHAP. VII.

TRIAL LEVELS.

In carrying out these levels, the only information furnished the leveller is the general route marked down upon the Ordnance Map.

In most cases, two or three different lines are levelled, in order that the engineer may select between them; and some engineers have at the same time, cross sections taken at cer-

tain intervals, so as to obtain all the requisite data for making a good selection. This may be, however, like many other good things in themselves, carried to an extreme.

The young leveller being furnished with the line, should at once proceed to identify it, by carefully measuring with a scale, upon the map the distances to the nearest corner of the several points where the line crosses the road, and determining these points upon the ground.

With respect to the practical carrying out of this, there seems to be considerable difference of opinion, as to the quickest and cheapest mode of so doing. Some, and I am myself of their opinion, advocate the propriety of working with two staves, and having an assistant whose sole office is to range and prepare the line for you. Others, on the contrary, consider that both the extra staff, and the assistant may be economically dispensed with, without materially delaying the progress of the leveller.

The whole difference of opinion hinges upon the *extent* of the delay. That the country may be oftentimes such as to be traced by a *very good and experienced hand* while levelling and that the localities may be in many cases easily distinguishable, I readily admit, though I must also, on the other part, maintain, that the contrary is frequently the case, that not only is the leveller frequently puzzled to find out his course; but that oftentimes he does not discover his error, until he has been some quarter of a mile or more out of his line. I say nothing of the greater liability to error where the attention is divided; though, from my own experience, I am convinced that it is no easy matter to level over the exact route as laid out upon the Ordnance Map, when the localities are not very decided, and at the same time to level correctly.

However, it does not always happen, that the party who does the work, has the alternative allowed him; in that case he must do the best he can; and that best is, after all, vir-

tually to adopt the plan I suggested, by leaving himself every evening an hour or two to get two or three miles ready ranged for the following day. Great time, there is no doubt, may be saved in this, as in all other matters, by judicious arrangement; and herein lies the difference between the young and the old leveller; the former, after he has left off work, thinks no more of it till he resumes it the following day; the other provides regularly of an evening for the judicious disposal of the next day's time.

If you are working *up* to your inn, leave off a little sooner and trace the line on your way home in the evening; and in the morning get upon the line at once, and follow it back till you join it to the place you finished tracing it to the evening before. In this way you have plenty of work laid out for the day, and you can go a-head rapidly.

If you are working *away* from your inn, range on the line from where you left off, for two or three miles (till you come to some well defined locality), and level the reverse way back again. You will only have to keep two field books; the usual one (No. 1), in which all the onward levels are taken, and this (No. 2) for the backward. These latter, as soon as you have levelled back to where you had arrived in the previous levels, can be easily transferred to No. 1, by beginning at the bottom and copying the No. 2 into book No. 1, making the foresets backsets, and the backsets foresets; the intermediates will of course remain as they are. With respect to the plotting, should the distances have been chained (which is not always the case in trial levelling) lay off the total distance, and measure the distances backwards; their heights will of course be the same.

A great deal of time is also lost, by young men, in their trial levels, by the quantity of luggage they take with them. When four miles at least are expected to be done, and in many cases may be done in a day, and that in a straight direction, as the crow flies, it rarely happens that you can sleep

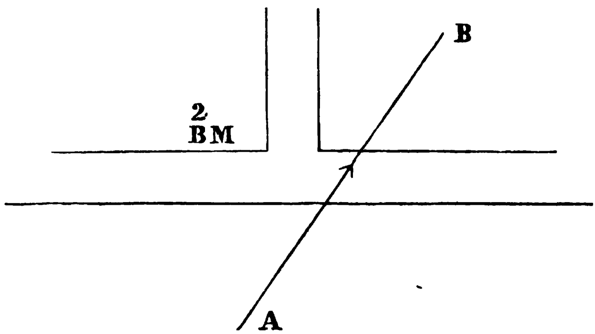
twice in the same house, without having four or five miles to walk to your work in the morning; and the same distance or more perhaps, home again at night. With three good hands, you might easily arrange one to carry a small knapsack, and another to strap the level box to his back; and the third might take his turn to relieve the other two. This is where you have but two chainmen and a staff-holder; if using two staves it would be still easier.

Much ground might be soon got over in this way; and it would not be compulsory upon a leveller to leave off at a fixed and early hour, to be in time for a conveyance back again to his inn, with the alternative of having perhaps half a dozen miles to walk after dark if he returned; or, if he went on, though perhaps with not a dry thread upon his back, of not having a change ready for him when he reaches his resting place.

Experience only can give you a clear idea of the inconvenience of having too much luggage with you, when out levelling. The only conveyance, perhaps, the carrier's cart once a week, left the day before you arrived, does not, even if in time for it, probably go straight to the place you desire, but is two days on its rounds, and leaves its parcels at the place you want to go to last. Some faint idea may be entertained of the importance that I myself personally attach to this; by this single circumstance, that I should never think of employing any one in levelling, who was in the habit of taking much luggage with him.

Another important point in trial levels is *a judicious selection*, and *accurate description of proper B. M's.* When cross roads are within one or two miles of each other, the B. M's. taken upon them for the purpose of the cross sections will be near enough. Where they are further apart, some intermediate bench mark must be taken. With respect to the cross roads, the lower hinge of the nearest gate should be taken, unless some other more desirable B. M. should present itself. These B. Ms.

should be carefully sketched in, thus; as in the annexed diagram:



AB gives the position and direction of the Main line; and B. M. 2, the position and number of the bench Mark, being numbered according to the number of the cross roads: so that the 5th cross road, should have a B. M. No. 5, and so on. All intermediate B. Ms. should be lettered *5a*, *5b*, *6a*, *6b*, depending upon the numbers of the cross roads on either side of them.

The plan should also be numbered with the corresponding numbers placed in their right places. Too much care cannot be taken in properly selecting and carefully describing these B. Ms. Another person has frequently to "take up" your B. Ms., for the finding of which, he has to depend entirely upon your notes, which ought to be made perfectly clear and intelligible, so that no one could possibly be mistaken as to what they referred.

CHAP. VIII.

CHECK LEVELS.

These, strictly speaking, are taken solely for the purpose of of testing other levels, by levelling again between certain fixed points, and determining their respective heights. They are usually taken more for the purpose of determining any considerable error such as may affect the Bill, than for any greater accuracy. Generally, they are very roughly taken, and being so, however much the parties taking them may, as they frequently do, intend them to check some 20 or 25 miles *to a foot*, and question the correctness of more careful levels, if these check levels differ that amount from them: they are only useful for the former purpose. How young men can fancy, and from my own experience among them, I know they do, that levels taken with the instrument placed any where between the staves, and with distances as long as they can read them, are good for any thing beyond an approximate check, is to me astonishing. Not that I think they are to blame so much as their employers. Quantity, not quality, has been the order of the day in this work, and he has ever been the best man, who could read off the longest sights, and run over the greatest distance in a day. Last year certainly, if not profiting from, at least compelled by, past experience, and dear bought proof of the inaccuracy of this mode, a different plan was among some parties adopted.

The levels were taken along the High Roads that ran near the line of railway from mile post to mile post, *with two levels side by side*: and, where the Turnpike Roads were too far off, then by the nearest Occupation or Bridle Roads; B.M's. being always taken within every two miles.

These, as well as the mile posts, which, of course, had their own particular designation, "*being such a mile from so and so*," were all regularly numbered. The readings also, besides being carefully checked at every observation, were compared at every B. M., and if found to differ above $\frac{4}{100}$ or .04 of a foot were gone over again.

Check levels should be carefully distinguished from trial levels. The latter cannot be done too carefully; and all the rules given for insuring accuracy should be strictly called into use in taking them; especially, that very important one, of always placing the instrument in the *middle* between the two staves.

CHAP. IX.

FINAL LEVELS.

By the term *final*, I mean the last levels previous to depositing. The levels taken after the Act is obtained are generally called "*permanent*" levels.

I have given an example of this kind of levelling in a subsequent Chapter, being a portion of a proposed line from "Hereford to Marden". To these I shall refer the reader for illustration of the remarks I may have to make respecting the best mode of conducting the Field Book.

At starting, the chief thing is to secure a good B. M., such as can be unobjectionably referred to, as a standard of reference for the DATUM LINE.

Several bills have been lost solely from imperfect standards of reference. The necessity of the point, to which all the levels

bear reference, being clearly defined and not easily mistaken, must be admitted to be great, and serious injury might be done to both public and private interests, were this point so vaguely described, as to admit of a range of some 5 or 6 feet. In the present case, had the description stopped at the word "bridge", and the bridge happened to have been a long one with a steep approach, there would have been a difference of some two or three feet, according as the staff had been placed upon the string course at the foot pier, or over the centre of the bridge.

The description in that case would have been imperfect and as it might have been so described purposely, or might have been misapplied afterwards, most probably an objection would have been taken to it, by the Opposition in Committee; and it might have shared the fate of many others, that I have alluded to. I would particularly impress the importance of this remark upon young Levellers. For I have sometimes had to send parties back from London, solely to take half a dozen levels to get a good point, from their not having taken sufficient care in the first instance in their own choice of one.

It may be depended upon, that in the description of a datum line, it is better to be too verbose, than too concise; the former may reflect upon the engineer; the latter may throw out the bill.

In the year 1846, there was rather an amusing illustration of this, in the case of a line, in which the datum line was referred to the "*soffit*" of a bridge, several Engineers on either side, asserting and denying the strict and unambiguous meaning of the term "*soffit*". The one side limiting the word to the intrado of the key stone; the other extending it to the whole intrados of the arch, from springing to springing, having a versed sine or rise, perhaps of 7 feet, to the crown of the arch. It is not for me to decide where "doctors disagree," but it would have been easy to have said *soffit at the crown;* this might (though I think not) have been unnecessary, but would not have been disputed.

Having decided upon this first point, the levels may be proceeded with.

When practicable, tracings of the survey, with the line marked upon them, should be furnished the leveller. Having these, he should either range the line first himself (as fully explained in the trial levels), or should be accompanied with an assistant to do it for him. My own opinion is in favour of the latter, as being certainly more expeditious, and as cheap. The assistant, who could of course proceed faster than the leveller, can be left to take the cross Sections of the roads.

I would also recommend that these final levels should be taken with two levels, side by side, as in the check levels of last year. *For the further consideration of this,* see page 223, &c. It is the only safe plan, and looking at the consequences of a single error in the reading of the red figures, I am surprised it has not been more generally adopted. It may be said, it is a matter of opinion: I think not: it may be *made* so, and so may every thing: time and experience however, generally settle these matters of opinion; and I have no doubt, they will do so in this case. The man who still uses the sliding vane levelling staff, when every man of experience of the present day has long abandoned it, considers it "a matter of opinion"; and, deeming himself to have as much right to have an "opinion" as any body else, clings faster to the prejudice of his youth, and goes on in his old way still.

When two levels are used, it will of course be necessary to use two staves, as in case of finding any discrepancy in the rise or fall of any observation between one level and another, if the back level has been removed to become the forward, the observation can not be repeated.

This being the case, it might be necessary to refer to the matter of opinion entertained by some, of its being better to use one staff than two: the reason they allege, is, that it is seldom that two staves are exactly divided alike.

There is no question that this is the fact; it cannot be

avoided; but what follows? Let there be two staves, A and B, A being one inch longer than B. If B be the back staff, then, A being longer, will give the forward point lower than it should be: the same point, however, becomes the next back station; and from the same staff is taken the next backward reading; this reading, therefore, will be greater than it should be in proportion to the next forward one. The last forward, and the present back reading being greater, and the errors equal, they will correct each other. In fact, wherein does this differ from a perfect staff being placed, not *upon* the ground, but *in a hole* or *rut*. The only consequence, that would result from this unavoidable disagreement between the staves, would be, that if on resuming work in the morning, the staff A was used, when the work was left off the night before with the staff B, there would be a differenee, not of an inch, but of a portion of an inch only. There might be this amount of error also in a B. M.: but no more.

So much on the side of the advocates of a "single staff." Against this, on the other hand, let the reader set the advantages of checking each observation, of not having to move the backward staff, until he has tried, if he wishes it, his backward reading again, and of the time saved by his not having to wait for the backward man, who, be it remembered, with a 5 chains sight only, would have to walk an $\frac{1}{8}$ of a mile, occupying, at a moderate pace, about 3 minutes, to say nothing of his having often to get over ploughed land; climb fences, leap ditches, go round buildings, and overcome various other causes of delay,—and then judge for himself.

I have said nothing of the immense advantage of the man, who is about to plant the forward staff, having the backward staff still left standing for him to line himself by.

I have omitted also the benefit to the leveller, when he has taken up his instrument, as he passes the forward man, in his finding the chaining completed, and booking the length of the observation at once; checking it himself on the ground by the

distance of the staff on the chain, and the pins in the follower's hands, and not being under the necessity of trusting the chaining to the men. These observations are not really wanted to prove the superiority of two staves over one: there are advantages, however, that naturally accrue from using the two, and the mentioning of them may therefore be serviceable to the reader.

The line being properly ranged for some distance a-head, you may proceed with the levelling.

Let your point of reference at startiug be termed B. M., No. 1; and be careful throughout, whenever you come to a road which requires a section, to take a B. M. and number it in your book, to correspond to the number of the road. Let these B. M's be as near as possible to where the line crosses, so as to serve easily for the Cross Levels. Whenever the main line crosses a road, cut a mark on each side the beaten Roadway *in the turf,* so that the direction of the line, and the exact point where it crosses the centre of the road, can be ascertained by the party who is taking the cross section.

When you are approaching a road in the main levels, be sure and place your level, where you can, if possible, command several points at once: the nigh side of the road, the centre of the road, the B. M., and some point beyond. This can only be done when the hedges are low, and the road is level with, or below the level of the country. If a raised road, the better plan is to fix your last forward staff, before you come to the road, on the nigh side of it, close to the hedge, and *opposite a gap;* then remove your instrument and place it *in* the road, where you can see the last station from, and at the same time command the centre of the road, the B. M., and the point in the field beyond. It is sometimes possible, when the road is pretty level and straight, to take the cross section at once, at the same time with the main levels. If the instrument is placed in the centre of the road where the line crosses 10 chains can be easily commanded on either side: the read-

ings thus obtained will be; the last point on the nigh side of the road, *the back set;*—the B. M.; the left side of the road, described as 0·00; and the right side, described as 20 chains; where the instrument is placed, as 10·00, (where the line crosses)—all belonging to *the Cross Levels,* and all *intermediates;* and the reading beyond the road, as *foreset.* It must be remembered, that the readings taken in the road, together with the B. M., must be again entered in the Cross Sections.

In crossing a river, much time is sometimes lost for want of arrangement. One of the staff-holders should be detached from the party, so as to get round to the other side by the time you arrive there, in order that, as soon as you get there, the foreset may be taken on the other side of the river, and the whole of the party may at once proceed round: very great care, of course, must be used, while reading from only one staff.

In all navigable or running waters, the surface of the water should be taken, and a special observation should be made of the previous state of the weather and the direction of the wind: this will frequently enable you to check your work roughly, by comparing the respective surface heights of the same river, crossed in different places. It must not be forgotten, also, to take each of the banks of the river.

It is not so correct to compare two sets of levels over the same ground by means of the "*surface of water,*" which has been taken at different times, and perhaps under different circumstances of wind and weather, as to refer both levels to the fixed banks which are subject to no such fluctuation. I really think, that if the advocates of either would consider this, they would see the advantage of taking both.

As the banks of tidal rivers are sometimes high, in levelling over a river thus circumstanced, the best plan is to place the level *half way up the slope,* so far from the top, that the level line will just come above it; the top of the bank can thus be ob-

tained as an intermediate, and the instrument will remain in comparatively low ground.

If the last backset has been a long one, place your forward staff beyond the river, half-way up that bank, if there is a bank there, which is not always the case. The surface of the water need not necessarily be taken, if the water is low, while the instrument is on this side of the river. By getting a lower point on the other side, which will, on the next remove of the instrument, be a backset, and can be made as much lower than the present backset, as the difference between that backset and the foreset, you may be enabled to command it then, which you cannot do now: in either case, the surface of the water will be an intermediate.

In rivers as well as at roads, B. M's should be always taken, if practicable. There is, however, sometimes much difficulty to obtain good B. M's: the country is generally flat, and the prevalent land-marks are dykes: the only thing near, perhaps, is a solitary bridge, a weir dam, a lock-gate: the last two make splendid B. M.: but the first stands too high for the purpose; the horizontal web of the telescope falls *below* the arch. In this case, instead of reading off the distance of the soffit or the string-course, whichever may be more convenient, *under* the web, by placing the bottom of the staff upon the string-course, ascertain its height *above* the web, by dropping the staff down, until the bottom of it coincides with the web. Where the distance is too great to admit of the staff-man distinguishing the motions of the leveller, let the staff be inverted, and held under the object, and let the reading of the web be taken as usual: this reading will be negative, and must be entered as such; thus:—

(*See notes on next page.*)

Back.	Inter.	Fore.	Rise.	Fall.	Reduced Levels.	Distance	Remarks.
8·20					96·60	3 miles	Bottom of bank.
	4·80		3·40		100·00		Surface of water.
	2·20		2·60		102·60	10·00	Top of nigh bank.
	–7·80		10·00		112·60	B. M.	Top of string-course centre of bridge on left.
	3·40			11·20	101·40		On far bank.
	4·60			1·20	100·20	15·40	On towing-path; at 10·20 and 15·00 river crosses.
		11·80		7·20	93·00	18·00	In field beyond.

In the example, the B. M., which was the string-course of the bridge was above the level line of the web: the other readings were taken as usual, but this one was read off the staff when inverted: the reading was 7·80, and was entered *minus.*

In obtaining the reduced levels, the difference between the consecutive readings was taken as usual, and entered in the proper columns of rises and falls; thus:—the first difference between 8·20 and 4·80, was 3·40, rise; between 4·80 and 2·20, 2·60, rise; between 2·20 and *minus* 7·80, was 10·00, *rise;* because, instead of the second point reading 2·20 *under* the level, it read minus or 7·80 *above* it. The next difference between minus 7·80 and 3·40 was 11·20, fall; because, to get to the latter point, it would not only fall 7·80, which the former was *above* the line, but 3·40 more; the next, between 3·40 and 4·60, was a fall of 1·20. To persons understanding algebra, it will be sufficient to say, that in this as in other cases, subtraction must be made by changing the sign, and adding: in the first case, between 2·20 and minus 7·80, the minus is changed, and becomes positive; and the rise being 10·00, is entered in the column of "*rise.*" In the latter case, the 3·40 being the second, is made *minus;* and both being *minus,* their sum 11·20 is entered in the column of "falls." With respect to

the observations; 8·20 is the backset, at 3 miles on the line; the river crosses at 3 miles 10·20 chains and 15·00 chains; and readings are taken on the nigh and far banks, on the towing-path, and on the surface of water; all which points are referable to the B. M., which was taken on the string-course, and lastly, the forward reading was taken in the field beyond, in the level of the country, so as to have a clear field for onward operations.

Where the river is wide, if the forward sight has to reach across it, care must be taken that the back observation should be proportionate to the width.

Should circumstances prevent this, which, from the steepness of the bank and the necessity of taking one or two observations to get down to the water, is often the case; then, as soon as you have passed the river, on the first opportunity, reverse any error that may have occurred, by taking a long backset and a short foreset. By this means, the error will be counterbalanced. The trifling difference requisite to allow for the correction, for curvature and refraction of the sight across the river, in itself is a matter of no moment, it is only the accumulation of such errors that affects the importance of the levels.

Where the width of the water is such, that an observation cannot be taken across, then by driving down a stake on each side of the river, exactly level with the water, and as near as possible at the *same time*, the approximate level can be obtained. The two stakes, of course, must be points in a line, intersecting the course of the stream at right angles; and there must not be any very strong *side* wind. If neither of these exceptions prevail, the stake on one side can be worked up to, as a foreset, and carried on from the stake on the other side, as the succeeding backset.

What the width of the stream must be to make it necessary to adopt this plan, must depend upon various circumstances. It does not follow, that, because you cannot distinguish the

the hundredths, you should not be satisfied with "guessing" the distances between the tens: a correct and experienced eye can do that very safely: When the tens are distinguishable, it would be safer to trust to the reading than to the stakes. Where the staff, however, is completely illegible, the use of the stakes is unavoidable. There will not be much error with these, when the line joining them is at right angles to the stream as mentioned above. It must be remembered, however, by the young leveller, that if the line of railway crosses the stream obliquely, the stake beyond the river will of course be a point *out of line*, from which levels must be taken along the bank to the place where the line of railway crosses, when the main levels can be resumed.

In levelling for a railway along side of a canal, in fair fine weather, the water between the locks may be taken as level. There is generally some very slight current downwards, owing to the unavoidable leakage at the lock gate, but not so much so, as to materially affect the result.

There must also have been no recent passage of a barge, or else it will not be level. A high wind will also have the same effect in destroying the level in a canal as in a running stream.

With respect to the best length for sights, where the country will admit of a choice, there seems to be some difference of opinion. I cannot myself subscribe to the theory, that long sights and few, are better than many and short: were the errors to be increased in the same way as by the Theodolite, I might be disposed to admit it; but in levelling, each observation is complete in itself, and independent of the rest; so that the fair inference is, that some errors one way may be counterbalanced by others the other way; and, were this not the case, being independent, they would only increase according to the number of sights: to say nothing of the more difficult and less accurate reading of the observation at a long distance. If the question be between very short sights, such as one chain each, and moderately long ones as 5 chains, taking all things into con-

sideration, I should prefer the longer. What any one's individual opinion may be, however, I do not consider of much moment. The person levelling has seldom much discretion in the matter; generally speaking, from 4 to 6 chains are the longest sights that can be taken, and even to take these, the ground must be pretty regular and even.

I am, however, satisfied, that 3 chains each way is the most favourable length for distinct and correct observation. Though it rarely happens, that the ground or intervening obstructions will let us take exactly the distance we desire.

Where the ground is very level and nearly equal lengths can be obtained, longer sights may be sometimes safely ventured upon, especially where the diagonal staff is used, the tenths of which are distinguishable wherever the red figures are; the points between, or the twentieths are distinguished by an angle on the opposite side to those of the tens; and the intermediate points can be easily guessed at. Long sights are specially available in the evening, when the staff is on the east side of the observer, and the contrary, when the telescope is turned towards the west. This arises from the greater rarefaction of the air, by which the reading is materially affected. An evil that is found seriously to prevail in the glare of the noon day sun, when the ebullition of the atmosphere almost makes a correct reading impossible.

In long sights much of the difficulty arises from not being able to distinguish certain *red* figures from each other: the 3 and 5 are much alike; and so are the 8 and 9; and the 9 and 6 may be easily mistaken at a distance. To remedy this I have had the red figures on my own staves marked thus: I. 2. III. 4. V. 6. 7. 8. IX. X. 11. XII. The fifties are distinguished by the angular point being filled up; and ·25 and ·75 in each foot, by a round dot. By this contrivance, it is almost impossible to mistake one red figure for another; in fact, it is

easy, at a very great distance, to read to a tenth of a foot, by means of the conspicuous subdivisions of each foot in quarters.

The consideration of the difficulty of reading long sights, brings us to that of the opposite kind, short sights, which are sometimes even more difficult than long. Where the sights are so close, that the red figures are not in the sphere of vision, the best plan is to mark down carefully first the "tens and hundreds," and then direct the staff holder to raise the staff gently from the ground, until the red figure becomes visible. Where you are too close to distinguish the subdivisions, direct the assistant to hold the field book open, close against the staff, and move it gently up or down, until the level line of the web coincides with the white edge of the paper. This is the only way to manage with an inferior instrument, which will seldom read within half a chain; while with a good one, the divisions will be visible within a couple of yards. The rack, upon which the object end runs, is in common instruments seldom made long enough. The reader, if he thinks for a moment, cannot be surprised that there should be so many bad instruments made, as few, very few makers are scientific men, and not being practical men either, they go on the same old system, as they have always done. The Leveller should therefore endeavour to make himself perfect master of the principles of construction of his instrument, as his own practice in the field will then suggest certain improvements that would never occur to the maker.

If the reader, therefore, should have the misfortune to be in possession of one of these instruments, his only plan will be to have a new eye tube, fitted to his instrument, with a rack and pinion attached, so as to move the eye end, as well as the object end. By this means he may read as close as he pleases.

It may not be out of place here, as we are talking of short sights, to say a few words to my young readers especially

Young men are all too apt to estimate their day's work, not by the difficulties overcome, but by the extent of line they have got over; and are, in consequence, too eager to commence, without regard to choosing a good B. M. first, and too heedless of localities, when they have commenced.

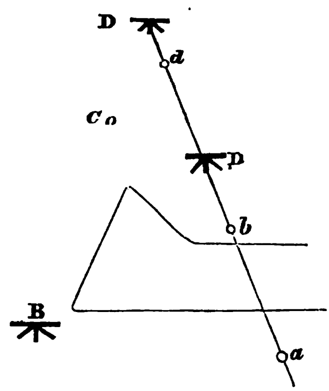
In going up or down a hill, for instance, the unavoidable slowness of the work, instead of making them more careful, renders them more impatient; and, in being so, they frequently defeat their own object. In going up a hill, they generally place the instrument too high, and after having levelled it, they find, on taking the observation, that they cannot see the staff, the level line is some feet above it: of course, the instrument has to be moved and set again. I have seen this sometimes occur three times, before they have got it right.

The staffholder too has generally got to be cured of the same mistake. Be assured, that the slowest plan on these occasions is the quickest. Errors accumulate rapidly at these times, and where discrepancies are found to exist between one set of levels and another, they will almost always be found to occur in the most hilly parts of the line. Go slowly over hills and make up for it where you come to lower and more level ground. Again, where any houses, woods, barns, or any other obstructions are in the way, the levels must of course be taken round them, all of which, in the book, will be marked "out of line". Perhaps the distance to the other side of the obstructions, may not be more than one observation, and several may be necessary to get round. Do not on that account, take your levels carelessly; in fact, the more observations, the more carefully they should be taken, as the chances of error increase with the number. Get the shortest way round, you possibly can, of course, and make use of any scheme, that your own judgment and experience may suggest to you to do so. Take out a plank of a clap-boarded fence; place the staff upon the hinge of a gate on one side as a foreset, and upon the other side, on the same hinge, as the

next backset. Take out a brick; drive one of the chain pins through a crevice; open both gates of a barn; in short, use your own good common sense, which you will generally find sufficient for every emergeney.

In overcoming such impediments as these, the young leveller may probably find much difficulty, unless he clearly understands the true definition of the terms, backward and forward readings: they do not mean backward or forward as to "direction," though in common cases they happen generally to be so; but the two ends of the link, which belong to the chain of levelling. These may or may not be points actually wanted for the section: they may be all, as in the case above, points "out of line;" but they have nothing to do with "direction."

This being understood, the reader will perceive, that in going round an obstacle, it may be necessary, in order to read the staff, to place it *beyond* the next point, whose height you require. On moving the instrument, this staff will be a backset, and the backward point either a foreset or intermediate.



Thus in the accompanying diagram: let B be the place of the instrument, and let (*a*) be the backset and (*c*) the foreset, the line running through *a b d*. Now let the instrument be moved to D; (*c*) will be backset, though it is further in the direction of the line than (*b*), which will be an intermediate,

and (*d*) will be the foreset. Again, it might have happened, that the point *d* might have been read off as the foreset, when the instrument was at B, and the instrument might have been placed beyond *d* from *b*: *d* would then be the backset, and (*b*) the *intermediate*, although back on the line, as compared with (*d*).

Some difficulty is also experienced as regards bench marks and intermediates. A B. M. may be, and generally is, an intermediate, but not so necessarily: it will sometimes fall better as a foreset. A foreset, also, is not necessarily a point in the section: in levelling across a hedge, no point in the line may be visible from where the instrument is placed; any point, therefore, must be taken which is visible: this point will be a foreset.

In permanent levels, where the heights at every chain have to be taken, this is specially useful in going up a hill: sometimes you cannot read more than one stake; it may be *nearly two*; by reading off the one stake as an intermediate, you will gain something in the ascent by taking the highest point you can see, as a foreset; with this advantage, you may succeed in reading two stakes next time.

Where young men might save time in their operations, would be in distinguishing between intermediates and foresets and backsets. While the last two form, as before observed, the chain of levelling, and any error goes on accumulating, errors in the first or intermediate are confined to themselves, and therefore need not be read off to such a nicety as the other two, especially where these intermediates are points upon the ground; of course, when they happen to be B. M.s, the same care must be taken with them as in backsets or foresets.

Additional Remarks (carried on from page 211).

As these levels are usually taken after the survey has been made, and the levellers are furnished with plans of the

country through which the line passes, and have nothing to do but to attend to their levels; the best plan of taking them is for two persons to level side by side, with two levels, but the same staves, in the same way as adopted last year in check levelling; so that each observation can be at once checked in the field. It is not possible to do without two staves, of course, as the last backward staff must not be removed, before comparing the different readings of the levels, and ascertaining whether they agree. The additional expense involved in this mode of levelling, may be objected to by some, but I do not think this difference will be found so great as may be expected, especially where the levellers are equally experienced in the matter, and are quick and ready in the use of their instruments. It must, at the same time, be recollected, that both *the main and check levels are in this way done at once*, and that there is but one set of "men" to pay, instead of two. In the common way, too, of proving the work by check levels; in case of any difference being found between the main and check levels, between any two given points, this distance has to be levelled over again. This is saved by this method: by each observation being checked at the time, the position of an error is immediately determined, and can be at once corrected.

In levelling this way, side by side, some curious facts have been elicited. First, two levels seldom find the same difference of height between the two fixed points, unless they are placed just midway between them; secondly, however much they may differ when they are not so placed, they will always give the same result, whatever the *length* of sight may be. The inference is, that though much has been said about the proper adjustment of the level, and many rules are given to secure it, still it is practically impossible to make the axis of the telescope perfectly parallel to that of the bubble: it may not be easy, perhaps, to detect any material error in any particular level; all the tests given may be apparently tried; but

two different levels, which are considered to be in adjustment are frequently found to read off differently, it is evident that these adjustments are not altogether correct. In going over twenty-five miles of country with a friend who had his own instrument, we found that when we were half-way between the staves, we read alike; when we were not so, the one was either above or below the other, according to whether we were going up or down hill; and that the amount of that discrepancy, depended upon the difference of distance of the instruments from the back and forward stations. That when the one rose higher than the other in ascending, it fell lower than the other in descending; so that, if the rise and fall of the hill was the same, the reduced levels obtained from the two instruments became the same as before. And at the end of the twenty-five miles, that the difference between the two was in proportion to the height or depth of the last point above or below the starting point; the intermediate ascents and descents having, in their results, counterbalanced each other throughout. The whole inference evidently to be drawn from this, is that, whatever the state of the instrument may be, it is better, where it is possible, to place it half-way between the staves. This, I am satisfied, is the only correct way to level; the only error in this case, to be guarded against, being misreading of the figures: all the other usual errors become neutralised.

As, in counting the lines upon a levelling staff, nothing is so difficult, as when there is nothing but straight horizontal lines one above the other; so, in levelling itself, nothing is so likely to beget errors as the mere fact of having little more to do than to read off a set of figures, one sight after another. The chances of making an error somewhere, increases with the number of sights, and the length of the line; and as this error *may occur anywhere*, the best check is that of having two levels.

That it is the best check, I am satisfied from experience;

as I have known many instances, by the usual way, where several errors were detected in various portions of a long line, and yet the result of the whole line was correct; the correctness of it depending upon the accident of the errors having corrected each other in various places; the following figures, on several occasions, having been taken the one for the other: 3 for 5, and 6 for 9; but having fortunately been taken an even number of times, and read in opposite directions.

As to the allowable limit of difference between main levels and their checks, much difference of opinion exists; but, generally speaking, there is scarcely sufficient allowed. Suppose that each observation averages 10 chains, there will be 8 of them in a mile: in each sight of this distance, it is not possible to read off to $\frac{1}{100}$ of a foot; so that if the differences should by chance fall the same way, there would be a difference in 10 chains of $\frac{2}{100}$, or, in a mile, of $\frac{16}{100}$, or ·16. Let the error be only half that, or 0·08 in a mile, that is, not one inch in a mile, (and I am satisfied that, at the usual rate at which levels are taken and run, this supposition is a low one,) this error in 25 miles will amount to nearly 2 feet.

It may, I admit, be urged against this, that the errors may in many places be contrary and destroy each other: there is no question, but what they will. In the above hypothesis however, I have not taken into account various other more serious errors that are found to result in practice: the amount of error deduced from the above, depends simply upon one thing; viz. the mechanical imperfection of the telescope, which is unavoidable. All other things have been assumed perfect. But where you take these as you really find them, and not as they ought to be, and throw them into the scale; to me, there is no question that, that ratio of error is as little as it ought to be. In the case of the staff, for instance, the staves are not all the same height; (they all profess to be, of course) one foot upon the same staff may not always measure the same: the one staff may be held more perpendicularly than the other: one staffholder may be

more careful in moving the staff round and not changing the place, nor pressing it more into the ground for a back reading than a forward one.

As regards the instrument itself; one instrument may point higher (and yet be considered in adjustment by the user) than another: one bubble may be sluggish; another may be sensitive; one bubble may be correctly turned, another badly: the centering of one instrument may be better than another; and in one case the parallax may be comparatively trifling, and in the other, considerable; to say nothing of the greater, but very common difference, of one level being regularly good for nothing, and there are many levels that I have met with literally in that state. Added to this, one Leveller may be more careful than another, may place his instrument more in the centre and hurry less over his work. These are the practical chances on the side of error; the only chance on the other side, against all these, is, that there is no reason why they should all fall one way: many of them *may balance* each other, and this is the fact. Still, looking at the question theoretically, and taking it as you find it practically, it will be found contrary to both theory and experience, that they should do so entirely: they may do so sometimes; but that agreement is purely accidental, and cannot be depended upon. The very agreement with me would make me question its correctness. Still this seems a matter of opinion, and I must leave each of my readers to think for himself. At the same time, I cannot but consider, and give it as my own opinion, that the parties who urge upon railway committees a discrepancy of two feet in a line of some 20 or 30 miles long, as a proof of non-compliance with standing orders, are arrogating to themselves a certainty they cannot attain, and misleading the committee as to the correctness of their opponents work.

A correctness greater than 2 feet in every 25 miles, could not be obtained with certainty, unless at a sacrifice of time and money, which never has been, and never would be *incurred in*

common practice, and which there are no such interests involved as may make it necessary, should be incurred, in the usual common practice of Railway Levelling.

I am speaking this advisedly, because the humbug (I can give it no better term) that I have heard endeavoured to be passed off upon some of the Railway Committee, is such, as ought to be put a stop to.

Allegations for instance of "non compliance", because the greatest "cutting" has measured 9 feet instead of its given height 7 feet. Roads objected to, because, by the cross section, they have to be raised 5 feet instead of 4 feet, which is stated in the Main section, and many other such instances of "non compliance". And this, when every professional man knows, that it is not *possible* to secure that accuracy upon the Lithographic Plans.

The vertical scale to which sections are plotted is so small, never less than 50 feet, and generally 60 or 80, or 100 feet to the inch; that it is possible to make any height read a foot more or less, according to whether the scale is placed in the centre or one side of the line. This is the case upon the paper, in the original section immediately it is completed, before the paper has had much time to contract. What discrepancies must not therefore naturally ensue, when the original section itself has contracted; and the tracing of this contracted original being transferred to the copper, its impression has contracted still further. The fact of contraction, and the extent of it, are admitted by all, but the inference deducible from it, appears either to have escaped the notice, or from a wish not to abandon so fatal a weapon of attack, against their opponents, to have been kept back by all, viz., that, as in the field work, there is (or ought to be, if the requisite certainty of all practical matters be a desideratum) an allowable amount of error, so also in the plotting, there are errors of manipulation which cannot be avoided, the existence of which in no way affects the corectness of the section, or militates against the object of the

Parliamentary deposits. It is in fact a mere waste of the time of the Committees, an abuse of the funds entrusted to their charge, for rival parties to bring and defend such allegations as these, and the result unfortunately is, not only that the time thus occupied, the money thus spent are unjustly thrown away; but that wearied and distracted with a multiplicity of meaningless and frivolous allegations, the committee are apt to lose sight of, or inadequately appreciate, the full importance of any serious errors that may be brought before them.

It is much to be regretted, that if the subcommittees are judges of matters of fact only, they do not, in the report to the standing orders committee, classify those facts; dividing them into frivolous, harmless and serious, so that the latter committee may decide upon the merits "not of the line" as a line, but of the drawings as to their *legitimate* correctness, or to the purposes for which that correctness is required.

I am dwelling long upon this subject, because it is really an important one, especially to those who have done their work best, but who have more able parliamentary agents and solicitors than their own to oppose them; in many cases, after all, it is a mere trial of skill between counsel, neither of them understanding the subject, and only furnishing from their mutual ignorance additional weapons of attack to each other. It would be far better, if the professional points could be managed directly by professional men, and not through general counsel. It is seldom the merit of the question so much as the comparative ability of the pleader that at present decides it.

Formerly, when all the learning of the age was limited to the bar, in fact, when there was no such profession as civil engineering, this system was unavoidable; but at the present day, when men are at once engineers, and at the same time, men of education and intellect, there is no occasion for it, certainly not as far as professional and technical questions are concerned,—cæteris paribus, a man is his own best pleader, as they say after a certain age, a man is "his own best physician,"

so it may be depended upon, that whether in attack or defence, the truth would be sooner elicited from the witness, and would be more clearly comprehended by the committee, direct from the engineers, than through the medium of a non conducting solicitor or parliamentary agent. I object not to their assistance or their surveillance of the case before them; this would be as absurd the other way; the leaving of legal questions, with which unfortunately all matters of business are at this day involved, to engineers, would be as bad, as entrusting engineering questions to solicitors: what I want, what I would suggest, is, that in all professional matters, the engineer should have the opportunity of putting his own questions direct to the several professional witnesses it might have been considered desirable to summon.

Were this done, the expenses of getting a bill through standing orders would be reduced one half. Engineers would only attack and defend upon good grounds; the humbug of allegations would be done away with, and the really serious errors, which are now lost among the others, would then have their due prominency and consideration.

CHAP. X.

EXAMPLE OF FINAL LEVELS.

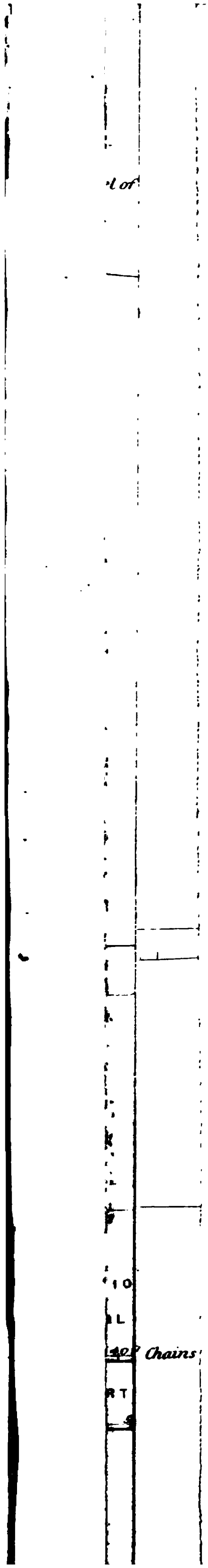
The first page of the following notes has been completed, the rises and falls calculated, and the reduced levels put in. The other pages contain only the several readings at the different stations, and the respective distances or observations to each: the rest must be filled in by the student.

FINAL LEVELS

TAKEN BETWEEN

HEREFORD AND MARDEN.

Back.	Inter.	Fore.	Rise.	Fall.	Reduced.	Dist.	Remarks.
9·96					100·00	B.M.	1. Taken in the centre on string course of Bridge over Canal.
	10·85			0·89	99·11		centre of road from Hereford to Mordiford. No. 1.
	4·81		6·04		105·15	0·67	bank of field. Road × at 0·00 and [0·65.
		6·38		1·57	103·58	1·50	
3·49		7·25		3·76	99·82	4·55	hedge × at 434
2·57		10·64		8·07	91·75	7·00	
4·46		6·54		2·08	89·67	11·00	12·30 and 1500 Mill Pond ×
4·63		3·12	1·51		91·18	12·10	on bank of Mill Pond. ×
6·33	7·33			1·00	90·18		surface of water in Pond.
		2·46	4·87		95·05	16·00	
10·18		1.08	9·10		104·15	17·50	
11·94		3·53	8·41		112·56	21·00	23·33 hedge.
4·48		4·75		0·27	112·29	25·00	26·72 hedge ×
6·08		6·43		0·35	111·94	28·00	
3·28		9·03		5·75	106·19	30·00	Road × 30 37 and 32·40 [cester.
6·75		3·14	3·61		109 80	31·65	centre of road from Hereford to Wor-
11·89	5·84		6·05		115·80	B.M.	2 on lower hinge of gate opposite
		1·83	4·01		119·86	33·50	hedge × [Crown Inn.
10·40		3·14	7·26		127·12	37·24	3724 and 3754 lane ×.
6·20	6·52			0·32	126·80		centre of lane.
		1·60	4·92		131·72	39·50	41·03 hedge ×
7·03		0·73	6·30		138 02	42·70	44·04 hedge ×
6·25		1·28	4·97		142·99	45·00	
8·13		2·67	5·46		148·45	47·00	
8·32		1·50	6·82		155·27	49·00	
828		1·48	6·80		162·07	51·00	
140·65		78·58	86·13	24·06	162·07		
78·58			24·06		62·07		
62·07			62·07		100·00		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10
L
Chains
RT

Back.	Inter.	Fore.	Distance	Remarks.
7·13				road × 52·25 and 52·85.
	0·92			centre of road from Aylston Hill to Wor-
		3·91	53·00	cester, No. 3. [height, 160·28.
0·92				
	5·90		B. M. 3	on stake driven down by side of road,
		0·37		out of line. [B. M., 160·31.
8·31		3·02		ditto
5·85		4·80	59·00	in line
3·26		9·67		out of line
6·00				
	5·25		65·50	in line
		4·03	67·00	
8·26		0·38	69·00	
7·90		2·22	70·00	70·05 hedge ×.
11·81		0·86	71·50	
10·63		1·94	73·00	
4·70	2-08		73·74	road × 73·83 and 74·15. [side of road.
		1·62	B.M.3 a	on lower hinge of gate left of line, near
4·66				[B. M., 204·02,
	4·68			centre of cart road from main road.
		5·26	75·00	
1·19		9·45	78·00	
1·17		10·57	80·00	1 mile.
1·17		10·16	1·00	
0·70		11·74	2·00	2·35 hedge ×.
1·59		10·05	2·54	
0·24		10·89	3·50	4·18 hedge ×
0·78		9·09	4·50	
0·59		10·48	5·50	
0·66		11·69	6·80	
0·56		10·16	8·00	
0·08		5·41	9·00	11·30 hedge ×
1·81		10·56	11·27	
4·91		8·41	12·00	
3·27		5·19	22·00	
5·44		5·47	30·00	ditch × 30·03 and 30·13.
6·35		5·49	38·00	
5·16		4·49	46·00	
5·54		4·22	48·67	dyke × 48·70 & 48·90.
7·66				road × 49·00 & 49·45.
	10·29			Height of water in dyke. [No. 4.
	4·63			Centre of road from Hereford to Worcester.
		9.89	50·00	

Back.	Inter.	Fore.	Distance	Remarks.
				[Height, 94·74.
9.83	2·14		B. M.4	On nearest post of railing to bridge right of
		8·22	50·55	line and far side of road, B.M. 97·57.
				50·50 hedge ×
5·10		5·83	55·21	hedges × at 52·82 & 54·59.
5·23		4·94	63·00	hedge × 55·31.
5·76		6·38	71·00	hedge × 69·06.
5·73		4·84	79·00	2 miles, [hedge × 79·39.
5·40		2·58	4·00	hedge × 3.65.
3·31		5·31	12·00	hedge × at 14·76 & 16·10.
5·74		4·29	16·50	
5·59		4·85	21·46	hedge × 1·9·54.
5·00		2·08	24·23	hedge × 25·51—ditch × 24·55 & 24·70.
4·71				road × 25·30 & 25·60. [B. M. 97·45.
	4·11		B.M. 5	on lower hinge of orchard gate left of lane
		5·24		centre of road to Shelwick, No. 5.
3·59		4·93	29·00	[Height, 96·32.
4·83		5 19	33·00	
5·75		4·74	37·56	37·67 hedge, &c.
5·83		4·75	38·62	
5·21		5·58	45·76	hedge × 43·72.
7·10		4·16	50·41	canal × at 51·00 & 51·80.
9·51	4·70		50·95	on bank of canal.
	5·74			height of water in canal.
	4·38		51·85	opposite bank of canal.
		8·97	52·50	hedge × 55·08.
6·48		6·36	59·00	road × 63·29 & 64·77. Height 101·48.
7·72	6·18			Centre of rd. from Hereford to Lem. No. 6.
		5·51	B.M. 6.	on lower hinge of gate left of lane, far side,
6·90		10·44	67·30	ditch × 71·63 & 71,90. [B. M. 102·15.
4·00		5·14	74·00	hedges × at 77·10 & 79·95.
5·50		5·15	1·00	3 miles. [Hedge × 3·10.
5·46		4·87	9·00	hedge × 6·08—fence × 10·65.
5·57		5·02	15·00	hedge × 16·00.
5·49		5·04	22·00	fence × 28·39.
5·43		4·52	30·00	
5·38		4·80	39·00	hedge × 47·22.
5·45		4·88	46·00	dyke × 47·70 & 48·00.
5·14	6·65			height of water in dyke.
		4·55	54·00	hedge × 54·47.
5·00		5·14	60·00	
5·58		5·04	68·00	hedge × 68·62.
5·82		4·65	74·00	4 miles.

Back	Inter.	Fore.	Distance.	Remarks.
5·40		4·77	2·00	
5·67		5·22	8·00	hedge × 8·37.
5·40		4·36	12·00	bank of river.
4·36				river × at 12·20 × 13·00.
	8·19			height of water in river Lug.
		4·68	13·20	opposite bank of river.
3·87		5·01	18·00	hedge × 18·53.
5·29		5·31	22·00	ditch × 23·45 & 23·60.
5·47		5·26	30·00	ditch × 25·30 & 25·40.
5·98		5·37	38·00	hedge × 39·58.
6·07		5·27	42·00	road crosses 45·88 & 46·52 [height 107 86.
7.80	5·83			centre of road from Moreton to Marden, No. 7.
	5·40		B.M. 7.	on lower hinge of gate, right of land far side.
		6·08	47·00	[B.M. 108·29.
5·33		4·77	55·00	fence × 54·68.
5·53		5·67	63·00	dyke × 64·31 & 64·50.
5·29		4·89	71·00	ditch × 75·45 and 75 57.
7·23		3·17	79·00	5 miles.
9·94		1·23	3·00	
9·35		5·97	11·00	road × 13 30 and 13·75.
3·64		5·95		centre of road to Marden Church, No. 8,
5·01	6·67		B.M. 8.	[height, 122·27.
		6·06	15·00	on highest point of stone opposite cottage,
7·64		7·21	16·70	[B. M., 120·60.
0·78		11·02	17·80	
4·26				hedge × 16·58.
	9·76			bank of river.
		5·33	20·00	river × 17·90 and 18·60.
4·41		5·29	28·00	height of water in river Lug.
5·08		4·89	34·00	hedge × 32·23.
5·77		5·00	40·00	hedge × 34·09.
5·74		4·79	48·00	hedge × 42·10.
5·82		4·92	56·00	hedge × 50·48.
5·58		4·85		road × 59·70 and 60·20. [No. 9,
4·90		5·10	61·00	centre of road from Wellington to Marden,
4·92				height, 113·60.
	4.94		B. M. 9.	stone on which a × is made, B.M., 112·78.
		4·94	63·00	right of line far side of road.

Explanations of the Field Notes, from Hereford to Marden.

Each reading, whether backset, foreset, or intermediate, will be found to have a remark; thus, the first, the backset, 9·96, has its reduced level, 100·00 which was assumed, being B. M. No. 1, which is described in the column of remarks. Of the next reading, 10·85—the reduced level is 99·11, and the point, the centre of the road from Hereford to Mordiford; the distance of the centre of this road is not given, but the two sides, 0·00 and 0·65, are given in the next line; the two following readings are 4·81, which is at 0·67, *just beyond* the road, and 6·38, which is at a distance of 1·50. Throughout these notes the same arrangement will be found to prevail. The height of the centre of the roads is taken and entered as *such*, and the distances of each side of the road given in the next line. This is also the case with rivers; the surface of the water, the nigh bank, and far bank, are noted in three lines; and the *points of crossing* of the banks are given upon *another* line. The object of this is, by means of the blanks left in the column of distances, to call the attention to these principal points. B. M's. will be found at these roads numbered 1, 2, 3, &c., to correspond with the roads, which, in the levels of approaches are called—cross sections, Nos. 1, 2, 3, &c. Immediately after B. M. No. 3, there are two observations out of line, and also two more out of line after 59 chs. The chaining goes on up to 80 chains or 1 mile, and then begins again. The distances being all measured within each mile from the starting point, according to the plan recommended at page 238.

At 1 mile 48·67 chs. the foreset is 4·22, which is the bank of the dyke. In most instances, the reader will find the first intermediate placed collaterally with the backset. This is not the case, however, with the 10·29, which is placed in the line below the 7·66. This is merely to allow space for the remarks, which, from the dyke and road coming together, are

somewhat numerous, and this arrangement has been adopted where necessary, throughout.

When not unavoidable, I much prefer the method of having no line without its distance and reduced level. The distance and description of each, it must be remembered, refer to the foresets or intermediates, and not to the backsets, which, being the same point as the previous foreset, have been already described as such. It is the referring to the same point (which is done by this means) in the adjacent links of levelling or settings of the instrument, that the connection between the two, and therefore the general connection of the whole is obtained.

With reference to the description of the B. M., No. 4, the reader will see clearly that in matters of right and left, of far and near, some fixed standard of direction must be adopted. We must, therefore, understand, that in all cases of the sort, the leveller is supposed to be standing in the direction, *in which* he is levelling, and consequently the side of the road that is nearest the beginning of the line is the "*nigh*", and the other the "far" side, and that it is also in relation to this position, that the terms right and left are spoken of. The young leveller will do well to consider carefully the force and certainty of this description, viz., *on nearest post of railway to bridge, right of line, and far side of road.* The next B. M., No. 5, is described as on lower hinge of (orchard) gate, left of line. There was, in this case, no need to mention which side of the road, as there being but an orchard on one side, it was limited to the side on which this orchard was. The following on No. 6, has "*left of line, far side of road.*"

Let not the young hand consider, because he can at once follow me, and admit the propriety of what I am stating, that all this must have occurred to him. It has not been so with young men generally, time and practice might, by its faults and omissions, have taught him this or perhaps some better plan, but he may be assured that it will be an economy of both, for him to thoroughly consider, and rigidly carry out the hints I have suggested.

CHAP. VIII.

PLOTTING OF THE MAIN SECTION.

DRAW first the DATUM LINE in ink, carefully and finely, then lay off the several miles upon it, by means of 12 inch ivory rule. The smallest scale allowed for railways sections, is 20 chains to the inch horizontal, formerly this scale used to be generally adopted, and the plan, of course, was made to the same scale. Latterly it has been in many cases abandoned; the plan being made to this small scale, whenever the line crossed any roads and came near any buildings, an enlarged plan to a scale of not less than 100 feet to a $\frac{1}{4}$ inch was required. On this account, it was thought better to plot the whole to some scale sufficiently large not to require any enlarged plan, hence, 5 chs., 6 chs. to the inch, have been frequently used, 100 feet to $\frac{1}{4}$ inch is 400 feet to the inch, and 6 chains is 396 feet to the inch, so that the scale of 6 chains answered all purposes. The vertical scale, too, was fixed at not less than 100 feet for the main section. This, however, the promoters of a line themselves found to be too small, as though it made real errors less easy to be detected, yet it magnified small ones much more than a larger scale. Vertical scales, therefore, of 60 and 80 feet to the inch have been usually adopted.

Whatever scale, however, may be used, the distances must be carefully gone over once or twice, to guard against error; and the number of miles must be marked against each. The method of chaining on a line from the beginning, and dividing the length into miles on the ground, confines every error of plotting within that distance, as the distances are always taken from the last mile point—so many miles, *plus* so many chains, till the next mile mark, when it begins again.

At every third mile, as well as the beginning and ending, erect perpendiculars to the line, geometrically by the compasses, to about twice the average height of the Section and draw a line through that distance parallel to the datum line; measure off upon this line the intermediate miles, and connect them with the corresponding points in the datum line, and divide each mile into furlongs.

Now, lay off, upon the datum line, the several distances in the column of distance for one mile, measuring them from the nearest furlong, then draw very fine lines therefrom, parallel to the perpendiculars. These parallels, or supposed perpendicular lines, being confined within every mile, cannot be far wrong. In fact, the correctness of the section line depends upon their being truly perpendicular, or the distances in the section line, though correctly measured on the datum line, might be incorrectly projected.

Upon these perpendiculars prick carefully off, with a fine needle, the several heights in the reduced levels corresponding to the distances, carefully distinguishing the roads and rivers, the position of hedges, &c., as in the accompanying section, and connect their several heights; *put this in ink at once,* while the plotting is fresh in the memory, marking off in pencil the "reduced level" of the centre, against every road, the surface of the water of rivers, canals, &c.

One mile being now finished, and not before, proceed to the next; finish that before you commence the following, and so on throughout.

CHAP. XI.

CROSS SECTIONS

Of roads between Hereford and Marden.

No. 1. From Hereford to Mordiford.

Backset.	Inter.	Foreset.	Rise.	Fall.	Reduced Levels.	Distance.
						0·00
4·86		7·19		2·33	100·00	6·00 *
0·58	1·47			0·89	99·11	B. M. 1.
		8·08		6·61	92·50	9·00
3·73		4·96		1·23	91·27	15·00
4·28		4·78		0·50	90·77	18·00

No. 2. From Hereford to Worcester.

Backset.	Inter.	Foreset.	Dist.
1.50			0·00
		6·13	6·00*
8·68	2 54		B. M. 2.
		2·07	11.00
5·78		4·96	16·00

No. 3. From Aylston Hill to Worcester.

Backset.	Inter.	Foreset.	Dist.
0·90			0·00
		10·53	2·00
0·87		10·16	4·00
1·07		8·58	10·00*
0·62	0·49		B. M. 3.
		10·68	20·00

No. 4. From Hereford to Worcester.

Backset,	Inter.	Foreset.	Dist.
5·33			0·00
		5·34	6·00*
4·62	2·03		B. M. 4.
		5·00	10·00
6.16		6·48	17·00

No. 5. From Moreton to Marden.

Backset.	Inter.	Foreset.	Inter.
4·70			0·00
		5·19	4·00*
5·88	4·00		B. M. 5.
		5·07	6·00
5·13		5·35	10·00
5·20		5·30	16·00

No. 6.
From Hereford to Leominster.

Backset.	Inter.	Foreset.	Dist.
6·40			0.00
		2·53	5·00
5·37	4·58		B. M. 6.
	5·25		6·00*
		7·02	9·00
5·40		6·90	15·00

No. 7.
From Moreton to Marden.

Backset.	Inter.	Foreset.	Dist.
6·26			0·00
		6·24	6·00*
4·83	4·40		B. M. 7.
		7·35	12·00
4·46		6·54	18·00

No. 8.
Road to Marden Church.

Backset.	Inter.	Foreset.	Dist.
2·66			0·00
		7·43	4·00*
7·91	6·25		B. M. 8.
		6·23	6 00
3·64		3 86	10·00
7·23		7·43	16·00

No. 9.
From Wellington to Marden.

Backset.	Inter.	Foreset.	Dist.
0·99			0·00
		6·59	6·00*
5·62	5·40		B. M. 9.
		5·43	12·00
4·54		5·40	20·00

* *The rail line crosses here.*

CHAP. XII.

LEVELLING AND PLOTTING OF THE CROSS SECTIONS.

These sections are taken for the purpose of showing the various slopes of the roads which are crossed by the line. Where the road has to be crossed on a level, and the surface of the rails happens to be on the same height as the road surface, no alteration will have to be made, and these sections will not be required. Again, should the line run at that

point through a cutting, and be so far beneath the road surface, as to admit a bridge of the proper height to be thrown across without raising the road; if in that case, also, these sections should be unnecessary. But, if on the other hand, in order to accommodate the line, the road should have to be altered, whether raised or lowered is a matter of no moment, a section of it would be indispensable. As it is not possible, before the engineer has decided upon his gradient, to know whether any one section will or will not be wanted, the sections of all the cross roads are always taken; if not required afterwards, they need not be inserted. It seems almost unnecessary to mention, that it is only by these sections, when a road has to be raised or lowered, that the extent of the approach and the nature of the arrangements can be determined. The necessity of raising or lowering the road, will of course depend upon the information to be obtained from the MAIN SECTION, as may be seen at the following roads: viz. No. 2, where the road is *raised*, and passed under; at No. 4, where it is *lowered*, and passed over; at No. 7, where it is raised, but crossed on a level; again, at No. 8, where it is raised, and passed under; but that the approach of No. 8 will extend farther on the right than on the left side, the slope remaining the same on either, depends upon the fact of the ground rising on the left and falling on the right of where the line crosses; which fact can only be obtained from the CROSS SECTION.

The object of these cross sections being now briefly explained we will proceed to show the best method of taking the levels necessary to obtain them.

Having, in the course of proceeding with the main section, carefully marked upon the ground the exact spot in the centre of the road, where the line crosses, and taken the height of the nearest bench mark to it, about 8 or 10 chains on either side of this spot are measured, *in chains' lengths*, along the road, for the purpose of obtaining the height of the road at every chain.

In taking the cross levels, be careful always to level one way,

from left to right, that is, measure always from the left side of the line (looking in the direction the main section has been levelled, or towards the termination of the line), to the centre, where the line crosses, onwards to the right side of the line. And be careful to designate the road in the same way, always from left to right; as, in case of your omitting to mention, that you have levelled the other way, the consequences ofn eglecting it, may be serious. For, unless the number of chains levelled on either side, where the line crosses, be always the same, which, depending upon the inequality of the road, is seldom the case, the line would cross in its wrong place, and the calculations for lowering or raising the surface, for the bridge across it, would be altogether incorrect.

Call these chains' lengths, therefore, in your level book (*beginning* as far, on the left of the line, as from the nature of the ground may be necessary, and numbering to the centre), 1 chain, 2 chains, and so on.

Place your level at a point, half way between 0·00 and where the line crosses, and take the several readings at every chain's full length, and also at the spot where the line crosses, and should the bench mark, before alluded to, be on that side, take that also; then remove your level to midway between the centre, where the line crosses, and the extreme point you intend levelling to, and take the several readings there also.

Enter these in your book, according to the examples given, making the reading at 0·00 your first backset, at 1 chain your first intermediate, placing it in the next lower line, so that you may be enabled to mark the height of the starting point 0·00 in the column of distances, thus, in the cross section No. 3 the back set, at 0·00 is 0·90. Mark all the subsequent readings, taken at the same place, *intermediate*, except that, where the line crosses, which should be the first *foreset*, saying in every case, in the column of remarks, line ×. The B. M. will be intermediate, as usual.

It may happen that the road slopes too rapidly to take the whole 10 chains at once, or it may be nearly level, and sufficiently straight to command the whole 20 chains at one setting of the instrument; in the former, the leveller has only to decrease, in the latter, to increase his intermediates, taking up the B. M. as he goes along. Thus at cross section No. 3 (which, however, being unaltered is not shewn among the sections), from 0·00 to 2·00 chains, there is a fall of nearly 10 feet, consequently the Level must be placed almost opposite to 1 chain; this is also the case with the next observation, which only commands 2 chains more.

Having now obtained all the Field data necessary, calculate carefully the rises and falls, using the ordinary checks to ensure accuracy. Then turn to the notes of your main section, and observe there the height of the B.M., and of that part of the road where the line crosses; enter these in the column of reduced levels, as the reduced levels of the same points taken in the cross section, and, determining the subsequent portion of the Section as usual, *reverse the calculations of the upper portion.* By this means you obtain the several heights of the road relatively to the whole section, and as the height of where the line crosses, obtained by the section levels, should be the same as that by the main section, you have a certain check upon the general accuracy of the levelling, and are enabled to ascertain whether the staff has on the cross levels been held in the same place for the centre of the road as in the main levels.

In case of the road being steep the student will see clearly that the wrong position of the staff in taking the cross sections may make a difference of a foot or two. By taking the *right* B. M., which is a matter *of correct determining,* this point can be easily ascertained; for if, on reducing the levels on the cross section, the height of the point on the road, where the line is supposed to cross, does not agree with that of the main levels, the proper point can be easily ascertained by a simple rule of proportion; for if the centre thus assumed is too

low and the next chain too high, it will lie between the two, at that distance from the one, which, reckoning at the rate of increase from one to the other, will rise to the point equal to the height on the main section. This point thus ascertained must be entered in the remarks as the point where the line crosses. Thus, suppose the known height is 12 ft. 50 inch., and the height at 10 chains, assumed as the point of the line crossing, is 12 feet, but at 11 chains it is 13 ft. then at 10·50 chs., the height will be the required height 12·50, enter therefore in the *remarks* "*at* 10·50, *line crosses.*"

In calculating the reduced levels of the cross sections, transfer the reduced level of the B. M. from the main levels to the cross section, wherever it may fall, and for all points below, treat them as usual, by deducting for the falls, and adding for the rises; but for the points *above* reverse this method, add for the falls and deduct for the rises. By consulting the first example of cross sections given in the book which has been fully worked out, this will be easily understood. Having worked out all the reduced levels, proceed to plot the several cross roads, numbering them carefully, No. 1, 2, 3, &c., to agree with the corresponding roads in the main section, marking in each where the line crosses. The *names* of the roads are given in the main sections without the numbers, in the cross sections only the numbers to correspond.

The scale for the *Cross* Sections is fixed by standing orders, being 5 chains to the inch horizontal, and 40 feet vertical.

Formerly all cross sections had the same DATUM LINE to them as the main sections, reduced, *to get them in*, by 50 or 100 feet; at present, the ground line is shewn without a DATUM by which means much trouble is saved, and less opportunity is given to opponents to detect errors. In the examples given, the first only is given complete; with all the requisite writing of "1 in 20," "present surface of road," "road when altered," &c., the others have merely the ground line, level of rails, and the proposed mode of alteration without any working

shown upon them. They are all exactly alike and therefore one example is sufficient for the whole.

The section being now complete, the field book is no longer wanted, and the whole is handed over to the engineer, for him to put in the gradients. The selection of them, to overcome such local difficulties, as may present themselves, of hills or vallies; of roads and rivers, must depend upon his judgment and experience, no *general* rule can be given respecting them.

CHAP. XIII.

APPROACHES.

Method of determining the height that the several roads should be raised or lowered, and whether they should be passed over or under.

In the explanation of this subject, the student must refer to the example of the Plan and Cross Section of the line from Hereford to Marden; which he had better plot for himself, and compare with the plate. The gradients to enable him to understand how these calculations are to be made, have been assumed as of certain heights at certain distances: the assumed data, being the first three columns, and the results will be found in the following schemes.

SCHEME THE FIRST.

No. of Gradients.	Distances on the Section. Mls.	Furl.	Chs.	Heights.	Rise.	Fall.	Lengths of Gradient in chains.	Rate of inclination.	Value in feet for each chain.
	0	0	0	100					
1					15	—	90 chains	1 in 396	$\frac{1}{6}$
	1	1	0	115					
2					—	5	70 —	1 in 924	$\frac{1}{14}$
	2	0	0	110					
3					5	—	76·60	1 in 1011	·0653
	2	7	6·60	115					
4					—	—	83·40	Level.	·00
	4	0	0	115					
5					6	—	90·00	1n 990	$\frac{1}{15}$
	5	1	0	121					
6					—	8	43·00	1 in 437	·186
	5	6	3	113					

SCHEME THE SECOND.

No. of Road.	No. of Gradient.	Distances on Main Line. Mil.	Fur.	Chs.	Distances on Gradient in chains.	Height of Gradient.	Height of Ground.	Above Ground.	Below Ground.	Mode of Alteration of Roads.
1	—	0	0	0	0·00	100·00	99·11	—·89	—	To be raised 10 in. and crossed on a level.
2	1st.	0	3	1·01	31·01	105·27	109·80	—	4·53	To be raised 14 ft. 6 in. and passed under.
3	—	0	5	2·55	52·55	108·76	160·28	—	51·92	Level unaltered.
4	2nd.	1	4	9·22	39·22	112·20	94·76	17·46	—	To be lowered 2 ft. 6 in. and passed over.
5	3rd.	2	2	5·45	25·45	111·66	96·32	15·34	—	To be lowered 4 ft. 8 in. and passed over.
6	3rd.	2	6	4·03	64·03	114·15	101·48	12·67	—	To be lowered 7 ft. 4 in. and passed over.
7	5th.	4	4	6·20	46·20	118·08	107·86	10·22	—	To be raised 10ft. 2 in. & crossed on a level.
8	6th.	5	1	3·52	2·52	120·47	122·27	—	1·80	To be raised 17 ft. 2 in. and passed under.
9	6th.	5	5	99 5	49·95	113·46	113·60	0·14	—	To be lowered 2 in. and crossed on a level.

The FIRST SCHEME gives the several gradients, where they begin and end, which is shown in the *second* column: their heights at either end, which are arbitrary, dependent upon the judgment of the engineer, are given in the *third* column. The *fourth* and *fifth* columns show whether the gradient rises or falls, and how much. The *sixth* gives the actual length of the gradient. These, of course, are obtained from the preceding columns—being the bases and perpendiculars of right-angled triangles—whose hypothenuses or rates of inclination have to be calculated. In the first gradient, for instance, which rises 15 feet in 90 chains, or 5940 feet, $\frac{5940}{15}=396$; or in every 396 feet of length, the gradient of the line of railway rises 1 foot. This is the *seventh* colnmn. The rate of inclina tion of each gradient must be entered upon the section. Again, in the same example, if the line rises 15 feet in 90 chains, it will rise $\frac{15}{90}$, or $\frac{1}{6}$ in every chain. This expression of the rate of inclination is not shown anywhere in the section, but is indispensable in obtaining for the cross section the height of the gradient at the crossing of the roads. It is given in the *eighth* column.

The SECOND SCHEME is prepared for the purpose of ascertaining the difference at any road between the ground line and the gradient line, and thence deciding upon the mode of conveniently altering the approach, to admit of the passing of the railway.

The *first* column gives the Nos. of the road; the *second* column shows what gradient it is in; the *third*, its distance upon the section, so many miles, furlongs, and chains. This distance is obtained from the Field Notes. The *fourth* column gives its distance upon the respective gradient, which can be obtained from the first scheme. Thus in the fifth road, by turning to the Field Note, you will find the second road comes at 2 miles, 2 furlongs, 5 chains, 45 links; which, on inspection of the first scheme, will be found to fall within the third gradient, which commences at 2 miles. The distance of

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No. of Road.	No. of Gradient.	Distances on Main Line. Mil.	Fur.	Chs.	Distances on Gradient in chains.	Height of Gradient.	Height of Ground.	Above Ground.	Below Ground.	Mode of Alteration of Roads.
1	—	0	0	0	0·00	100·00	99·11	—·89	—	To be raised 10 in. and crossed on a level.
2	1st. (1–3)	0	3	1·01	31·01	105·27	109·80	—	4·53	To be raised 14 ft. 6 in. and passed under.
3	—	0	5	2·55	52·55	108·76	160·28	—	51·92	Level unaltered.
4	2nd.	1	4	9·22	39·22	112·20	94·76	17·46	—	To be lowered 2 ft. 6 in. and passed over.
5	3rd. (5–6)	2	2	5·45	25·45	111·66	96·32	15·34	—	To be lowered 4 ft. 8 in. and passed over.
6		2	6	4·03	64·03	114·15	101·48	12·67	—	To be lowered 7 ft. 4 in. and passed over.
7	5th.	4	4	6·20	46·20	118·08	107·86	10·22	—	To be raised 10ft. 2 in. & crossed on a level.
8	6th. (8–9)	5	1	3·52	2·52	120·47	122·27	—	1·80	To be raised 17 ft. 2 in. and passed under.
9		5	5	99 5	49·95	113·46	113·60	0·14	—	To be lowered 2 in. and crossed on a level.

The FIRST SCHEME gives the several gradients, where they begin and end, which is shown in the *second* column: their heights at either end, which are arbitrary, dependent upon the judgment of the engineer, are given in the *third* column. The *fourth* and *fifth* columns show whether the gradient rises or falls, and how much. The *sixth* gives the actual length of the gradient. These, of course, are obtained from the preceding columns—being the bases and perpendiculars of right-angled triangles—whose hypothenuses or rates of inclination have to be calculated. In the first gradient, for instance, which rises 15 feet in 90 chains, or 5940 feet, $\frac{5940}{15}=396$; or in every 396 feet of length, the gradient of the line of railway rises 1 foot. This is the *seventh* colnmn. The rate of inclina tion of each gradient must be entered upon the section. Again, in the same example, if the line rises 15 feet in 90 chains, it will rise $\frac{15}{90}$, or $\frac{1}{6}$ in every chain. This expression of the rate of inclination is not shown anywhere in the section, but is indispensable in obtaining for the cross section the height of the gradient at the crossing of the roads. It is given in the *eighth* column.

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1	—	0	0	0	0·00	100·00	99·11	—·89	—	To be raised 10 in. and crossed on a level.
2	1st.	0	3	1·01	31·01	105·27	109·80	—	4·53	To be raised 14 ft. 6 in. and passed under.
3	—	0	5	2·55	52·55	108·76	160·28	—	51·92	Level unaltered.
4	2nd.	1	4	9·22	39·22	112·20	94·76	17·46	—	To be lowered 2 ft. 6 in. and passed over.
5	3rd.	2	2	5·45	25·45	111·66	96·32	15·34	—	To be lowered 4 ft. 8 in. and passed over.
6		2	6	4·03	64·03	114·15	101·48	12·67	—	To be lowered 7 ft. 4 in. and passed over.
7	5th.	4	4	6·20	46·20	118·08	107·86	10·22	—	To be raised 10 ft. 2 in. & crossed on a level.
8	6th.	5	1	3·52	2·52	120·47	122·27	—	1·80	To be raised 17 ft. 2 in. and passed under.
9		5	5	99 5	49·95	113·46	113·60	0·14	—	To be lowered 2 in. and crossed on a level.

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1	—	0	0	0	0·00	100·00	99·11	—·89	—	To be raised 10 in. and crossed on a level.
2	1st.	0	3	1·01	31·01	105·27	109·80	—	4·53	To be raised 14 ft. 6 in. and passed under.
3	—	0	5	2·55	52·55	108·76	160·28	—	51·92	Level unaltered.
4	2nd.	1	4	9·22	39·22	112·20	94·76	17·46	—	To be lowered 2 ft. 6 in. and passed over.
5	3rd.	2	2	5·45	25·45	111·66	96·32	15·34	—	To be lowered 4 ft. 8 in. and passed over.
6		2	6	4·03	64·03	114·15	101·48	12·67	—	To be lowered 7 ft. 4 in. and passed over.
7	5th.	4	4	6·20	46·20	118·08	107·86	10·22	—	To be raised 10ft. 2 in. & crossed on a level.
8	6th.	5	1	3·52	2·52	120·47	122·27	—	1·80	To be raised 17 ft. 2 in. and passed under.
9		5	5	99 5	49·95	113·46	113·60	0·14	—	To be lowered 2 in. and crossed on a level.

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1	—	0	0	0	0·00	100·00	99·11	—·89	—	To be raised 10 in. and crossed on a level.
2	1st.	0	3	1·01	31·01	105·27	109·80	—	4·53	To be raised 14 ft. 6 in. and passed under.
3	—	0	5	2·55	52·55	108·76	160·28	—	51·92	Level unaltered.
4	2nd.	1	4	9·22	39·22	112·20	94·76	17·46	—	To be lowered 2 ft. 6 in. and passed over.
5	3rd.	2	2	5·45	25·45	111·66	96·32	15·34	—	To be lowered 4 ft. 8 in. and passed over.
6		2	6	4·03	64·03	114·15	101·48	12·67	—	To be lowered 7 ft. 4 in. and passed over.
7	5th.	4	4	6·20	46·20	118·08	107·86	10·22	—	To be raised 10ft. 2 in. & crossed on a level.
8	6th.	5	1	3·52	2·52	120·47	122·27	—	1·80	To be raised 17 ft. 2 in. and passed under.
9		5	5	99 5	49·95	113·46	113·60	0·14	—	To be lowered 2 in. and crossed on a level.

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this point, therefore, upon its own gradient, will be 25·45 chs. This distance will be put into the *fourth* column. The next column, which is the *fifth*, contains the height of the gradient line at the point 25·45 chs., being the centre of the road No. 5. This is obtained from the previous column, No. 4, multiplied into the *last* column of the FIRST SCHEME, which gives the rise in feet of every chain of this distance; thus, on turning to the first scheme, the rise for the third gradient will be ·0653 of a foot per chain: 25·45 chs. multiplied by ·0653 will give 1·66 feet; which, as the gradients rise, (denoted by the column into which the amount of rise, viz. 5 feet, is placed—see first scheme,) will have to be added to 110, the height of the gradient line at its commencement, thus making the height of it at the road 111·66 feet. This height is entered in the *fifth* column. Again, on turning to the Field Notes, you will find the height of this road to be 96·32, which is entered in the next column, No. 6. The two following columns merely give *the difference* between the gradient line and ground line—entered on one or the other, according as the *gradient line* is above or below the ground line. In the case of the road, No. 5, which we have been considering, it is 15·34 feet *above* the ground line.

It must be clearly understood here, that thus far these columns are all means to an end: the end or object of the whole is the result to be obtained in the last column, No. 9; which explains *whether the road is to be lowered or raised—how much—aud whether it is to be passed over, or under, or on a level.*

"Whether it is to be raised or lowered," depends upon the judgment of the Engineer, applied to the consideration of local difficulties, who, in his decision, must be guided by the cross section, which has been for this purpose prepared for him, and by the nature of the adjoining main section, and the character of the country on either side of the line.

It might happen that the present road surface was already raised, and it might not be desirable to increase the ascent, as the approaches, in doing so, might become too long. On the other hand, it might be already a sunken road, or on a level, with a flat and marshy district, and if lowered could not easily be drained. Again, in the particular case under consideration, it might be both inconvenient and expensive to do either; while by passing on a level, which the slight traffic in the neighbourhood could not make objectionable, every difficulty might be got rid of. As to the power of establishing the fact of want of traffic, in case of opposition, and therefore satisfying the Committee to allow of its being passed upon a level, that would, of course, depend upon the Promoters or Solicitor of the Bill. In my own opinion, I think the Committee are not acting with sufficient forethought, in allowing any roads to be passed on a level, unconditionally. The Company should be bound, at any future time to throw a bridge over the railway, whenever the increased traffic of the neighbourhood required it.

As to *how much* the road has to be raised or lowered, a few words need only be said. It is a mere matter of calculation, depending upon the previous decision.

When a road approach is passed over by the line, the usual distance allowed from the surface of the road to the gradient line or surface of rails, is 20 feet. The 20 feet are obtained in this way: 16 feet from the road to the soffit of the arch; 1½ feet for the voussoirs; ½ foot for the puddling; and 2 feet for the ballasting of the railway. The height may be reduced by using strong iron girders.

When the approach is passed under—as one foot of metalling, as it is termed, is quite sufficient for a common road approach—whereas 2 feet of ballasting are wanted for the sleepers, 19 feet will be sufficient: this also can be reduced by the same means. In the examples, however, and the calculations given in the book, when the road goes *under* the

railway, 20 feet are allowed. When the road goes *over* the railway, 19 feet are sufficient.

These being premised, let us proceed to the consideration of the last column; and, as an example of the mode of determining the alteration to be made, we will continue to use the 5th road. On turning to the second scheme, to the seventh column, and running the eye down to the 5th road, the gradient line will be found at this road to be 15·34 feet above the ground line. Now for the *railway* to go *over* the road, this distance should be 20 feet; deduct, therefore, 15·34 from 20 feet, which will leave 4·66 feet or 4 feet 8 inches. This distance, as the gradient is fixed, must be obtained by lowering the road. The following entry will therefore have to be made in the ninth column; viz. *Road to be lowered* 4 *feet* 8 *inches, and passed over.*

Take, as another example, Road No. 8. The gradient line of the road, on inspection of the 8th column, will be found 1·80 feet *below* the ground line. Now, whenever the railway has to go under the road, it should be 19 feet under it; at present it is only 1·80: deduct 1·80 from 19 feet, this will leave 17·20 feet, or 17 feet 2 inches; which is the height that the road will have to be raised, to allow the railway to go under it. This remark upon Road No. 8, will be found in the last column; viz. "Road to be raised 17 feet 2 inches, and passed under." The other roads, the reader is recommended to work out for himself. The only explanation that may be useful to him in doing so, is, that when the surface of the rails crosses the road 20 feet, or more than 20 feet above or below it, the road remains the same, *the level is unaltered.* This is the case with Road No. 3. And that when, as at Road No. 7, the gradient line is so much above the road, that it would not be prudent to raise the road high enough for the railway to pass under, and yet, at the same time, requiring the road to be so much lowered, in order to pass over it, that it would be beneath the bed of the river; it becomes neces-

sary to raise the road to the height of the railway, and cross it on a level.

The column (No. 9) thus obtained has to be transferred to the main section with the remarks to eaeh road entered at their proper places, as at road, No. 2, for instance, and the same plan prevails throughout, there is entered "*Road from Hereford to Worcester to be raised* 14 *feet* 6 *inches, and passed under.*" *See Cross Section, No.* 2.

This will now bring us to the consideration of the Cross Sections themselves, and the mode of representing these arrangements upon them. Thus still referring to Road No. 2, as a example, on looking to the 8th column of the Second Scheme, the level of rails will be found 4 ft. 6in. under the ground line. It has been already said, that previously to delivering the sections to the engineer, the point where the rail line crosses the approach, should be marked by a vertical dotted line. Mark off now therefore upon this line the required position of the level of the rails; represent this as shewn in the example, 4 feet 6 inches, under the ground line, and from this point upwards always upon the same vertical line lay off 19 feet, for the centre point of the road surface.

Then to obtain the slope of the approach, refer back to the main section, and observe whether it is a turnpike road, a high road, or occupation road. A high way, a public road are synonomous with a high road. The turnpike road must not be steeper, than 1 in 30; the high road, than 1 in 20; and the occupation road, than 1 in 15. The quickest plan to lay off this slope upon the sections is to calculate in each case what will be the length of the base of a right angled triangle, whose height is 19 (or 20 feet as the case may be,) to make the hypothenuse the required slope; a slope of 1 in 30 (in the case of the turnpike road) will make this base $30 \times 19 = 570$ feet $= 8{\cdot}64$ chains. This distance must be laid off upon the level of rails line on each side of the vertical dotted, or centre of road line, and the points thus obtained joined to the centre

point of the road. These lines will represent the surface of the road, and will be limited as to their length by the ground line. In the given example No. 2, as the line of railway happens to cross in the lowest part of the road and the ground rises in each side, the slopes fall short. The same system should be adopted with the other roads. The distance for the public road, or highway will be $20 \times 19 = 380$ feet = 5.75 chs. Roads 2 and 4 in the section are examples of the former, being turnpike roads, and are therefore 1 in 30, having their distance on each side 8·64 chs. The others are all high roads, of a slope 1 in 20, with the distance of 5·75 chains on each side. Of course, it will make no difference, the reader must be aware, whether the level of rails be above or below the ground line, as in the former case in roads No. 4, 5, and 6, or in the latter as in the example just referred to No. 2. It will only be the same triangle turned upside down, the lines of the slopes will still be bounded by the ground line, only in the former case they will be beneath the ground line; in the latter case, above it. In this example of cross section No. 2 will be seen all the lines and remarks required by "standing orders." It is imperative to shew the ground line, the level of roads, the height, and approach of the proposed road, to distinguish them as the present and altered snrface of road, and to enter upon each their different slopes. Thus in the same example, the reader will find the present surface of road marked 1 in 25 on the left and 1 in 23 on the right, and the surface of road when altered 1 in 30. All the sections are required to be done in this way. In the plate however the reader will find the lines only shewn upon the other sections. The repetition of the same terms would have been of little benefit and from the quantity of writing would been added materially to the expence of the work. The writing, therefore, has been omitted.

CHAP. XIV.

SUPERFICIAL AREAS.

On the mode of calculating the superficial areas, occupied by the railroad.

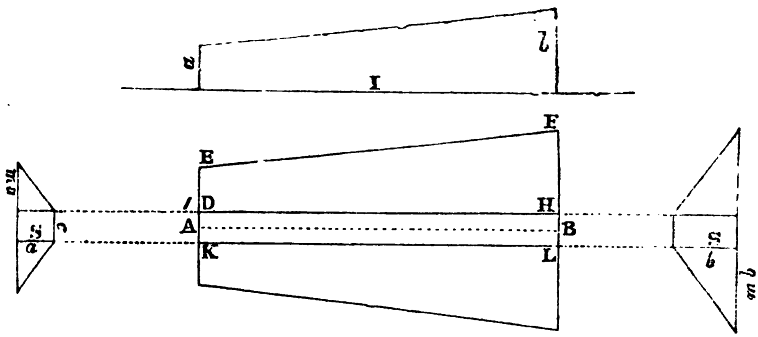
Having determined the effects of the line in the altering of the approaches, the next thing to be ascertained, is the superficial area that it will occupy, so as to estimate the expence of purchasing the ground. This will, of course, depend upon the heights or depths of the cuttings and embankments, upon the width of the line, and the proportion of the slopes.

In making these calculations, much depends upon the object in view and the pressure of time. In preparing for deposit, the difference of heights between the section line and the *gradient* line are taken by the compasses; while, in preparing for the contracts, it is requisite they should be calculated from the field book, and the rates of inclination in the section. Again, for deposit, these differences are only taken at any apparent change of inclination in the ground line, while for the contracts they must be made for every two chains.

Before proceeding to calculate the superficial quantities occupied by the Rail Road in the example given below of the Great Wigston Survey, let us take some general case, and explain the best method of calculating the areas of the several blocks, into which any length of line must necessarely be divided.

Let a, b, be the several heights; let c be the width of the railway; and let the slopes be as m to 1 (by the term "*slopes*" is meant, technically, that the base of the slope is to the

height of the slope as m: 1); therefore, if the heights be a, b, the bases of the slopes will be ma, mb.



Let AB be any length of line, whose height at A is (a), and at B is (b). Then, if AD be taken as the half of the line, and DE and HF be the extent of the slopes on the one side, then DE and HF will be respectively equal to ma and mb; and therefore, the area of AEFB will be $\frac{c+ma+mb}{2}\times$ H, where H equals the length, but AEFB is only half the area; therefore the whole area of the block equals $(c+ma+mb)$. H

EXAMPLE 1.—Required the superficial area of a portion of a portion of railways, 20 chains long and 12 feet high at one end, and 10 feet at the other, the slopes being 2 to 1 (the width of line being 33 feet); now $c=33$, $a=12$, $b=10$, and $m=2$, and H $=20$ chs.; therefore,

$$\frac{33+34+20}{66}\times 20 = 23{\cdot}333 \text{ sq. chains} = \overset{\text{A.}}{2}.\ \overset{\text{R.}}{1}.\ \overset{\text{P.}}{13}.$$

In this example c, a, and b, being in feet, it becomes necessary to divide their sum by 66, to bring them into chains. These multiplied into the 20 chains long, make the 23·333 sq. chains.

EXAMPLE 2.—What will be the superficial area of a cutting of a railroad, whose depths at every chain are 0, 2, 3, 5, 7, 9, 8, 6, 4, 2, 1, 0 feet; the other data being the same as in the last example?

The formula for the area is $(c+ma+mb)$ II; in the two end areas, in the present example, where $a=0$, and $b=0$, ma and mb would be 0, and the areas will be $(c+mb)$ II, in the one case, and $(c+ma)$ II in the other.

Now the whole area $= \frac{\text{II}}{66} \{(c+mb)+(c+mb+mc)+\&c.)$

or, by taking separately the whole of the *central area*, which will be $\frac{11}{66}$. II c., and applying the formula for calculating the areas of offsets, page 53 of the Tent Book, viz. Area $= d$ $(bg+ch+dl$, &c.), that is, the area equals the sum of the perpendiculars into the common distance, to the areas of the *slopes*, we have for their area $\frac{2\,\text{II}}{66}(mb+mc+md$, &c.) sq. chs.; therefore,

$$\text{the whole area} = \frac{\text{II}}{66}. \left\{33\times 11+2(4+6+13+\&c.)\right\}$$

		A.	R.	P.
$=8{\cdot}34$ square chains	=	0	3	13.

EXAMPLE.—What will be the area of a mile of railway, whose heights of embankment for the first quarter of a mile, at every 5 chains, are 0, 12, 25, 50, and 20 feet; for the next 25 chains, the line is level; and then, to the end, runs through a cutting, which rises gradually to the height of 40 feet, at half way, and descends to the end, having the other data as the preceding?

		A.	R.	P.
Ans.: $245{\cdot}45$ sq. chs.	=	24	2	7.

Where, however, the area required is a portion only of a cutting or embankment.

$$\text{area of slopes} = \frac{2\,\text{II}}{66}\left(\frac{ma}{2}+mb+mc\ \&c. + \frac{m\omega}{2}\right)$$

$m\omega$ being the last height.

Great Wigston Notes and Section.

The following Field Notes are taken from the levels upon the direct Manchester, they begin at 9 miles 50 chains, and end at 11 miles 15·42 chs. on the centre of the road from Newton Harcourt to Great Wigston, being the same portion as was selected for the second example of railway surveying. The position of the levelled line is shewn upon the plan. As this portion was selected from the middle of a longer section, the reduced level of the starting point was of course borowed from the MAIN SECTION, viz. 277·64 feet above the datum line. There is nothing particular in the notes themselves to call the Student's attention to, they were selected for the purpose of explaining the mode of calculating the superficial quantities, required for the purposes of the railway, out of each field, which could only be done when the plan accompanied the section. The plan, to make it to a sufficient scale to answer the purpose of explanation would not admit of its being very long. The section on the other hand required some considerable length to take in the proper subjects of remark and explanation. The present small section, therefore, has only been introduced to explain the superficial quantities.

The only remarks that need be made upon it, are that at the 3 roads which are crossed by the line, the B. M's. are numbered the same as the roads, viz. 17, 18, and 19. These roads occur at 9 miles 64·08 chs.; 10 miles, 69 chs.; and 11 miles, 15·42 chs. The level of the water in the brook at 10 miles 3·15 chs and 11 miles 1·92 chs is given, and the distances upon the line. As to the arrangement of the notes themselves, the first 3 columns are given, the "rises" and "falls" are left out, but so much of the "reduced levels" as is required to check the main points is inserted among the remarks. The distances and remarks are, of course, all inserted.

In addition to these Notes being used for the superficial con-

tents of land taken out of each field, they have also been used for the purpose of explaining the calculation of Cuttings and Embankments, the larger section being confined to its more legitimate purpose of explaining the mode of crossing the roads and rivers, and of determining the gradients or rates of inclination.

GREAT WIGSTON LEVELS.

Back.	Inter.	Fore	Distance.	Remarks.
			50·00	Hedge at 50·05. [Red. Level 277·64.
2·99		13·20	54·00	Footpath crosses.
2·44		12·59	58·00	Hedge at 55·96.
1·22		7·21	60·00	
7·21		11·40	64·08	Centre of rd. from Kilby to Gt. Wigston
2·21	4·58		64·45	Hedge at 64·50. Cross Section 17.
		7·18	64·56	B. M. 17 on lower hinge of gate, in line.
2·60		7·36	75·00	
6·89	6·95		77·30	
		6·00	77·50	Hedge at 78·90.
6·00	3·64			10 Miles.
		0·50	1·00	Hedge at 0·40.
9·45	11·92		level of water in brook at 3·15 and 3·27.	
	4·61		3·62	
		1·97	5·00	
11·32		1·63	10·00	Hedge at 8·48.
5·40	4·30		10·60	Hedge at 10·50.
		3·41	out of line	
6·22	4·64		11·65	
	4·16		13·47	
		7·40	out of line	Hedge at 14·85.
8·61	6·77		15·40	Fence at 15·76.
		1·84	17·50	Hedge at 16·30.
12·87		3·42	20·00	
11·44		0·28	23·50	
8·30	3·50		25·58	Hedge at 26·48.
		2·78	25·80	
8·28	5·46		30·00	
		5.60	32·00	

Back.	Inter.	Fore.	Distance.	Remarks.
				Reduced Level 297·32.
5·60		5·80	32·30	
2·68	6·88		35·00	
		12·77	37·00	hedge at 37·70.
1·93	11·17		40·00	hedge at 39·76.
	7·76		43·00	
	5·10		45·00	
		1·98	49·00	hedge at 46·12.
5·10		8·00	56·12	hedge at 52·31.
6·25		6·01	out of line	
4·45		11·85	61·00	hedge at 61·56.
1·28		13·20	64·00	
1·06		13·18	69·00	Occupation Road.
1·85	525		72·15	B. M. 18 on lower hinge of gate in line.
		6·52	out of line	hedge at 72 15.
2·22		3·86	75·60	
10·00	8 98		76·70	
		1·23		11 Miles Fence crosses.
9·67	11·43		level of water in brook at 1·92 and 2·05.	
	10·70		5·94	hedge at 2·10.
		8·42	6·60	hedge at 15·00 and 15·82.
8·42	8·80		15·42	Centre of road from Newton Harcourt to Great Wigston. No. 19.
		7·58	B.M. 19	on lower hook of gate right of line across road. Red. Level, 257·4[illegible].

Superficial Quantities continued.

Having plotted these Notes of the Great Wigston Section, it would be as well for the student to calculate the superficial qualities both ways, viz., in the way they would be done *before* and *after* the Act.

The best way to do the former would be to prepare a Scheme which is most generally adopted in offices.

Previous to this however it will be necessary to erect perpendiculars throughout the section, wherever the ground line changes, and to measure these heights between the ground and gradient lines. Enter these in the first column, measure the

distances between the perpendiculars, and enter them in the second column, the two heights at either end, as a fraction, in the first, with the distance between them, in the second. The same data will also form the first two columns of the solid quantities. Now by referring to the explanation at page 256, each of these portions into which the whole line has been divided will be found to have their area equal to $(c+ma+mb)$ II where $c =$ the width of railway. The value of c must not now be assumed as 2 (15) feet, or double the half width of the line simply—which was done at first to facilitate the explanations,, but as 2 (15 + 10, an allowance for the side ditches) or 50 feet. Enter therefore the several values of the formula, $(c+ma+mb)$ or 50 + twice the sums of the heights, supposing the rates of slopes to be 2 to 1, in the third column, which will be in feet. Then multiply the second and third columns together for the fourth column, which being added together, due care being had of the proper decimal points throughout, and divided by 66, will give the fifth column or the whole area of the cutting or enbankment in square chains.

The reader will perceive that it would have been a waste of time to have ascertained the square chains in each of the several portions that formed the embankment, as instead of having only to divide the *sum* of the fourth column, *each* portion must then have been divided by 66.

The superficial quantities of the Great Wigston Section calculated in this way, which I have given for practice to the reader, stand thus:

1st embankment . .	9 acres.	1 rood.	30 perches.
1st cutting	9 acres.	1 rood.	16 perches.
2nd embankment .	3 acres.	1 rood.	20 perches.
Total. . . .	23 acres.	0 roods.	26 perches.

After the Act.

When it is required to ascertain the quantity of land required to be taken from each field along the line for the purposes of the railway, a different plan must be adopted. First calculate the gradient heights at equal short distances, say five chains—thus the inclination of gradient being 1 in 387; now by reducing the 5 chains into feet, and saying as 387 : 1 :: 330 feet : $x = \cdot 8527$ feet—we have as the ground falls from the starting point, the value in decimals of a foot to be substracted from every five chains, because the height at starting is according to the levels previously taken, 277·64ft. above the datum line—the height at 5 chains will be 277·64—·85 or 276·79 feet, at 10 chains will be 276·79—·85 or 275·94 feet, and so on. These gradient heights being afterwards compared with the heights of the ground at corresponding distances found in the level book or measured on the plan, their difference (h) must be noted down. These several heights and distances should then be entered in separate columns, &c., distinguishing the embankments and cuttings.

Now as the railway is to be 30 feet wide, and 10 feet allowed on each side for ditches, the general width at surface of rails will be 50 feet, and as the slopes are two to 1, the whole width required for the railway will be $50+4h$, where h represents the difference in feet between the gradient and ground heights—these quantities should be entered in a separate column. Now lay off half of these widths on the plan on each side of the red line at the end of every 5 chains, or as often as may be requisite. The first half-width at starting will be 25 feet, which must be measured off on each side of the Rail line perpendicular to it. The next at 5 chains, 52·58ft.; at 10chs. 75·88ft.; at 15chs. 88·18ft.; will be the half widths, and so on till the whole is complete.

This must afterwards be drawn in carefully in ink, the centre or actual width of the railway must also be shewn on

each side in dotted lines, and the cuttings, then coloured lightly in lake, and the embankments in green.

The portion of ground required to be taken from each field for the railway, will then have to be calculated, by dividing those portions into trapeziums and triangles, and finding the areas, from the rules given at page 18. Finally, these areas thus calculated, are marked conspicuously on the plan, on or as near the rail line as may be found most convenient. The several areas required for each field in the example given for the student's practice, will be found marked upon the plan, their total amounting to 22 acres, 0 roods, and 8 perches.

CHAP. XV.

CUTTINGS AND EMBANKMENTS.

It is requisite that the reader should understand, that these solid quantities have reference, not to the gradient, or surface of rails line, but to what is called the *balance* line; that is, to the lower line of the ballasting. There are generally 2 feet of ballasting upon every line of railway in which the sleepers are placed. This ballasting forms a portion of the permanent way, and is not mixed up with the earthwork, which is invariably reckoned separately. As regards the line of rails, therefore, which is the line shown upon the section, this balance line will be two feet beneath it throughout; and the

heights measured off from the section will be two feet too much for the embankments, and two feet too little for the cuttings. In getting out the quantities, therefore, for the earthwork of a line of railway, having obtained the heights at either end of the several blocks, (as explained at page 256, in the chapter on Superficial Quantities,) deduct two feet for the embankment heights, and add two for the cuttings. These, with the distances and heights of the several blocks, will form the first and second columns of the Scheme, pages 247 & 248.

There are two or three methods adopted in practice, in the calculation of these quantities; viz., the Prismoidal formula, Bidder's Tables, M'Niel's, &c., each of which shall be briefly explained.

1. *Prismoidal Formula.*

Let c be any width of cutting, having, at one end, the height $= a$, at the other $= b$; length of cutting $= \text{II}$, slope 2 to 1, or generally m to 1.

Now the solid, thus cut off, assumes the form of an imperfect prism, which may properly be divided into three divisions; the *central* one, being a solid, generated by a plane of the given heights and length; moving along a plane, at right angles to it, to a distance, equal to the width of the railway; and the two slopes, equal to each other, being strictly frustra of pyramids; the height of the frustra being equal to the length of the cutting, and the sides of either end, being the rides of a right-angled triangle, whose base = the height of the cutting at that end; and perpendicular, the proportion of of the given slope to it.

Having the same data as above, (see Diagram, page 256,)

the central contents $= \text{II}.\ \frac{c}{2}(a+b)$, where c = width of railway.

$$\text{slopes} = \frac{\text{areas of the two ends} + \text{the mean area}}{3} \times \text{the length},$$

$$=\frac{\Pi}{3}(ma^2+mb^2+mab).$$

$$\therefore \text{ The whole contents } =\frac{\Pi}{2}\cdot c(a+b)+\frac{\Pi m}{3}(a^2+b^2+ab)$$

$$=\frac{\Pi}{6}.(3ac+3bc+2a^2m+2b^2m+2abm).$$

$$=\frac{\Pi}{6}(ac+a^2m+bc+b^2m+2ac+2bc+m(a+b)^2)$$

$$=\frac{\Pi}{6}\cdot(\overline{c+ma}.\ a+\overline{c+bm}.\ b+4\left(\frac{a+b}{2}\right).\ (c+\frac{m}{2}.\ \overline{a+b})$$

which is the *prismoidal formula;* where $\overline{c+ma}.\ a$ and $\overline{c+mb}.\ b$ are areas of the two ends, and $4\ \left(\frac{a+b}{2}\right)(c+\frac{m}{2}.\ \overline{a+b})$ is 4 times the area of a section midway; the whole area being equal to the sum of these three into $\frac{1}{6}$ the length, whether of cutting or embankment.

2. *Moseley's Formula.*

This formula, for which I am indebted to the late Professor Moseley, of King's College London, I have had several opportunities of testing, and have found it exceedingly useful. The calculations from the Prismoidal Formula, which is geometrically deduced, are strictly accurate; but they are too tedious for general purposes. Bidder's Tables, on the other hand, though convenient for common practice, are still, from being only calculated to full feet and limited to 50 feet, unfitted for contract estimates, Whereas Moseley's Formula, being applicable to any height and any subdivision, and embracing any extent of cutting, whose heights are taken at any equal distances whatever, becomes especially useful for the final calculations, which are usually made at every two chains of distance, and to heights of one hundredth of a foot.

Because (*See previous page.*)

the central area of any block $=\Pi.\frac{c}{2}(a+b)=\frac{\Pi m}{6}\left(\frac{3c}{m}.\overline{a+b}\right)$

and the slopes $=\frac{\Pi.}{3}m.(a^2+b^2+ab)=\frac{\Pi m}{6}(2a^2+2b^2+2ab)$

$$\therefore \text{ whole contents } =\frac{\Pi m}{6}\left(\frac{3c}{m}(a+b)+2a^2+2b^2+2ab\right)$$

Now let $a=y_1$, and $b=y_2$, we have

1st contents $=\frac{\Pi m}{6}\left(\frac{3c}{m}(y_1+y_2)+2y_1{}^2+2y_2{}^2+2y_1y_2\right)$.

And, as the adjoining cutting must commence with the same height with which this terminates, supposing the other height of the second block $=y_3$, and next height y_4, we have—

area of 1st block $=\frac{\Pi m}{6}\left(\frac{3c}{m}(y_1+y_2)+2y_1{}^2+2y_2{}^2+2y_1y_2\right)$

of 2nd block $=\frac{\Pi m}{6}\left(\frac{3c}{m}(y_2+y_3)+2y_2{}^2+2y_3{}^2+2y_2y_3\right)$

and the series $=\frac{\Pi m}{6}\left(\frac{3c}{m}(y_1+2y_2+2y_3 \ldots\ldots 2y_{n-1}+y_n)+2(y_1{}^2+\right.$ $2y_2{}^2+2y_3{}^2 \ldots\ldots\ldots 2y^2_{n-1}+y^2_n)+2(y_1y_2+y_2y_3+y_3y_4+\ldots\ldots\ldots y_{n-1}.$ $y_n)$or β,

and $\therefore =\frac{\Pi m}{6}\left(\frac{3c}{m}(2\Sigma y_n-(y_1+y_n)+2(2\Sigma y_n{}^2-(y_1{}^2+y_n{}^2)+\right.$ $2(\beta)$.

Now, when y_1 and y_n are $=0$, which is the case in a complete cutting or embankment, we have—

$$\text{Total contents} =\frac{1}{6}\Pi m\left(\frac{3c}{m}\ 2\Sigma y_n+4\Sigma y_n{}^2+2(\beta)\right),$$

where Σy_n = the sum of the simple quantities,

and $\Sigma y_n{}^2=$ the sum of their squares,

and $\beta=$ the sum of the continued products of the simple quantities.

3. *Bidder's Formula,*

for computing the solid contents, is—

$$\left.\begin{array}{l}\text{for the slopes} = \frac{22}{27}\left((a+b)^2 - ab\right) \\ \text{for the centre} = \frac{11}{9}(a+b)\end{array}\right\} \text{ in yards.}$$

This is for a chain in length, and in the centre for a foot wide; the slopes being one to one.

Now, it has been previously shown (page 256)

that $\frac{\text{n}}{2}c(a+b)$ = contents of centre.

Let $c = 1$ foot, which is the width to which the tables are calculated.

and $\text{n} = 1$ chain or 66 feet, $\therefore \frac{66}{2}(a+b)$ in cubic feet = centre.

$\therefore \frac{66}{2}$ of $\frac{1}{27}$ or $\frac{11}{9}(a+b)$ in cubic yards = centre.

And that the slopes $= \frac{\text{n}m}{3}(a^2+b^2+ab)$

Let $m = 1$, and $\text{n} = 66$ feet as before.

$\therefore$ slopes $= \frac{66}{3}$ feet, or $\frac{22}{27}(a^2+b^2+ab)$ in yards.

and $\therefore = \frac{22}{27}(\overline{a+b}^2 - ab)$ cubic yards.

The *central* contents, thus obtained, would have to be multiplied by the length of the blocks in chains, and the width of the line in feet; and the contents of the *slopes,* by the ratio of m to 1, and by the length of the block in chains.

M'Neil's Tables are also for slopes of 1 to 1, and for base of 1 foot; but not for lengths of 1 chain.

The quantities obtained, therefore, in M'Neil's tables, would have to be multiplied, for the centre, by the width of railway in feet; and for the slopes, by the length n (in feet).

These two tables, however, are only calculated for long

sections, and for full feet, (for which they are invaluable) principally, for computing the whole contents of a line previous to going to Parliament, to form the first estimate of the expense.

Example.

(Which is fully worked out, both by Bidder's Tables and Moseley's Formula.)

Required the solid contents of a cutting or embankment, whose several heights at each chain's length are, 0, 5, 10, 15, 20, 25, 30, 30, 24, 18, 15, 10, 0—slopes 2 to 1, and the width of railway, 30 feet.

BY BIDDER'S TABLES.

Lengths.	Height at either end.	Central Contents.	Contents of Slopes.
1·00	$\frac{0}{5}$	6·1	20
1·00	$\frac{5}{10}$	18·3	143
1·00	$\frac{10}{15}$	30·6	387
1·00	$\frac{15}{20}$	42·8	754
1·00	$\frac{20}{25}$	55·0	1243
1·00	$\frac{25}{30}$	67·2	1824
1·00	$\frac{30}{30}$	73·3	2200
1·00	$\frac{30}{24}$	66·0	1789
1·00	$\frac{24}{18}$	51·3	1085
1·00	$\frac{18}{15}$	40·3	667
1·00	$\frac{15}{10}$	30·6	387
1·00	$\frac{10}{0}$	12·2	82
		493·7	10611
		30	2
		14811·1	21222
			14811
			36033

493·7 being the number of cubic feet in the central column, for the width of one foot, which must, therefore, be multiplied by 30 for the whole width, and 10611 being the contents of the slopes for the ratio of one to one, which must, in this case, at the ratio of two to one, be doubled; the sum of the two, 36033, will be the number of cubic yards required.

BY MOSELEY'S FORMULA.

The sum of the simple quantities $= 202 = \Sigma\, y_n$
the sum of their squares $= 4400 = \Sigma\, y_n^2$
the sum of their products
(of each with its preceding) $= 4400 = \beta$

Now the formula $= \frac{\text{II}m}{6}\left\{\frac{3c}{m}.\, 2\,\Sigma\, y_n + 4\,\Sigma\, y_n^2 + 2\,\beta\right\}$ and

$$2\,\Sigma\, y_n = 404; \frac{3c}{m} = 45;\ 4\,\Sigma\, y_n^2 = 176000;$$

$$2\,\beta = 8444 \text{ and } \frac{\text{II}m}{6} = \frac{2}{6}.\ 66$$

$\therefore \frac{2}{6}\cdot 66\ (404 \times 45 + 17600 + 844) =$ cubical contents.
$= 22\ (18180 + 17600 + 8444)$
$= 972928$ cubic feet $= 36034$ cubic yards,

Example.—Calculate, by Moseley's formula and Bidder's tables, the cubical contents of an embankment, whose heights at every chain are 0, 2, 3, 5, 7, 9, 8, 6, 4, 2, 1, 0, slopes 2 to 1; width of centre, 33 feet. *Ans.* 5178 cubic yards.

Example by the Prismoidal Formula.—What are the contents in cubic yards of a cutting, where the heights at every 2 chains are, 0, 3, 5, 7·5, 8, 9·5, 8, 6, 4, 0, the width of the line being 33 feet, and the slopes 2 to 1.

because $\overline{c + ma}.\ a =$ area at one end.
and $\overline{c + mb}.\ b =$ area at the other.
and $\frac{a+b}{2}\left(c + \frac{m}{2}\overline{a+b}\right) =$ area midway.
$\therefore 0,\ \overline{33+6} \times 3,\ \overline{33+10} \times 5,\ \overline{33+15} \times 7\cdot 5,\ \overline{33+16} \times 8$ &c., - - -0,

are the several end areas, the first and last in a complete cutting being always zero.

$$\text{and } \frac{3}{2}(33+3),\ \frac{3+5}{2}.(33+3+5), \frac{5+7\cdot5}{2}.(33+5+7\cdot5), \text{ \&c.,}$$

are the several areas midway.

Now the sum of the end areas $=2404$ square feet, and that of the middle area $=2380$.

But four times the middle area, *plus* twice the end areas (as the areas at every height serve for end areas to two sections, the first and last, which are the only two single ones having, as above shewn, no value at all), multiplied into $\frac{1}{6}$ the common length, will be the contents in cubic feet.

$$\therefore \frac{2\,(2404)+4\,(2380)\times 132}{3} = 315216 \text{ cubic feet,}$$

$=11675$ cubic yards, $=$ the solid contents required.

The student is now recommended to take the Great Wigston Section, and with the usual data of 30 feet for the width of centre, and with the slope of 2 to 1, to calculate the solid contents of the cuttings and embankments, the following scheme, being the first portion of it, viz., Embankment No 1, has been added, fully worked out for an example.

In the first column are the heights at either end, with the two feet deducted, on account of its being an embankment, from the height on the section. The second column contains the length. These two form the data. From the first column, the third and fourth column are obtained from Bidder's Tables; the third, being the central contents to one chain long and one foot wide; and the fourth, the contents of the slopes, to the respective heights as given in the first column, to one chain long, but to a slope of 1 to 1. Now, by multiplying the lengths into these two columns, viz., the third and fourth, the fifth and sixth are obtained, which are termed the "reduced centres" and "reduced slopes." Both columns must now be added up;—the former, as it is only calculated to one foot wide, must be multiplied by thirty, the width of the railway; an

the latter, as the tabular slope is only 1 to 1, must be doubled, to bring it to the slope of 2 to 1. The results thus obtained must be added together, and their sum will be the required solid contents in cubic yards.

Heights.	Distances.	Centres.	Slopes.	Reduced Centres.	Reduced Slopes.	Cuttings and Embankments.
$\frac{0}{4}$	2·50	4·9	13	·250	32·50	
$\frac{4}{11}$	3·25	18·3	147	59·475	477·75	
$\frac{11}{21}$	4·10	39·1	646	160·310	2648·60	
$\frac{21}{27}$	4·50	58·7	1415	264·150	6367·50	
$\frac{27}{30}$	5·50	69·7	1987	383·350	10928·50	
$\frac{30}{35}$	5·10	79·5	2587	405·450	13193·70	
$\frac{35}{28}$	5·00	77·0	2436	385·000	12180·00	
$\frac{28}{18}$	5·00	56·2	1313	281·000	6565·00	Embt. No. 1 173,350 cubic yards.
$\frac{18}{2}$	6·80	24·4	297	165·920	2019·60	
$\frac{2}{3}$	3·80	6·1	16	23·180	60·80	
$\frac{3}{0}$	1 60	3·7	7	5·920	11·20	
				2146·005	54485·15	
				30	2	
				64380	108970·30	
					64380	
					173350·30	

The solid contents of the whole section given as an example are—

The 1st Embankment . . . . . 173·350 cubic yards.
1st Cutting . . . . 196·115 " "
2nd Embankment . . . . 11·840 " "

Instead of the preceding, however, there are two methods in common use among contractors, which, though apparently correct, are really far from being so.

The first is by taking the MEAN OF THE TWO END AREAS, which makes the results *too much.*

The second, by taking THE AREA OF THE MEAN HEIGHT, which, on the other hand, makes it *too little.*

Now, in the proof of Bidder's formula, it has been shown that the contents of slopes $= \frac{\text{II}}{3}\ (m\,(\overline{a+b}^2 - ab\,)$, and of the centre$=\frac{\text{II}}{2}.c(\overline{a+b}$ therefore the whole contents are $=$ $\text{II}\,(\frac{m}{3}\,\overline{a+b}^2 - ab) + \frac{a+b}{2}.c)$ and by assuming x and y, as the unknown values of the respective excess and deficiency in the two erroneous methods above, and equating them with the true formula, we can obtain their respective values.

Of these two incorrect formulæ

the first $\text{II}\,\frac{(\overline{c+ma}.a+\overline{c+mb}\cdot b)}{2} = \frac{3\,ac + 3\,ma^2 + 3\,bc + 3\,mb^2}{6}$

$$\text{the second} = \text{II}\frac{a+b}{2}\,(c + \frac{m}{2}.\,\overline{a+b})$$

$$= \frac{6\,ac + 6\,bc + 3ma^2 + 6mab + 3mb^2}{12}.$$

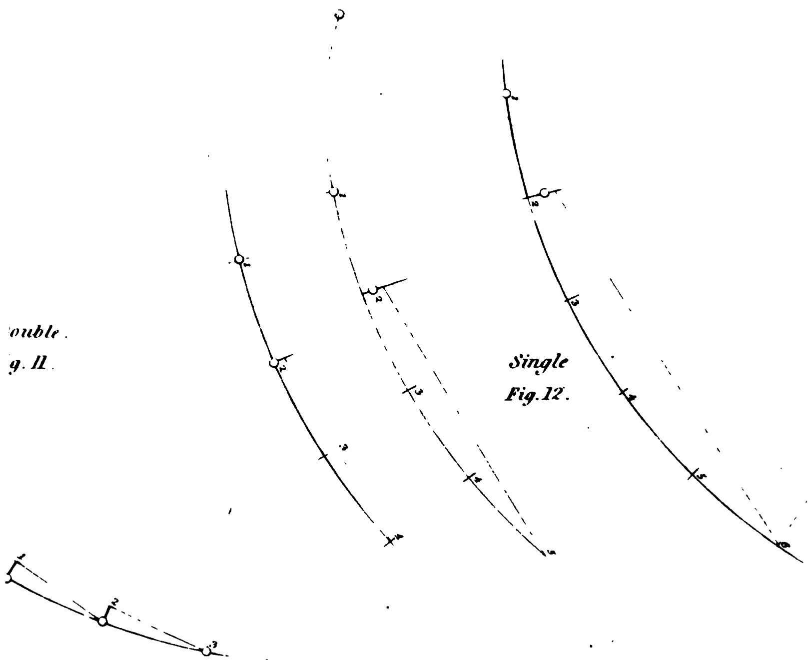
Finding the differences between these and the correct formula, which are the values of x and y respectively, we obtain

$$\text{an } excess \text{ for the first of } \frac{m}{6}.\overline{a-b}^2$$

$$\text{a } deficiency \text{ for the second of } \frac{m}{12}.\,\overline{a-b}^2$$

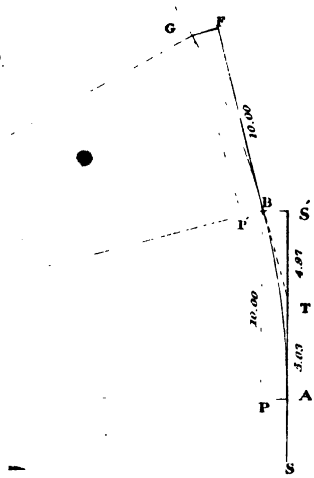
where a and b, as usual, represent the heights at the ends, and m the ratio of the slopes.

These corrections have to be multiplied by II, the length of the section.



ouble.
g. 11.

Single
Fig. 12.



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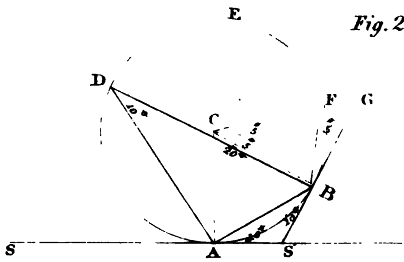


Fig. 20.

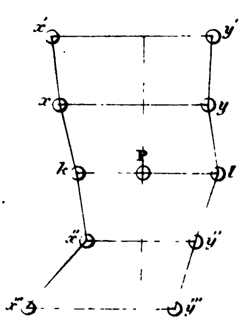


Fig. 21.

F. Mansell sc.

RAILWAY CURVES.

Part the Fifth.

CHAP. I.

The chief, in fact the only, object in introducing curves upon railways (for there are many objections attending them) is to avoid some obstacle, to pass round some town, or escape tunnelling through some hill or other. The particular object in view will of course determine the points of starting and ending, as well as the length of the radius,—the curve, which leaves the previous straight line at the given point, entering also at the given point upon the subsequent straight line, having both the previous and subsequent straight lines, tangents to it at their respective points of contact.

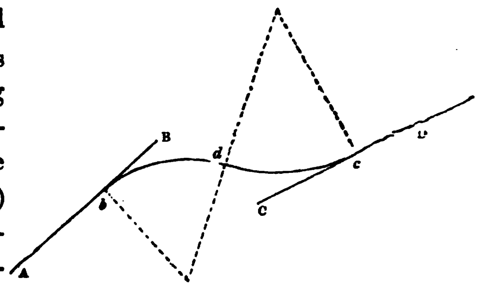
Straight lines upon railways are generally fixed first, being selected both for their distance and their gradients, and the curves are strictly subordinate to them.

The relative position of these lines (which are connected by a curve, to which each is respectively a tangent) being perfectly arbitrary, dependent upon various circumstances, a few examples are subjoined.

First, let the two lines run parallel or nearly so to each other

intersecting at a point considerably beyond the required points of junction, having their directions represented by the lines AB, CD (fig. 1.)

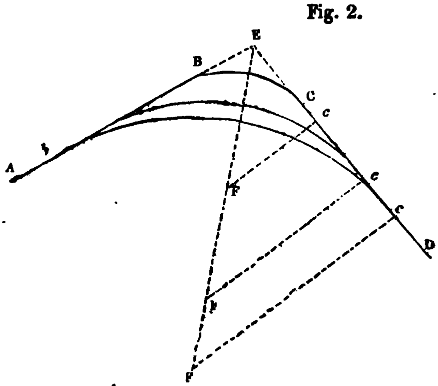
Fig. 1.



They will be connected by the convex arc *bd*, and the concave *dc*, or what is called the S curve.

If not parallel, and if when produced they will soon meet, the angle made between them being necessarily less than 180 *degrees.*

Fig. 2.



Let AB and CD (fig. 2) be not parallel, but meeting when produced at the point E. Then will the angle AED between them be less than two right angles. The curves that connect them will now fall within the angle, and will of course be convex towards E.

The connection of these two lines may be made by

A single curve, to which AB and CD are respectively tangents.

In this problem, there are various conditions depending upon the peculiar circumstances of the case.

First: AB and DC can be produced to E, the angle AED bisected, and *any point* F taken upon the line that bisects it, as a centre of a circle to which the given lines will be tan-

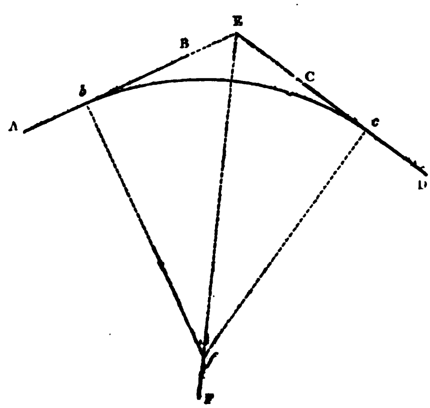
gents: the number of circles, that will comply with the condition, is as infinite as the points upon the line of bisection; that point, therefore, will be regulated by the special object in view, of avoiding the obstacle (whether it be a hill, or town,) that may lie within the angle AED; the perpendicular F*c*, let fall upon DC, being the radius of the curve, increasing as the point F moves along EF, and flattening the curve, of course, as it increases.

Thus a small or large radius can be taken as desired, limited only (as to its *minimum*) by the centrifugal force, or by the sanction of Parliament (generally no curve should have a less radius than 80 chains); and as to its *maximum*, by the position of the intervening obstruction.

Within the above limits, however, circumstances may sometimes render it desirable that the *point of contact* (*b*) in AB *should be fixed* (fig. 3.). By drawing *bf* perpendicular to AB till it intersects the line EF, *bf* becomes the radius, which is now no longer arbitrary, and *f* the centre of the circle.

Again, *the points of contact* (*b*) *and* (*c*) *upon both lines are sometimes given.*

Fig. 3.



*Now, if b*E *and c*E *are equal,* the perpendiculars *bf* and *cf* will meet in *f*; and AB and CD will be connected by a curve whose radius is *bf* or *cf*.

By a double curve.

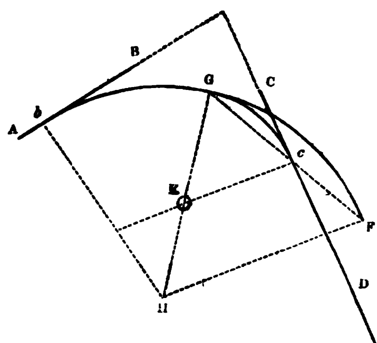
*If b*E *be not equal to Ec,* these points must be connected by two curves of different radii, but having the same normal, that is, the same straight line being a tangent to each at the point of junction.

This problem admits of several solutions, dependent upon the length of the radius H*b*.

Let therefore any perpendicular whatever H*b* be taken, (Fig. 4.) such however, that the curve *b*GF will pass beyond the other tangent.

From H the centre let fall the perpendicular HF, intersecting the curve at F; join F*c* and produce it to G; join GH, intersecting the perpendicular from the given point *c* in K; K will be the centre of the other curve, and will have the same normal.

Fig. 4.



For H being the centre, HF will be equal to HG, and HF and K*c* being parallel, KG will be equal to K*c*, and because K*c* is perpendicular to CD; therefore CD is a tangent to this new curve G*c*. —Q. E. D.

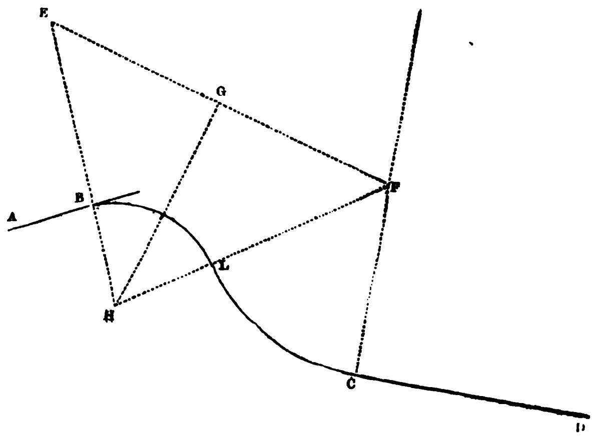
(*For a continuation of this subject see page* 296.)

In the case of the junction of the two lines by an S curve :—

There are two conditions attached,—

1. *When the points of contact are given.* Let AB and CD be the two tangents, and B and C the points of contact: it is required to connect them by an S curve. There is an infinite number of cases in which this can be done. As a general rule, take any equal perpendiculars whatever at B and C, (fig. 5,) viz.: BE and CF, join FE and bisect it in G; erect the perpendicular GH, intersecting EB produced in H, join HF. F being the centre of one circle, H will be that of the other. For HF=HE, and BE=FC and ∴ FL, and ∴ HL=HB, and they have the same normal. FC can be taken any length that the relative position of the tangent will admit.

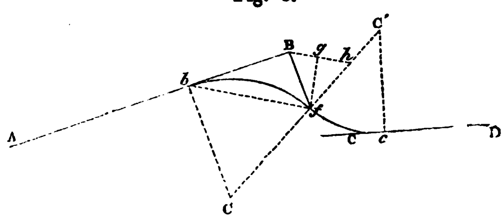
Fig. 5.



2. *When the radius and one of the points are given.* Let AB and CD be the tangents, *c* the given point, and *c*C′ the given radius.

This problem admits of several solutions, restricted only by the angle *c*C′C, which may be drawn in any position CC′: describe the

Fig. 6.



curve *cf*, and at *f* let fall a perpendicular to AB; bisect the angle B*f*C′, and from B let fall a perpendicular to it B*g*, and through *f* draw *fh* parallel to B*g*; *b* shall be the point of contact upon AB. From *b* erect *b*C perpendicular to AB, intersecting C′*f* produced; *b*C shall be similar and similarly situated in relation to *f*C, as B*f* is to *fh*, and therefore shall be equal to it.

The two circles will therefore touch at *f*.

CHAP. II.

STAKING OUT THE CURVE.

HAVING briefly touched upon the several cases that are likely to occur in practice, and the general principles of the solution of each, let us proceed to the consideration of the usual methods adopted to stake these curves upon the ground.

There are several methods of so doing. Each of them, perhaps, finds its advocate among professional men, and therefore, without assigning any preference to any one, I will endeavour to explain them all.

First, that which is called the *old method*, by curve frames, which, with certain checks that are indispensable, has certainly its advantages.

A, B, C, are three curve frames exactly alike, *bc* being the frame (of oak), which has a vertical slit to see through, nearly throughout its whole length: *de* and *fg* are moveable arms, which pass through the frames *bc*. They are connected by the string *s*, and can be fixed in any position by the screws at *m* and *n*. Plate VIII., fig. 9.

These arms are to be so adjusted that the distance from the centre of the string *s* to that of the slit, measured on *de* and *fg*, must be exactly equal to what is termed among Engineers the required '*ordinate*.'

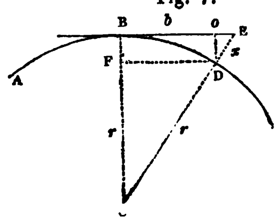
There are two ordinates used, the single and double ordinate.

This has frequently been productive of serious mistakes, as sometimes (which is certainly the more correct of the two) the single ordinate is called the "ordinate," and the other the "double." At other times (with those who generally use the double ordinate) the 'double' becomes 'the ordinate,' and the

other, when referred to, is called the *half* ordinate. The term *single ordinate* is intended to express what mathematicians call the versed sine. This versed sine, which is the same measure (on a different scale) as the allowance for curvature, may be calculated in arcs of the common radii in the same manner.

Let ABD be an arc of a given circle, whose centre is C, (fig. 7.) Let BE be a tangent of any distance (not greater than one-twentieth of the radius), it is required to find the ordinate oD. This, within the limits above stated, can be assumed as DE (x), which is equal to sec.—rad.; and Bo and BE also may be considered the same length. Now, by right-angled triangles,

Fig. 7.



$$\overline{r+x}^{2}=r^2+t^2, \text{ where } t=\text{BE or B}o$$
$$r^2+2rx+x^2=r^2+t^2$$
$$2rx+x^2=t^2$$

throwing x^2 away as indefinitely small in relation to $2rx$, we have

$$2rx=t^2$$

$$\text{and } x=\frac{t^2}{2r} \qquad \text{or}$$

the single ordinate is equal to the square of the tangent divided by *twice* the radius; and as this divisor is a constant quantity, the ordinates (of a given circle) are, within the above limits, proportionate to the squares of the tangents.

Beyond these limits, however, when Bo could neither be safely assumed as equal to BE, nor Do to DE, Bo and Do would have to be differently calculated.

Bo, though strictly the sine, being still *termed* the tangent:—

Now Do, being the versed sine, is equal to rad.—cos., and the cosine equals the $\sqrt{\text{rad.}^2-\text{sin.}^2}$, or preserving the same letter

for the new tangent as before, t, we have cos. $=\sqrt{r^2-t^2}$, and the versed sine or single ord. $= r-\sqrt{r^2-t^2}$

1st. *To run a Curve by a* Single *Ordinate.*

This is not so correct practically as that of the double ordinate, the frame being covered by the string in this case, and the two strings being covered in the other; yet it has the advantage of being able to run curves of shorter radii, the arms *de* and *fg* (Plate VIII. fig. 9,) being only required to extend half the distance.

Let SA be a tangent, (Plate VIII. fig. 10,) A the point of contact where the curve begins; produce the tangent one chain towards S′, and measure off the single ordinate above calculated with a rule towards the centre of the circle, at right angles to AS′; this will give you a point in the curve. Place the curve frame (2) here, and another (1) at the point of contact A, turning the strings inward; then measure a chain carefully towards (3) in the direction which covers the frame (1) by the string (2). Then take the curve frame (3), and placing it where the arrow is, seeing that it be perfectly perpendicular by a plummet, look through the slit in the centre and observe whether the string (2) covers exactly the middle of the frame (1). If not, move this curve frame (3) until it does; this will be a point in the curve.

Then, measuring another chain in the direction, that covers the frame (2) with the string (3), place the frame (1) in that point, using the same precautions as before, to make the frame perpendicular, and to cover exactly the points of direction.

The same process continued will give you as many points as you wish, and the arcs measured between these several points will be subtended by chords of a chain each.

The principal *objection* to this method is, that the curve

frame is too large an object to be distinctly covered in the centre.

2nd. *To run a Curve by a* Double *Ordinate*; *that is, with the string set at* twice *the versed sine.*

The point of contact and the tangent being assumed as the same, obtain the first point in the curve as before, viz., by measuring off the versed sine, or what would be termed, by those who generally use this ordinate, the *half* ordinate, at the distance of one chain from the point of contact. Then, having the curve frames set at the double ordinate, place No. 1 at the point of contact (Plate VIII., fig. 11,) and No. 2 in the first point of the curve, both strings turned inwards; then, measuring one chain carefully in the direction that covers the two strings, look through the slit in the frame, (No. 3,) and cover the two strings. This will give you a second point in the curve. Then, by taking up the frame, (No. 1,) and at another chain's distance onward covering the strings (2 and 3), you will obtain a third point. Any number of points may be obtained in the same way.

The common objection to each of the above methods is that, practically, there are many sources of error; for with perfect correctness in every detail—viz., as to the exact distance of the string from the slit,—as to the absolute perpendicularity of the frames,—as to the string being always turned exactly towards the centre of the circle, and the careful covering of the points of direction,—and all these are scarcely possible in an absolute sense—there can be but correctness after all.

Overcoming obstacles by the two methods above.

In the consideration of the preceding use of the curve frames, it has been assumed throughout that the ground was

perfectly flat and clear, and that no obstacle of any kind intervened either to secure the direction or to mark the point.

This, however, is anything but the case practically. Sometimes the line of direction is intercepted by a tree, sometimes by the steepness of the ground, so that from the third point of the curve the points 1 and 2 are not both visible, or are so placed that the one is seen altogether above or below the other: it is not easy to cover them in the latter case; impossible, of course, in the former. Sometimes, again, a hedge or river falls just where the point of the curve would come; and though that point *might* be determined for itself, yet it destroys the line of direction for the next point. And, therefore, some plan becomes necessary in practice to obtain an onward point from the position and direction of two points some 3 or 4 chains back.

1. *By the single ordinate.*

Let the points 1 and 2 be known, to find point 4, 3 not being obtainable. (Plate VIII. fig. 12.)

Cover the frame of (1) with the string of (2), having the frame set where the string is, and the string turned inwards, so as to make the string mark off a *double ordinate*, and measure two chains in this direction; or turn the string of (1) outwards, the string of (2) remaining in its right position, and cover the two strings: measure 2 chains in this direction as before.

Points 1 and 2 being known, to find point 5.

Set the frame (No. 2) in the same place, and the string in the same direction as in the last case, so that it marks off a double ordinate; and cover the *string* of No. 1, *turned outwards*, by the string of No. 2; this direction, measured to the distance of 3 chains, will be the required point in the curve.

Points 1 and 2 being known, to find point 6.

Cover the string of No. 1, turned outwards, by the string of

No. 2, (marking a point of three times the ordinate inwards,) and measure 4 chains.

2. *By the double ordinate.*

Points 1 and 2 being given, to find point 4. Fig. 13.

Cover the frame of No. 1 by the string of No. 2, and measure 2 chains.

Points 1 and 2 being given, to find point 5. Fig. 14.

Cover the string of No. 1, so placed outwards as to form a single ordinate, by the string of No. 2, and measure 3 chains.

Points 1 and 2 being given, to find point 6. Fig. 14*a*.

Cover the string of No. 1, turning outwards, by the string of No. 2, and measure 4 chains.

In case of doubt on the field as to the exact method of determining any of the above unknown points, the safer plan would be to find a tangent (the method of doing which will be explained below) at the point No. 2, *and calculate the ordinate to the tangent, at the point you desire.*

To find the next point in the curve, points 1, 2, *and* 6 *only, being given.*

From No. 6 (fig. 14 *a*,) measure towards No. 2 one chain, and marking outwards from there a single ordinate, you will obtain that point where the string of No. 5 (set to a double ordinate) would come, so that by placing the curve frame in its proper place, you will obtain point No. 5 of the curve.

Where points 1, 2, and 5, are given, (fig. 14,) measure towards 2 one chain, as before; this will be the point where the string (to a double ordinate) of No. 4 should be placed.

Where points 1, 2, and 4 are given, (fig. 13,) measure towards 1, and do the same as in the last, and No. 3 will be found.

The same principle will hold good for the *single ordinate*, only that where the points of the string (to the double ordinate) are found, twice the single ordinate will have to be measured outwards for the points of the curve.

Two contiguous points being thus obtained, the next point will be found as before described.

To run a Curve into a Straight Line.

1. *By a single Ordinate.*

Let No. 3 (fig 15) be the point of contact, turn the string of No. 2 *outwards* instead of inwards, and covering the string of No. 2 with the frame of No. 3, measure 1 chain onwards; this will be a point in the tangent.

Or, continuing the curve 1 chain beyond the point of contact, reverse the string at this point, turning it outwards; the string will be a point in the tangent.

2. *By a double ordinate.* Fig. 16.

No. 3 being the point of contact, as before, the frames Nos. 1 and 2 still remaining, take up No. 2, and measure onwards from No. 3 one chain, in a direction that covers the string of No. 1 with the string of No 3, and you will obtain a point in the tangent. The line that joins this with the point of contact, No. 3, will be the direction of tangent.

To run an S Curve.

1. *Single ordinate.* Fig. 17.

Cover the frame No. 2 by the frame No. 3, at the distance of 1 chain, and you will obtain a point in the new curve; then by reversing the string No. 1, and turning it inwards towards the new centre, and covering the frame of No. 3 by the string of No. 1, you reverse the curve.

This only holds good, of course, where the new curve is of the same radius as the old.

When it is of a different radius, a tangent must be found to the old one, at the point of contact; and the new curve, with the strings fresh set, must be run from that.

2. *Double ordinate.* Fig. 18.

No. 3 being the point of contact, cover the string of No. 2 by the string of No. 3, at the distance of a chain from the point of contact, and reverse the frame No. 1, so that the string shall point towards the centre of the new curve; the string of No. 1 is the first point in the curve; then reverse the string No. 3, and proceed as usual.

CHAP. III.

PRACTICAL CHECKS UPON THE CURVE FRAMES.

In using the curve frames, the chief difficulty in practice consists not so much in the curve that is run not being a perfect curve, as that from practical inaccuracy the ordinate may be somewhat more or less than that of the given curve.

The following is the usual check:—

Let the tangent be produced from the point of contact,—say 10 chains,—and erect a perpendicular till it intersects the curve; this line is the single ordinate, or, to speak mathematically, this line is the versed sine to that portion of the tangent measured, viz. 10 chains, which is the sine—the radius being any length, (*a*,) measure the requisite length of the or-

dinate, and you will have a point in the curve. If the stakes put down from the curve frames agree with this first distance, you may proceed. To do this, suppose another tangent drawn at this point, either way, it will intersect the former tangent at a certain point, which can be determined beforehand, and measured on the first tangent.

Thus, in fig. 19.—

Let SA be the tangent, and let A be the point of contact. At A erect a perpendicular AC = 80 chains; from C, at the distance CA, describe a circle; produce SA to S′, making AS′ 10 chains; at S′ erect a perpendicular to B, making S′B the proper length; B will be a point in the curve.

Again, join BC, and through B draw FBT at right angles to BC; FBT will be another tangent, and will intersect the former at T.

Now because vers. sin. $= r - \sqrt{r^2 - \text{sin.}^2}$,
(for what is here termed the tangent is the sine,)

and $r = 80$ chains, and sin. $= 10$ chains,

therefore vers. sin. or single ordinate $= 62\frac{1}{2}$ links.

Next, to find the angle at the centre, the rad. and sin. being given, we have the following proportion:—

As 80 chains : rad. : : 10 chains : sin. θ or $\angle$ at centre, which $= 7° \; 11'$.

Now the $\angle$CBT=S′BP, being both right angles; by taking away the common angle PBT,

the remaining $\angle$ CBP=$\angle$S′BT,

but $\angle$CBP is the complement of 7° 11′, and ∴ = 82° 49′, and ∴ $\angle$S′BT=82° 49′.

Again, to find S′T.

Assuming S′B as the radius in the right-angled triangle S′BT, which is $62\frac{1}{2}$ links; S′T will be the tangent to the angle at the base, 82° 49′, and can be obtained by the following proportion:

As rad. : $62\frac{3}{4}$ links : : tan. 82° 49′ : S′T, which $=4{\cdot}97$ chains, (therefore AT$=10{\cdot}00-4{\cdot}97=5{\cdot}03$ chains.)

We shall, therefore, have the ordinate $= 62\frac{3}{4}$ links, the angle S′BT=82° 49′, and the distance AT, from the first point of contact to where the second intersects it $= 5{\cdot}03$ chains.

In staking, therefore, these check lines upon the ground, having obtained the direction of the tangent AS′, measure AT 5 chains 3 links, and, placing a flag there, continue the measurement to 10 chains.

At 10 chains, erect, by means of a theodolite, the perpendicular S′B=$62\frac{3}{4}$ links; and at B, which will be the second point of contact, place the theodolite, and from the line BS, lay off the angle S′BT=82° 49′; if the check-work thus far be correct, the web of the telescope will cut the flag previously placed at T; if not, the whole must be done over again till it does.

When this has been correctly done, reverse the telescope for the direction of the new tangent, and proceed as before, marking off always as you go along where the next tangent will intersect the actual one, viz., at 5 chains 3 links.

These several points will be checks upon the staking out of the curve frames, and the stakes so put down should be altered to coincide with the first of these points, before proceeding to the rest.

This method, by the curve frames, is one of the many methods adopted by Engineers in running their curves, and perhaps one, with proper care and attention, as correct as any. In connection with the check above explained, it combines the two objects which are really indispensable, but which are not always looked at, viz., that of correctly connecting the tangents which are already given in position and at their points

of contact, and also that of an uniform and continuous curve, (that is, with a constant differential throughout.)

Other methods secure the former, but neglect the latter.

CHAP. IV.

BY A POCKET SEXTANT AND TABLE OF CHORDS.

ANOTHER method which, under certain circumstances, has also its advantages, is that, which is based upon some of the propositions in the Third Book of Euclid, viz.—

That the angle made between any chord and the tangent is equal to the angles in the alternate segment,—that all angles subtended by the same chord are equal,—and that the angles in alternate segments are supplemental to each other.

If SA be a tangent, (fig. 20, Plate VIII.) and A the point of contact where the curve ABED begins, and from the point A any convenient chord AB be drawn, then the angle S′AB will be equal to any angle ADB whatever, in the segment ADB, and the angle SAB will be equal to any angle in the segment AB.

If, therefore, flags be placed at A and B, and any number of points required be taken between A and B whence the angle made between the two flags is equal to the angle SAB, these points will be points in the curve.

Suppose, for example's sake, we have to run a curve of 80 chains rad. from A. Take the chord of any arc, say 20°, the length AB of this will be double the sine 10° (for the chord

of an arc are equals twice the sine of half the arc); but the sin. 10° equals the natural sine of 10°, multiplied by 80, and twice this product will be the length of the chord in chains.

Now the angle ACB (20°), which is the angle at the centre C, is double the angle ADB at the circumference, and (the angle ADB being equal to S'AB) is also double the angle S'AB, which is therefore 10°; and the angle SAB, being the supplemental angle, is equal to 170°. All the angles, therefore, in the supplemental arc from A to B will be 170°.

There is an useful little work on this method by Mr. R. C. May, Associate of the Institution of Civil Engineers, explanatory of the uses of an instrument which he has invented, for the purpose of finding out these points, which are usually taken by a pocket or box sextant. This instrument itself, however, differs little from the box sextant, except that the arc is graduated only from 130° to 180°; and that at the bottom of the box, underneath, there is a kind of trigger, for letting fall a pointed rod, which, being released by pressure on a spring, just as the two poles are seen to coincide, strikes in the ground exactly under the centre of the instrument.

This work is furnished with tables of chords, and the angles they make with the tangent, calculated to the radii that are generally used, and also with the distances upon the old tangents, and the angles at which the new ones intersect them, up to the radius of 240 chains.

Having thus staked out the several points in the curve to B, a new chord, BF, will have to be taken. Consider what angle at the centre you would have it subtend, and the angle FBG will be half of it. (Fig. 19.) Say that this angle be 10°, then the angle FBG will be 5°.

Now, because the angles S'AB and S'BA are each equal to the angles in the alternate segment ADB, they will be equal to each other, and the angle S'BA will be 10°; and therefore the angle ABF, which is the angle made at the end of the old

chord, between the old and the new chord, which has to be measured by the theodolite, will be the supplemental angle of the sum of the two, ABS′ and FBG,

$$\text{that is} = 180°-15°, \text{ or } 165°,$$

and the length of it will be,

$$\text{twice the given rad., i. e., } 2\,(80)\,.\,\text{nat. sin. } 5° = BF.$$

Again, FBG being 5°, and therefore the angle SBF=175°, these will be the measure of all the points of the curve between B and F.

CHAP. V.

Having now briefly explained some of the different methods adopted in practice to guard against errors, we will proceed to consider some of those which are best calculated to remedy them.

S Curves.

In staking out a line upon the ground, it may, and does, sometimes occur that the curve has been commenced from a wrong point, or that though a perfect curve, yet, in consequence of the strings of the curve frames not being set exactly to the fractional part of the inch, the curves which ought to meet are found either to fall short, or to overlap each other, but yet so triflingly that it may not be desirable to run them again. Under either of these circumstances, it is easier to connect them with another curve, or still better with a tangent.

There are various conditions attached to this problem, viz.

1. *When the two circles cross,* these can be connected by a curve, which starts,

First. From a point in the convex curve. (Fig. 21.)

Secondly. From a point in the concave curve, with any radius. (Fig. 22.)

Thirdly. The radius being given. (Fig. 23.)

2. *When these circles fall short of each other,* they can then be connected—

First. By a common tangent to the two. (Fig. 24.)

Secondly. By moving the whole curve along the tangent, that is, by commencing at a new point upon it. (Fig. 25.)

PROBLEM I.

When the two Circles cross.

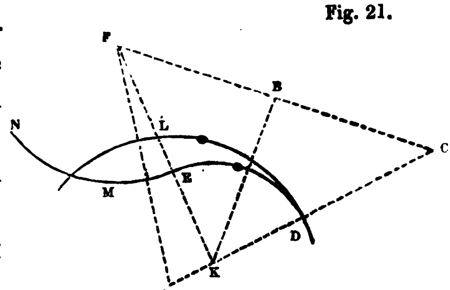
CASE I.

(From a point in the convex curve.)

Let NM and DL be the two curves which cross each other. Let *D be the point in the convex curve where the new curve is to begin.* Let A and F be the centres of the two *given* curves.

Join AD, produce it to C, so that AC shall be equal to the sum of the two given radii. Join CF, and bisect it in B; erect BK perpendicular to it, K will be the centre of the required curve.

Fig. 21.

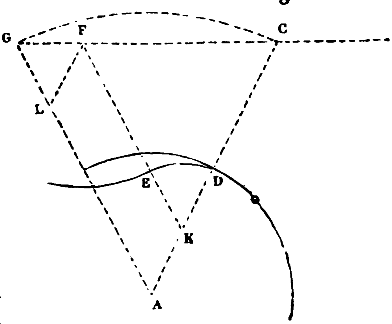


For AC being equal to the sum of the two radii, and AD being one, DC equals the other, and equals FE; but KC and KF are equal ∴ KE=KD, and as the line KEF, which joins their centres, passes through the point of contact, they have the same normal.

CASE II.

(From a point in the concave curve, with any radius, as before.)

Fig. 22



Let A and F be the centres, as before, and *let E be the point given in the concave urve.*

From the centre F draw FE indefinitely, passing through the given point E; draw AG parallel to FE, and equal to the two given radii; join GF, and produce it indefinitely; then from the centre A, at the distance AG describe a circle intersecting the line GC in C. Join AC and through F draw FL parallel to AC.

The point K will be the centre required, and D will be the point of contact.

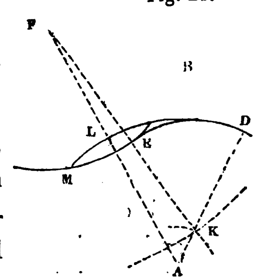
Because AC=AG, LF=LG, and therefore AC—LF or KC=AG—LG, or KF.

Again, DC was made equal to the rad. FE, and therefore DK equals EK, and K is the required centre.

CASE III.

(When the radius is given.)

Fig. 23.



Let B be the given radius, and A and F the centres, as before.

Join AF, and from the centre F describe the arc at K equal to the radius of one of the given curves (EM), plus the given radius. Again, from A, describe an arc equal to the difference between the given radius and and that of the other curve (DE), K is the centre.

For FK=the sum of the radius of the curve EM, and the given curve, and ∴ EK=given radius B; and AK=the diff. between the radius of the curve DE and the given curve, and ∴ KD=given radius B. K therefore is the centre required.—Q. E. D.

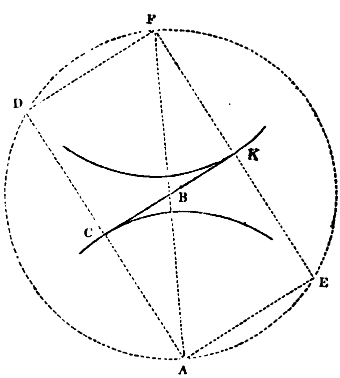
PROBLEM II.

When these circles miss each other.

CASE I.

(By a common tangent to the two.)

Fig. 24.



Join AF, and upon AF as a diameter describe the circle; then from F and A, as centres with radii equal to the sum of the radii of the given curves, describe arcs intersecting the circle at E and D. Join EF and AD, intersecting at K and C. KC shall be the tangent required.

For the triangles ADF, AEF are equal, and the trapezium ADFE, being inscribed in a circle, has its opposite angles DFE and DAE equal to two right angles; but they are equal to each other, each therefore is a right angle, and therefore AD and EF are parallel; and because EF is equal to the sum of the radii, therefore KE equals AC; and therefore ACKE is a parallelogram, and AEF being a right angle, CKE is a right angle, and CK is a tangent to the curve DK.

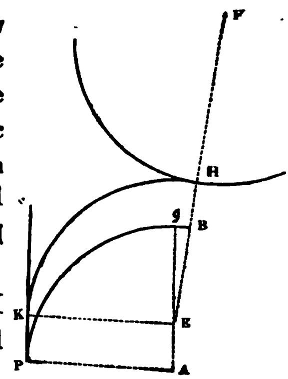
For the same reason DC=FK, and ∴ CK becomes the tangent to the other curve.

Thus being a common tangent to the two.—Q. E. D.

CASE II.

(By moving the whole curve along the tangent or determining the correct point of starting.)

Fig 25.



Let PN be the tangent, and P the point of contact; from A draw AEg parallel to the tangent, and from the centre F, with a distance equal to the sum of the two radii, describe an arc intersecting this line AEg in E. From E draw EK parallel to AP; E will be the centre, and EH will be equal to EK.

FE=their sum, and therefore FH being one, EH equals the other, and touches in H.

Again, AP equals EH, and AP equals KE, therefore KE equals EH, and K will be the new point in the tangent.—Q. E. D.

Explanatory Remarks.

In applying these problems to practice, it must be understood, that though strictly correct theoretically, yet in practice — as the whole error originates not in the theory, but in the imperfect carrying out of the practice—theory alone will be insufficient to remedy it.

The only data that an Engineer has upon the ground are two curves staked out, that ought to touch each other, but do not. The error may be either in incorrectly running the curve, or in starting from the wrong point. The remedy of the former is that of running the curve over again; of the a tter, as it can only have arisen from previous incorrectness of survey, is that of starting from the right point.

As the curve, however, may be a true one, though with a slight error in the radius of curvature, (and in fact this may be the case with both curves,) it may be deemed desirable to connect them with a tangent: this of course will be a practical remedy, a correction of the curves as they are, and not as they ought to be. The first thing, therefore, to be done, is to determine the relative positions of these curves, not from their tangents, but from the curves themselves.

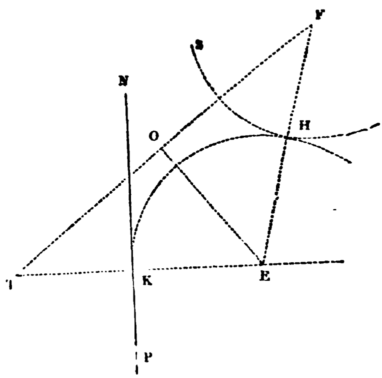
At any point K of one of the curves, by means of the curve frames, run the curve into a tangent, selecting the point such that the tangent, if produced towards the other curve, will come near to it, but not cut it. On this tangent, at a point nearest the second curve, erect a perpendicular to it, measure the length of the tangent thus taken, as well as the perpendicular; the relative position of two points, one in each curve, will be thus obtained. And by measuring other lengths in the tangent and other perpendiculars to each curve, the relative position of the curves will be determined. If the correctness of the radius of either curve be questioned, a tangent and a few ordinates taken to each will determine it.

Now the positions being thus known practically, as well as their actual radius of curvature, and a point being fixed upon each, the principles of the problem, (Prob. 2, Case I., page 293,) for finding the points C and K, may be safely and practically brought to bear.

In case II., where it is proposed to start at another point upon the tangents, the curve being supposed incorrect, or a different radius being desired, —

Adopt the same plan upon the ground as before; viz., run any point H in the first curve into a tangent, take ordinates upon it to determine the position of the curve, produce the tangent till it intersects the tangent PN produced, if necessary, and measure both tangents and the angle between them; all the

Fig. 26.

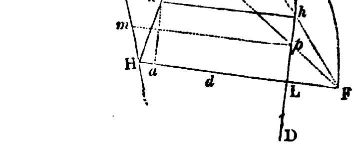


required data will be thus practically obtained. Then, *if the radius be given*, the required point of contact can be obtained by the figure, Prob. 2, (Case II., page 294;) and *if the point* K *be given*, and the radius be required, then through K draw the perpendicular TKE, TK being equal to FH, the radius of the first curve, and KE, indefinite; join TF, bisect it in O, erect the perpendicular OE, intersecting TE in E, E will be the centre, and EK and EH the radius of the new curve.

For TO=FO, and OE is perpendicular, ∴ TE=FE, and TK=FH, ∴ KE=HE, and KE is perpendicular to NP.—

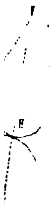
Fig. 4, *page* 276, *practically explained.*

Fig 4*a*.



Produce A*b* and DC till they intersect in E, measure the angle AED, then from the given point *b* erect a perpendicular *b*H of the desired distance, and at H construct the angle *b*HF, which will be the supplemental angle to AED; make HF=H*b*. Any lines drawn from F, cutting ED anywhere between L and C, and produced to the circumference between *b*C, will comply with the conditions of the problem; for let GF be drawn in any direction, cutting LC in *h*, through *h* draw *hk* parallel to HF, and therefore perpendicular to ED, the triangles FHG and *hk*G are similar, and ∴ G*k*=*kh*, and ∴ *k* and H are the two centres, and the line

g. 26.

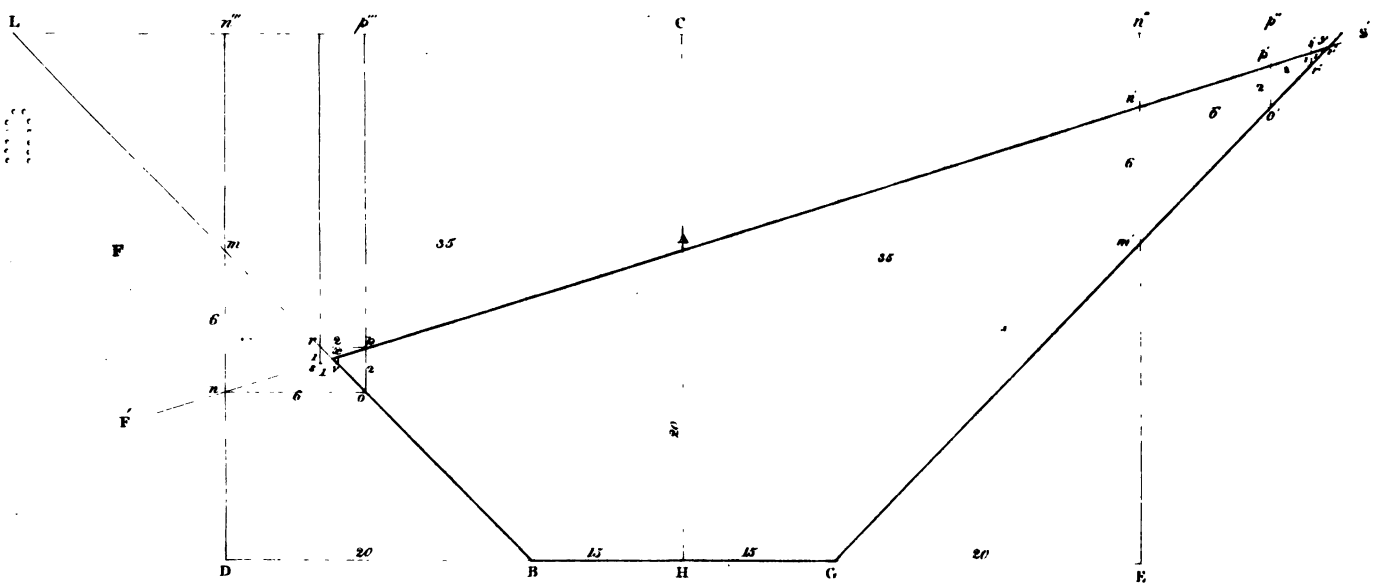


licular
d EK

; and
P. –

Plate 9

Fig 22



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HkG, which joins them, passes through the point of contact, G. The circles therefore touch, and hk being perpendicular to ED, Dh is a tangent to the curve. Any other point H might have been taken, and theoretically equally true; but as the first radius, BH, is constant, the other, kh, keeps decreasing as the point G moves towards C, and increasing toward b. There would be a practical limit, therefore, to its approach towards C, and towards b, where Fb intersected DC in p, the perpendicular, mp, would be the greatest, as mp would be then $=$ to mb. This would, however, destroy the conditions of the question, as it would reduce the problem to one curve instead of two, and shorten the given radius bH by mH. Within these two limits, therefore,—the one, of a minimum of radius, the other, of retaining the two curves,—it may be desirable to suppose one given. This must, of course, be less than FH; let it be Fd; from HL cut off $La = Fd$, then through a draw an indefinite line parallel to CD; from bH cut off $bm = La$, and from H, at the distance Hd, describe a circle cutting ak in k; join Hk, and produce it to G, k shall be the centre of the second circle, and kG and kh shall be each equal to aL. For $HG = HF$, and $Hk = Hd$ $\therefore$ $kG = dF$; but $kh = aL = dF$ $\therefore$ $kG = kh$, which is parallel to HF, and therefore perpendicular to CD.— Q. E. D.

CHAP. VI.

STAKING OUT THE SIDE STAKES.

Before giving out the contracts, after obtaining the Act, in addition to the centre stakes, which are usually planted at chain distances throughout the line, side stakes have to be

driven in on each side of these centre stakes, in order to mark the proper width of the railway.

In doing so. due allowance must be made for the height of the cutting or embankment, as well as for the ratio of the slopes, and the widths of the side and surface ditches. In determining the number of acres that the company would be likely to require for the line, and in estimating the probable expense of it, *previous* to DEPOSITING, these things would have to be taken into aceount; but then they would be based upon the hypothesis, that the cross sections would be level. And for the object in view, this approximation would be sufficiently correct. *After* the Act, however, as the contractors must know where to extend their excavating, and the company what quantity out of each field they shall have to purchase from the respective proprietors. it becomes necessary to determine the exact position of these side stakes, as they are affected by the *lateral* inclination of the ground, so as to calculate accurately the solid and superficial measurements required.

Before going out to determine these points, the Surveyor should furnish himself with the differences between the reduced levels of the ground-line and those of the gradient line at every chain distance, one of which, in fig. 22, Plate IX., is represented by AH. Let these differences be denoted by h, h', h'', &c.

Now if the width of the railway be taken as c, (see page 252,) *including the whole width required for the line, side and surface ditches, hedges,* &c., and the ratio of slopes as m, we shall have $\frac{c}{2}+mh'$, $\frac{c}{2}+mh''$, $\frac{c}{2}+mh''$, &c., as the half-widths of the railway, supposing that the cross sections of the country be in every case perfectly level; that is, (fig. 22,) if A be taken as the point where the centre stake is driven down, and BG be the cross section of the gradiant line, (the railway being supposed to be in a cutting,) and if Bm and Gm' be the direction of the slopes; then, if the cross section of the ground be

taken as level, viz., in the position of FS, then Am, Am' will be the distances upon the surface, and they will be equal to each other, and in every case the formula $\frac{c}{2} + m$ (h, or h', or h'' or h''') will be the required distances, to be measured off on each side of the centre stakes.

The Surveyor must therefore calculate these distances, and insert them in his field-book, placing them in a column alongside of the number of chains that the centre stakes, to which they refer, number respectively on the main section. These, as has been above stated, will be the distances that the side stakes would have to be placed from the centre stakes, if the cross sections were level. In most cases, *unless the side slope is considerable, they may be safely assumed as such,* and the Surveyor will then merely have to measure these distances. Should it, however, be so; not being, for instance, in the directien FS, in figure 22, but in that of FS′, then, as the other lines will be unaltered, BG being the width of the railway, and Bm and Gm' the direction of its slopes, it will be perceived, on inspecting the figure, that, in order to obtain the points where these slopes will come out upon the surface, Gm' on the right will have to be *produced* to y, and Bm on the left will *stop short* in x; so that the side distances will be, on the right Ay, on the left Ax, Ay being longer, and Ax shorter, than the constant distance Am.

It is required to determine practically Ay *and* Ax.

This could be done by taking the cross levels wherever they happen to be sloping, and laying them down upon paper, and then, AH of course being known, drawing the several lines BG, Bm, and Gm' of the cutting, and thus determining the respective positions on the surface of x and y.

But the following method, which is adopted by many En-

gineers, and which is done at once in the field, seems from its simplicity and practical accuracy, to be infinitely preferable.

Let fig. 21, Plate VIII., represent the ground plan of a portion of a line, showing the position of the side stakes, x, y; x', y'; x'', y'', &c., and taking in four chains. Let P be the point where the spirit-level is placed, (it should be so placed as to command, if possible, the whole of the four chains at once.) Let A, fig. 22, be the centre stake, and x and y (fig. 21) the side stakes; F′ S′ being the slope of the ground,

Their position is required.

Let LS′ be the level line, as seen through the telescope, and supposed to be higher than any one of the side stakes. Then, considering the angle of the ground slope n'AS to be such that An' can be safely taken as equal to Am', and HG being $=15$ feet, AH$=20$ feet, and the slope 1 to 1, we shall have

$$\begin{aligned} Ay &= An'+n'p'+p's'+s'v'+v'y \\ &= 35+6+2+1+0=44, \\ \text{and } Ax &= An-np+ps-sv+vx \\ &= 35-6+2-1+0=30. \end{aligned}$$

That is, in general terms,

$$Ay=\frac{c}{2}+m(h)+n'o'+p'r'+s'v' \text{ (see Plate IX.)}$$

$$\text{and } Ax=\frac{c}{2}+m(h)-no+pr-sv+$$

$$\text{but } n'm'=n'o';\ p'o'=p'r', \text{ and } s'r'=s'v',$$
$$\text{the slopes being 1 to 1;}$$

and generally $n'o'=m(n'm')$ where $m=$ ratio, and $n'm'=$ AC$-n'n''$ and $p'o'=n'n''-p'p''$, &c.; where AC, $n'n''$, $p'p''$, &c., are the several readings of the staff. Let these readings be r, r', r'', r''', &c.

Now, if the spirit level be fixed and turned to the centre stake at A, and the reading AC (r) be found, say 10 feet, and then turned to n' (the distance An' being previously measured,

such that An' would be the point where, if the ground were level, the side stake would come), and the reading $n'n''$ (r') taken there, 4 feet; $n'm'$ would $=10-4$, or $r-r'=6$ feet; measure 6 feet further *away* from A to p', and without moving the level, read off $p'p''$ (r'') $=2$ feet.

Now $p'o'$ or $p'r'=n'n''-p'p''=4-2=2$ feet, measure 2 feet still further from A. Let the next reading be 1 foot, then 2—1 will be the distance still further to be taken, thus making the whole distance $Ay=44$ feet. This will be in the direction *up* the slope; Ax in the direction downwards will be *less* than An. AC (r) will of course read as before, 10 feet, [should the ground, however, fall so much as to be more than 12 feet under the level line of the telescope LS', then it will be necessary to take, if possible, all the readings on the *right* of the centre line first, and move the instrument for those on the left,] and let the reading at n, n''' be 16 feet (r) $nm=nn'''-Ac=r'-r=16-10=6=no=np$. Now, because the second reading is greater than the first, measure $np=6$ feet *towards* A, and read pp''' (r'') 14 feet; $nn'''-pp'''=po=r'-r''=16-14=2$: because the third reading is less than the second, measure this two feet *away* from A; the next reading will be found greater than r''; let $r'''=15$ feet; measure this therefore $r'''-r''$, or $15-14=1$ back towards A to x. The distance Ax thus taken will be found to be

$$=35-(16-10)+(16-14)-(15-14)=30 \text{ feet},$$

and generally where r, r', r'', r'''. &c., are the respective readings, and Ay represents the rising ground, and Ax the falling: then in a cutting, and *vice versa,* in an embankment,

$$Ay=\frac{e}{2}+m(h)+m(r-r')+m(r'-r'')+m(r''-r'''), \text{ \&c.}$$

$$\text{and } Ax=\frac{c}{2}+m(h)-m(r'-r)+m(r'-r'')-m(r'''-r''), \text{ \&c.}$$

Now $\frac{c}{2}+m(h)$ will probably vary with every chain; they

must therefore be calculated to each before you go into the field, and then entered in a column in your field-book, with the distances to which they refer.

The following plan I should recommend as a convenient one to a beginner: to keep the left-hand leaf for the left side

on the left-hand leaf, thus:

— + —

A*x* · $r'''—r''$ | $r'—r''$ | $r'—r$ | chains.

the + and — above the columns showing the direction the

A*x*	— $r'''—r''$	+ $r'—r''$	— $r'—r$	Ma Distan Chain
47·30	9·20—9 00	9·40—9·00	9·40—8·50	35·
48·80	8·30—8·20	8·40—8·20	8·40 -7·60	34
43·60	8·60—8·20	10·20—8·20	10·20—2·40	33
47·00	6·40—6·10	10·40—6·10	10·40—1·80	32
53·00	9·30—8·90	10·80—8·90	10·80—2·30	31·

As the ground falls very ropidly here, it becomes necessary to mo
It sometimes happens also, that the ground on the right rises s
(in the second line), and the point where r'' should be taken is to
fore set out of line must be taken, and the instrument carried u
will still be the difference between the *reduced levels* of r' and r''
moved; let the 0·30 now read 8·60, and r'' 5·00; then $r'—r''=3·2$
be of course 5·00, the last placing of the instrument. Should r''' b
the difference between the two readings, is less than a foot.

of the line, and *vice versa*, and commencing the distances on the main section from the bottom upwards;—the right-hand column of the left leaf being the main distances in chains; the left-hand one of the right leaf being the side distances belonging to them. The other columns would stand thus:—

on the right-hand leaf, thus:

$$\frac{c}{2}+mh \mid \overset{+}{r-r'} \mid \overset{+}{r'-r''} \mid \overset{+}{r''-r'''} \mid Ay$$

distances have to be measured, whether towards or away from A.

Side Distance. Feet.	+ $r-r'$	+ $r'-r''$	+ $r''-r'''$	Ay in feet.
48·00	8·50— 6·70	6·7C— 5·80	5·80— 5·00	51·50
49·50	7·60— 3·20	3·20— 1.00	1·00— 0·20	56·90
49·80	11·00— 4·20	4·20— ·3·60	3·60— 0·40	60·40
51·60	11·00— 3·90	3·90— 1·50	1·50— 0·30	62·30
60·00	11·50—11·00	11·00—10·20	10·20—10·00	61·50

the instrument, and take r again for the left side.
rapidly, that in the $r'—r''$ column the reading of r' may be, say 3·20
high to be observed without moving the instrument: in this case a
higher. The point where the next reading r''' will have to be taken
Thus, let r' read 3·20 and the foreset be 0·30, and the instrument be
$-0·30+8·60—5·00=6·50$; the reading of r'' in the third column will
more than a foot less than r'', continue the reading till $(r^n—r^{n\times 1})$, or

APPENDIX.

FIELD NOTES

OF THE

HENDON SURVEY.

End of the left side of Base Line.

	32	1·40	to 76·43 on Base Line
	32	0·00	
From 1630 on 1660			

100		16·60	Strt to 68·57 on B. L.
	100	16·30	Δ
	100	15·00	
	85+40	9·80	
	30	6·00	pond
	40	3·00	20 + 30
	60	0·20	
		0·14	
From 092 on 962			

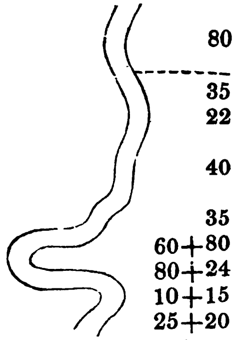
		8·04	to 25 back from 400 on 888
Brook		7·48	
		7·38	×
	50	7·00	
	18	6·30	
	27	5·00	
	9	3·50	
	5	0·00	
From 56·37 on B. L.			

	12·57	0
	10·00	63
	3·00	54
	2·00	30
Brook	0·92	
	0·86	×
From 310 on 888		

	10·19	to 093 on 2492
	0·80	30
From 4500 on B. L.		

	6·76	to 412 back from 1655 on 1655
	6·10	15
	5·00	26
	3·00	36
	0·00	20
From 093 on 2492		

	8·88	to 95 back from 5060 on B. L.
	8·00	
	7·70	
	7·34	
	7·15	
	7·00	
45	6·85	
10	6·50	

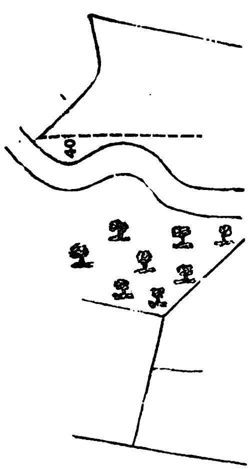


	6·30	77
80	5·70	
	5·50	
35	4·88	
22	4·17	
	4·00	△
40	3·16	
	3·10	△
35	3·00	
60+80	2·40	
80+24	2·20	
10+15	1·30	
25+20	0·72	

From 1300 on 2518

	9·62	to 18 back from 6038 on B. L.
36	8·57	
30	7·80	
44	6·92	
40	6·66	
46	6·39	
	5·91	
30	5·86	△ on gate (3 notches)
20	5·55	
60	4·40	
50	3·90	
70	3·30	
24	2·00	
10	1·40	
7	0·60	

From 2492 on 2518



80	25·78	
	25·18	
70	24·92	△
85	23·90	
116	20·00	
135	14·50	
40	13·70	230
	13·56	
	13·40	
	13·00	
0	12·73	
40	10·50	
77	9·12	
	6·29	△
80	6·05	× straight to offset 60 at 4495 on B. L.
75	5·73	
111	2·48	
105	0·93	
	0·73	×

From 245 on 245

70	2·45	in range with 11·40 on 16·55	
75	0 00		

From 11·69 on 12·44

40	12·44		
40	11·69	Δ	
50	9·13		
50	8·41		
15	0·00		

From 6·16 on 6·16

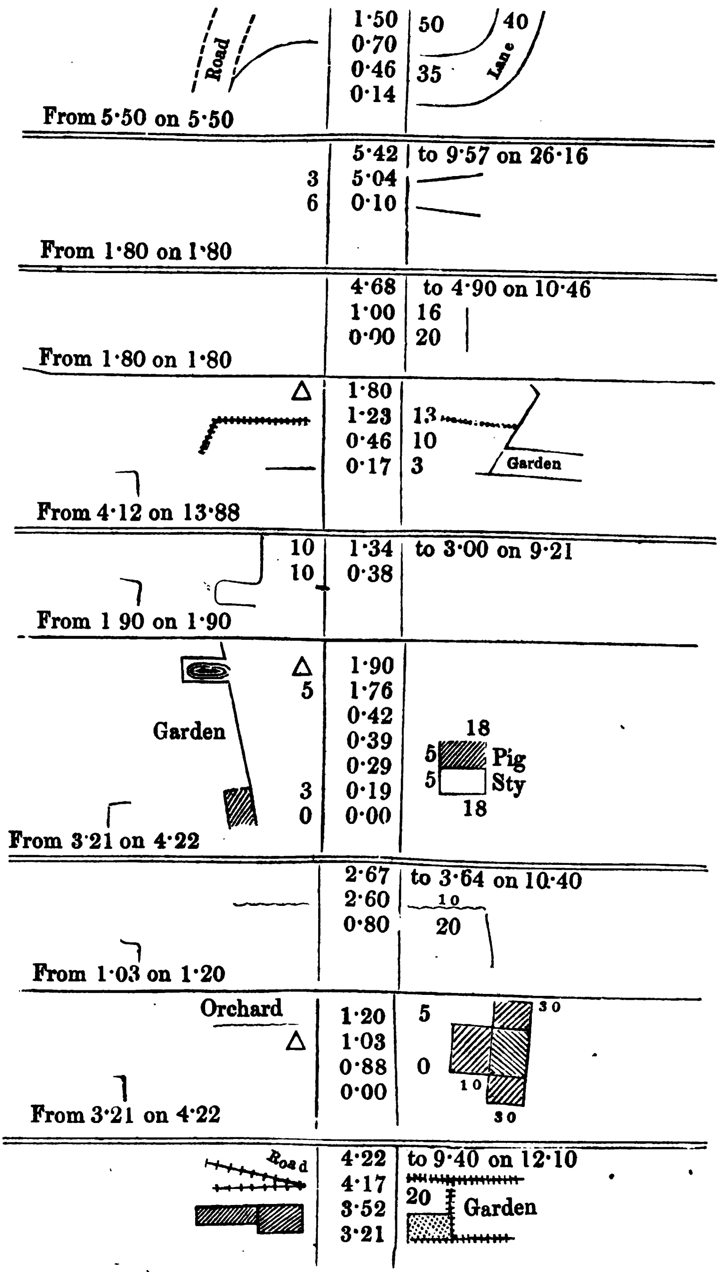
20	6·16	Δ	
30	5·50		
37	4·00		
35	1·90		
Road.	1·40		
	0·30		

From 1·40 on 16·65.

	4·08	o 4·00 on 16·55	
	3·92		
	3·80		
45	3·30		
15	2·00		
15	1·40		
30	0·90		

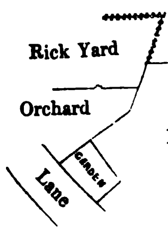
From 29·52 on B L.

	16 55	to 25 on from 43·28 on B. L.	
60	15·55	50	70
	14·25	90	40
60	13·10	75	50
	12·30	50	36
Δ	11·40	10	20
	9·00	0+35	
	6·40	20	
0	6·00	0 +30	
60 + 10	5 00	10	
	4·00	Δ17	Lane 25
	2·25	Δ45	


Road
1·50
0·70
0·46
0·14
50
40
Lane
35
From 5·50 on 5·50
5·42
to 9·57 on 26·16
3
5·04
6
0·10
From 1·80 on 1·80
4·68
to 4·90 on 10·46
1·00
16
0·00
20
From 1·80 on 1·80
1·80
1·23
13
0·46
10
0·17
3
Garden
From 4·12 on 13·88
10
1·34
to 3·00 on 9·21
10
0·38
From 1 90 on 1·90
1·90
5
1·76
0·42
Garden
0·39
18
0·29
5
Pig
Sty
3
0·19
5
0
0·00
18
From 3·21 on 4·22
2·67
to 3·64 on 10·40
2·60
10
0·80
20
From 1·03 on 1·20
Orchard
1·20
5
30
1·03
0·88
0
10
0·00
30
From 3·21 on 4·22
Road
4·22
to 9·40 on 12·10
4·17
20
Garden
3·52
3·21

15	3·06	
0	2·69	
23	0·42	

From 531 on 1040

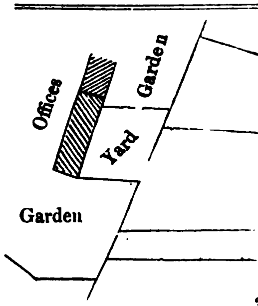


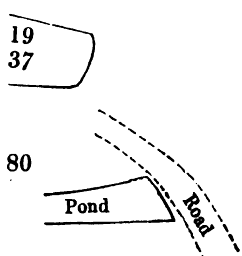
	10·40	to 60 back from 1704 on B. L.
△	5·31	
3	4·98	
△	3·64	
	3·61	Straight to 970 on B. L.
3	3·53	
35	2·00	
120	1·15	
0	0·00	

From 0·98 on 442

	4·42	to 98 back from 970 on B. L.
△	0·98	
	0·90	
	0·22	

From 576 on 1388





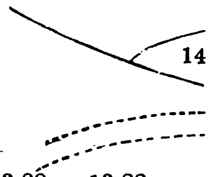
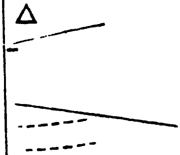
	12·10	to 904 on 1388
	11·45	
	11·00	19
11	10·14	37
	9·47	
19	9·40	
11	9·25	
	8·56	80
20	7·97	
15	7·62	
30	6·36	
35	2·00	

From 724 on 7·24

	7·24	to 24·00 on 2616
	7·05	
	6·79	
	6·60	65
	6·00	80
	5·44	96
	4·00	67
	2·00	15
	1·67	10
	0·46	7

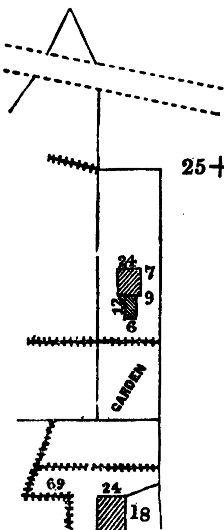
From 1787 on B. L.

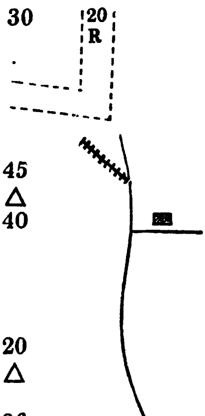
	9·21	to 27 back from 17·87 on B.L.
	3·00	△
	2·98	
14	2·84	
	2·53	
	0·71	
	0·48	

From 13·88 on 13·88

	13·88	to 16·04 on 26·16
60	13·24	
	12·20	30
20	11·97	
	11·92	
15	11·82	
7	11·38	
25+10	10·00	
	9·10	45
△	9·04	△
	7·70	40
16	7·11	
	6·77	
	6·58	
19	6·09	
	5·88	20
	5·76	△
20	4·12	
	2·00	36
16	1·40	
16	1·14	





From 0 85 on B. L.

	9·17	to 26·16 on 26·16

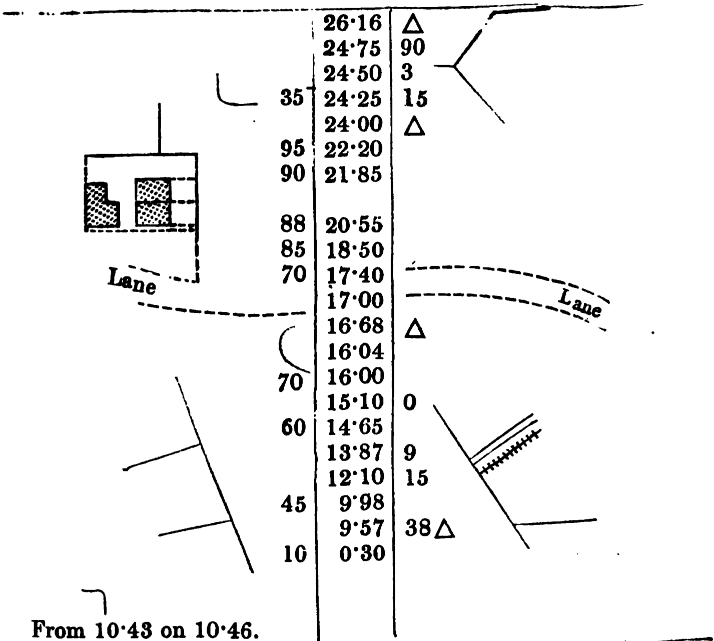
From 5·22 on 5·22

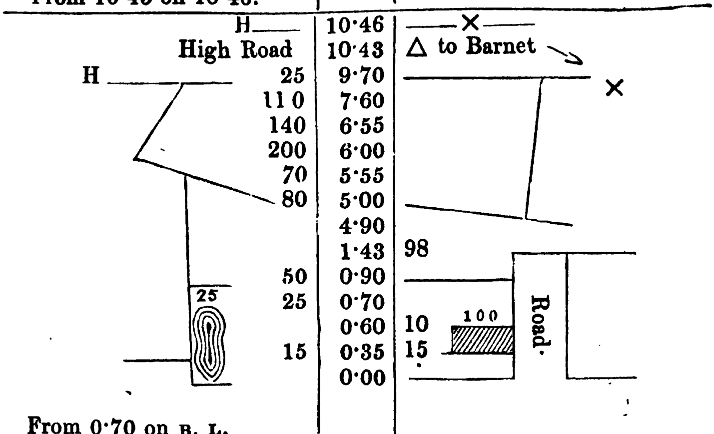
	5·22	to 25 back from 29·52 on B.L.
△	1·40	
40	1·00	
10	0·00	

From 5·50 on 5·50

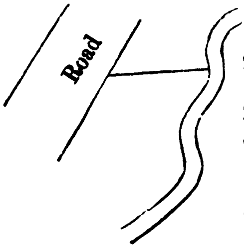
5	5·50	△
14	4·00	
15	3·75	

From 26·16 on 26·16





End of right		Side of Base Line.
	19·82	to 55·00 on B. L.
12	19·00	
	18·90	50
Occ. 22	18·50	Road
23	18·32	
18	18·00	32
13	15·00	23



8	13·00	30	
0	11·60	62	
5	11·25	△60	to 8·05 on 12·02
20	8·30		
15	7·00	65	
25	6·00	60	
30	5·00	50	
15	4·00	60	
25	3·00	70	
40	2·00	40	
70	1·00	12	
	0·60	20	
	0·30		
9	0·00		

From 6·65 on 16·46

	16·46	25	to 76·43 on B. L.
D	10·98	55	
	9·00	52	
	8·00	35	
	7·30		
Straight 60	6·90		
	6·65	△ on top of fence	
	6·56		
	5·70	10	
Brook	5·45		
H	4.15	×	
High		Barnet Road	
H	3·30	×	

From 9·83 on 10·00

	10·00	200
	9·83	△
12	9·30	
	8·05	△
	5·00	
30+40	1·80	
40	1·30	
	1·00	×

From 2·06 on 12·02

35	12·02	to 22 on from 63·08 on B. L.
25	10·00	
27	9·00	
D	8·60	×
D	8·05	×
H	5·60	×
High Road		to Barnet
H	4·70	×
	4·00	200

	4·00	200
	3·00	
	2·06	△
	2·00	120+45
	1·20	45+20
From 5·57 on 5·70		

D——	5·70	
	5·57	△
	5·27	27
	4·50	63
	3·50	25
	3·00	18
	2·60	6
	2·20	30
	1·00	30
From 15·28 on 15·28		nearly in the same direction.

	18·59	to 11·48 on 17·25
D——	18·30	——×
	18·00	14
	15·00	33
Hendon	11·00	42 Parish
	9·55	20
	9·08	
	8·88	95
195	8·80	
	7·90	150
Finchley	7·14	136 Parish
	2·00	90
	1·00	72
From 11·12 on 15·28		

	13·70	to 89 on from 55·00 on B. L.
H——	10·14	——×
High	9·50	Barnet Road
H—60	9·28	——×
55	9·00	
50	6·00	
40	3·35	
25	2·80	
18	1·00	
D——10	0·40	——×
From 13·40 on 15·28		

	15·28	△
	15·15	—— ×
12	14·72	△
45	13·40	△
△	11·12	
110	10·72	
	1·00	45

From 9·67 on 9·67

	21·33	to 9·67 on 9·67
	22·45	80
	18·65	90
	18·35	117
	17·25	36
△	14·54	△
	12·95	105
	12·05	58+40
	11·45	32
	10·71	45
Finchley	10·45	22
Parish	10·05	50
	9·80	
~~10~~	9·62	Hendon
	9·30	Parish
~~25~~	8·99	
	8·45	40
	8·05	80+63
H——	4·65	—— ×
H——	3·72	—— ×
	0·52	Centre of Brook
	0·45	Parish Boundary

From 89 on from 55·00 on B.L.

	9·67	△ 20 + 8
Hendon	9·26	18
+24	0·10	Parish
	8·98	
	8·90	
	8·80	
13	1·00	
Finchley △	0·49	Parish
12	0·40	
14	0·00	

From 7·24 on 17·25 page 3

	8·69	to 4·85 on 23·20
25+40	7·55	
55	4·00	
25	2·38	
	1·90	65
	1·00	60

Occ. Road.	0·70	along the hedge
	0·65	
	0·60	25 35
D	0·30	25
From 18·68 on B. L.		

	7·52	to 7·24 on 17·25
35	6·30	
30	6·00	
25	2·20	
18	1·20	
From 5·65 on 13·78		

	13·78	to 15·73 on 23·20
	13·00	87
	9·20	17
△	5·65	St. to off. 35 at 18·68 on B. L.
	5·64	27
	5·48	25
	4·70	20
	4·50	30
	4·00	25
	3·00	15
	2·00	25
	1·00	40
From 45 on from 29·52 on B. L.		

	17·25	to 38·20 on B. L.
	11·60	30
	11·48	△
	11·00	35
	8·00	45
	7·40	
	7·24	△40
	5·60	45
	4·00	45
	2·00	25
From 23·17 on 23·20		

	23·20	
△	23·17	△40
	20·00	32
	16·00	20
	15·73	△
10	14·72	25
12	13·00	
34	11·16	
	10·00	
	5·17	

100	5·17	
	5·07	
	5·00	40
△	4·85	
	4·20	20
	4·10	
0	4·00	Pond
20	3·50	40
35	3·00	
42	2·00	
From 10·79 on 19·27		

	8·11	to 7·80 on 19·27
Straight to 1140 on 1927	4·35	
	4·27	15
	3·00	9
	1·76	0
	1·00	10
From 9·70 on B. L.		

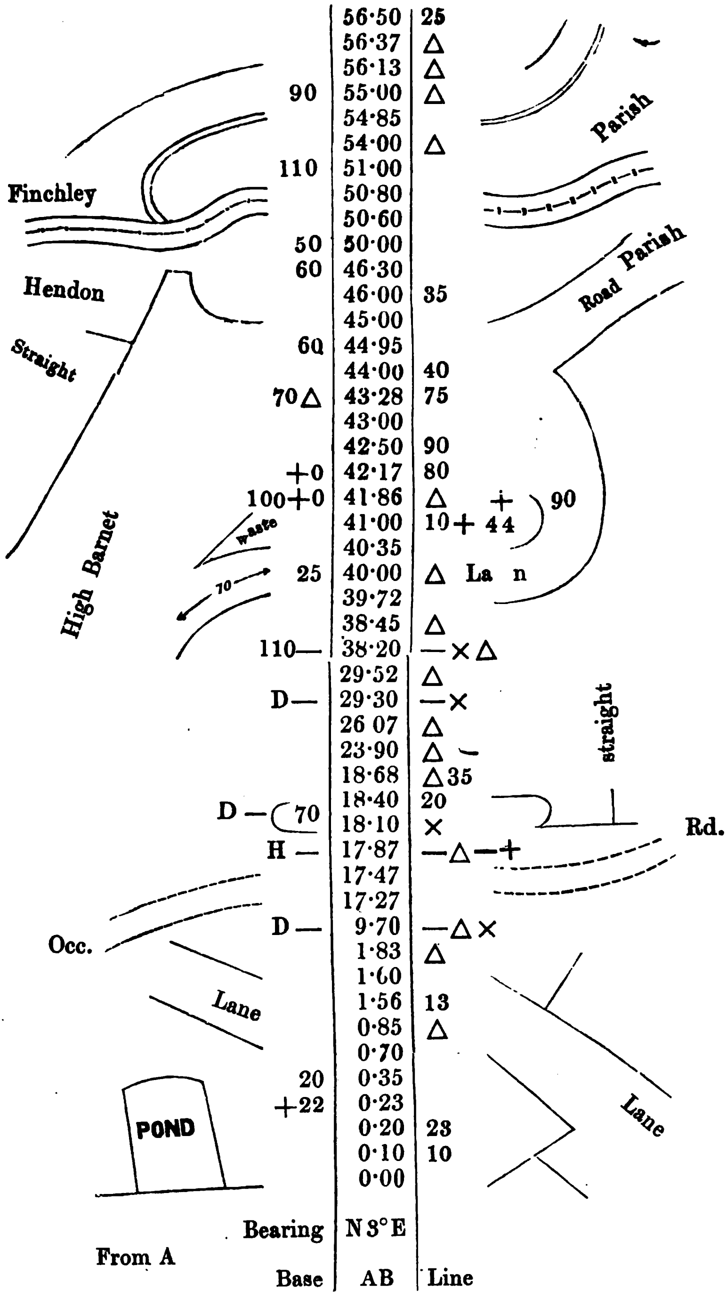
	19·27	to 82 back from 17·87 on B. L.
D——	11·40	——× △
	10·79	△
△	7·80	
95	6·00	
35	5·75	
Straight	4·95	
to 978	4·81	3
on 1398	3·00	72
From 13·98 on 13·98		

△	13·93	
+ 0	9·78	——× △
30 30	3·00	
40	2·50	
	0·90	——×
D——	0·20	Lane
	0·00	10
From 0·23 on B. L.		

D——	76·70	——×
	76·43	to B. on B. L.
D——	68 57	△
	68·29	△
	67·55	120
	65·73	
	65·63	△
85	63·20	
	63·08	——× △

HENDON SURVEY.

End of the left side of Base Line.

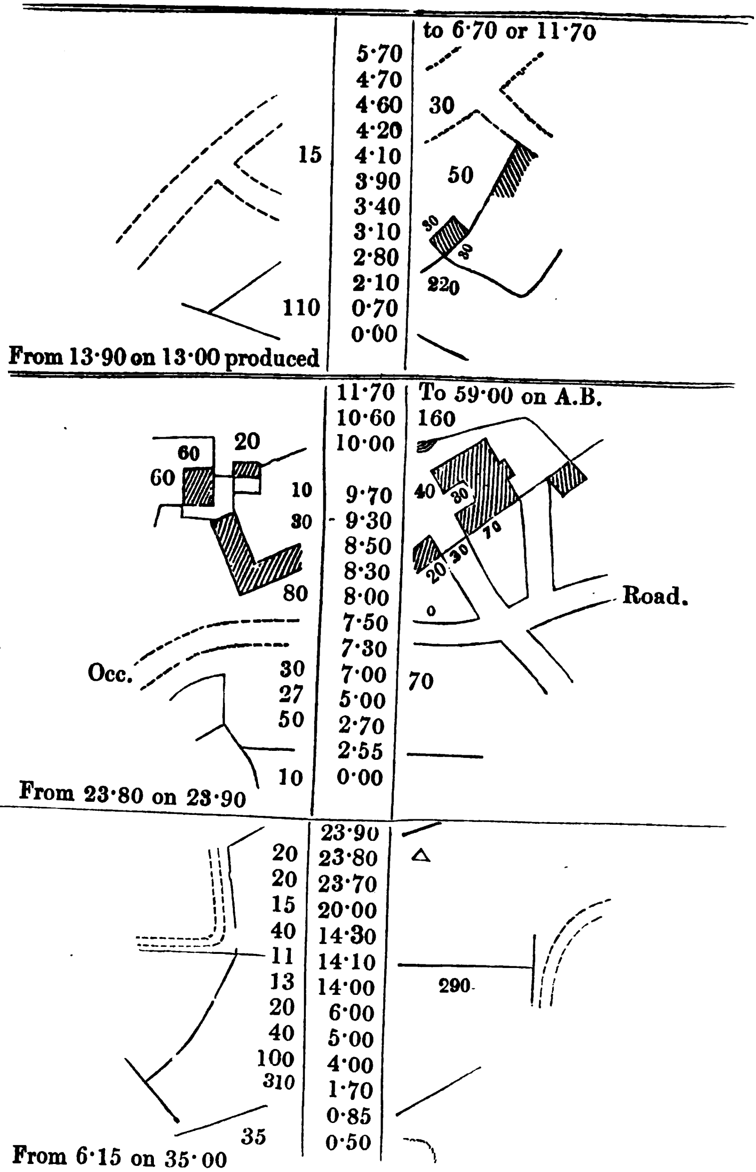


HENDON FIELD NOTES.

FIELD NOTES

OF THE

WIGSTON SURVEY.


to 6·70 or 11·70
5·70
4·70
4·60
4·20
4·10
3·90
3·40
3·10
2·80
2·10
0·70
0·00
15
110
30
50
30
30
220
From 13·90 on 13·00 produced
11·70
10·60
10·00
9·70
9·30
8·50
8·30
8·00
7·50
7·30
7·00
5·00
2·70
2·55
0·00
To 59·00 on A.B.
160
60
60
20
10
30
80
40
30
70
20
30
0
Road.
Occ.
30
27
50
10
70
From 23·80 on 23·90
23·90
23·80
23·70
20·00
14·30
14·10
14·00
6·00
5·00
4·00
1·70
0·85
0·50
20
20
15
40
11
13
20
40
100
310
35
290
From 6·15 on 35·00

			to 8·53 on 15·60	
		13·50	30	page 6
		13·00	30	
Lines found to have been omitted.		9·80		
	55	5·00		
	20	3·90		
	60	3·60		
		2·50		
		1·70	55	
		1·30	75	
		0·70		
	80	0·25		
From 600 on 16·80, page 5				

		11·00		to 33·70 on 35·00
		10·80	5	
		10·50		
	60 20	9·80	10	
		8·00	10	
Lock	20 40	7·50	10	
	40	6·00		
	60 20	5·50	15	
		0·00	10	
From 8·50 on 9·40				

		9·40		Canal
		8·70		
		8·50	△	
	10	8·40		
		7·20		
	60	5·90		
	60	5.30		
	30	3·00		
	0	0·00		
From 13·90 on 13·90				

		13·90	to 64·00 on A.B.	
	230	12·50		
		1·90		
Occ.		0·60		
		0·40		Road.
From 33·70 on 35·00				

		18·90	to 10·45 on 19·50	
	30	16·00		
	60 10	11·40		
	10	11·10		
	10	9·90		
	50	8·50		
Newton	70	7·00	Harcourt	
	20	4·00		
	30	2·75	50	
From 2·15 on 11·00				

	11·00	to 19·30 on 33·45
60 40	9.90	30
	9·50	
60	4·00	
50	1·35	
40	1·00	

From 94·00 on A. B.

	33·45	to 25·80 on 35·00
	33·00	80
	28·50	35
	26·00	35
	23·30	60
	19·50	10
	18·60	
40	17·00	
	12·50	
70	12·30	
70	9·60	
60	7·00	
	5·30	
	4·40	50
	1·00	70

From 18·65 on 19·50

	14·00	
	12·80	
50	11·60	
30	8·00	
20	0·90	
20	0·30	

From 4·00 on 35·00

20	35·00	
	34·50	
Canal	34·00	Canal
40	33·90	
△	33·70	
160	27·40	
90	27·30	
	26·70	
90	25·50	
60	20·20	to 79·30 on A. B.
	14·00	
10	12·90	△
10	12·80	
	12·60	70
120	11·00	
	7·20	
	6·40	

to Kilby

10 20 30 30

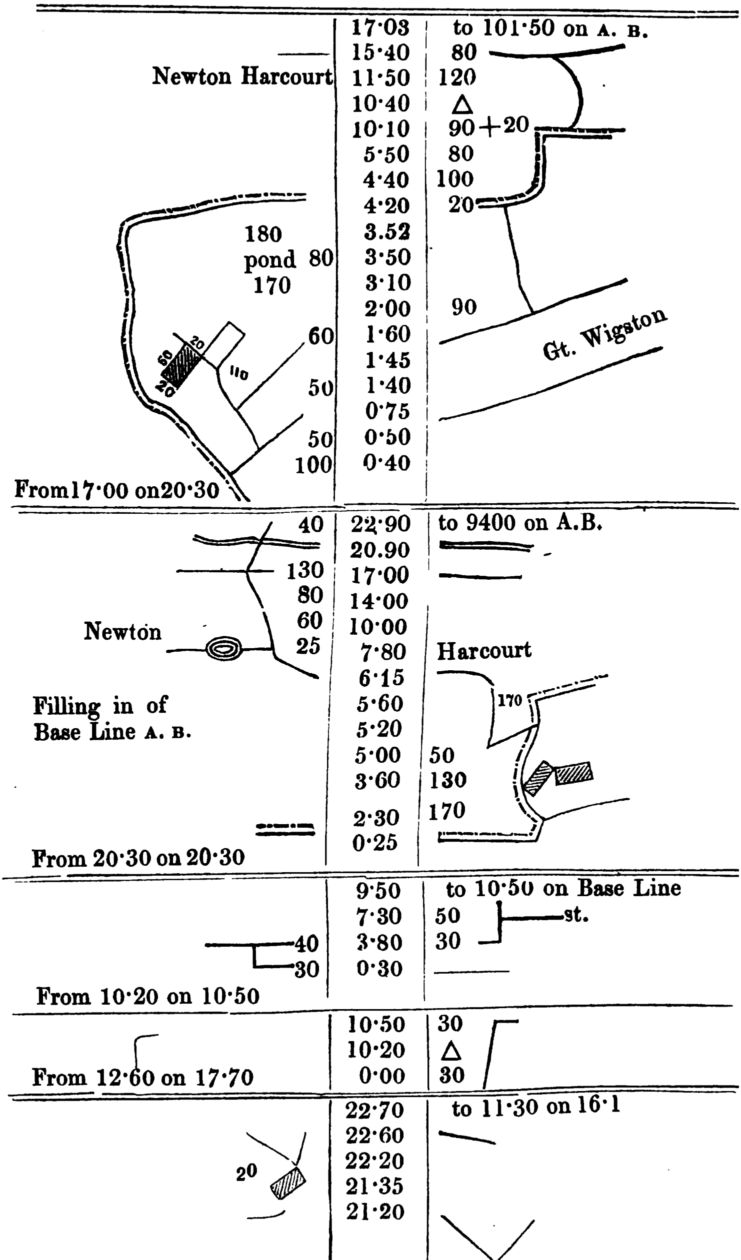
70	6·40	
60	5·00	
	4·50	90
	4·30	30
	4·00	△
	0·90	
90 10	0.00	
From right side of base line		

10	30·90	to 21·10 on 22·90
	27·50	
290	23·80	
	18·70	40
	18·00	40
	17·50	
120	16.50	
60	15·50	
100	14·50	
70	14·00	
70	13·00	
	12·70	
20	12·40	to 6·60 on 19·50
30	11·80	
40	9·50	
Check line 110	7·00	
140	5·80	
160	3·00	
150	0·60	
From 5·00 on 10·90		

	13.20	to 16·43 on 16·43
From 18·65 on 19·50		page 2.

	19·50	70
	18·65	△
	12·00	25
	10·90	
	9·00	50
	6·30	
	5·90	90
Path	5·00	
From B on base line		

	12·00	90	to B., on Base Line
	10·00	50	
Great	9·70	150	Wigston Parish
	8·00	140	
	5·00		
	4·25	65	
	1·10	40	
	0·80	40	
From 10·10 on 17·03			



17·03 to 101·50 on A. B.
15·40 80
Newton Harcourt 11·50 120
10·40 △
10·10 90+20
5·50 80
4·40 100
4·20 20
180
3.52
pond 80 3·50
170
3·10
2·00 90
60 1·60
Gt. Wigston
1·45
50 1·40
0·75
50 0·50
100 0·40
From 17·00 on 20·30
40 22·90 to 9400 on A.B.
20.90
130 17·00
80 14·00
60 10·00
Newton
25 7·80 Harcourt
6·15
170
Filling in of
Base Line A. B.
5·60
5·20
5·00 50
3·60 130
2·30 170
0·25
From 20·30 on 20·30
9·50 to 10·50 on Base Line
7·30 50 st.
40 3·80 30
30 0·30
From 10·20 on 10·50
10·50 30
10·20 △
From 12·60 on 17·70 0·00 30
22·70 to 11·30 on 16·1
22·60
22·20
20
21·35
21·20

70	16.00	V
160	14·50	
80	13·90	
130	13·50	
80	12·00	
70	10·00	
20	00	
10	0·00	
From 17·60 (page 4) on 17·70		

30	10·60	
	9·52	to 6·40 on 16·10
50	2·00	
From 31·40 on B. C.		

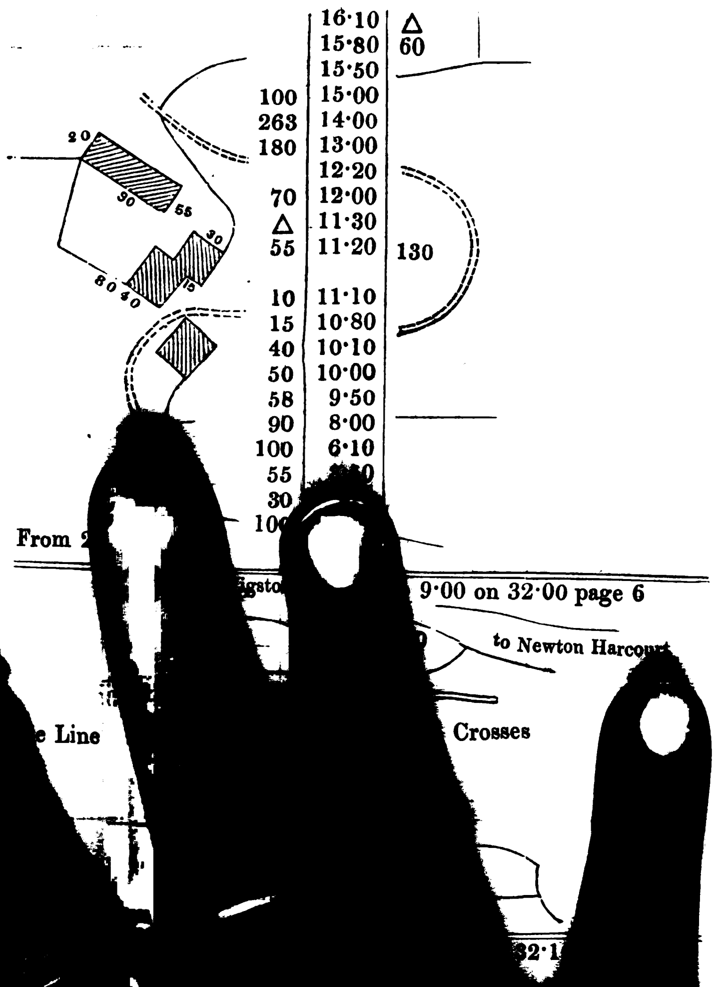
	10·22	to 9·45 on 13·00
	9·82	to 33·40 on B. C.
50	9·00	
12	7·50	
15	6·00	
Occ.	4·00	Road.
20	1·00	
30	0·30	
From 9·40 on 19·35		

	6·85	to 9·80 on 19·35.
40	4·00	
20	0·20	
From 8·72 on 17·80		

	7·40	to 0·70 on 19·35
	4·00	50
	2·00	35
	0·80	20
From 17·40 on 17·80		

85	17·80	
	17·62	
△	17·40	
6	13·50	
20	8·52	40
	3·00	25
From 11·20 on 24·35	0·00	20

	19·35	to 5·00 on 24·35
	18·35	20
	13·00	
	9·85	
	9·65	30
	5·00	15
	0·60	
From 16·10 on 16·10		


16·10
15·80
15·50
15·00
14·00
13·00
12·20
12·00
11·30
11·20
11·10
10·80
10·10
10·00
9·50
8·00
6·10
60
100
263
180
70
55
10
15
40
50
58
90
100
55
30
130
20
90
55
30
15
80
40
From 2
9·00 on 32·00 page 6
to Newton Harcou
e Line
Crosses
32·1

Δ	32·10	
	30·32	30
LEFT	26·00	30 BASE
	21·50	20
	19·80	
SIDE OF	19·70	20 LINE.
	15·80	30
Brook	12·70	
	11·00	
80	5·00	
120	0·60	
	0·15	
From 80·00 on B. C.		

	7·80	to 10·00 on 14·20
	6·00	
80	3·50	
100	2·00	
90	1·20	
	0·50	
From 11·40 on 11·40		

Δ	11·40	to 31·40 on B. C.
45	11·00	
35	8·22	
0	6 00	
20	4·00	
40	0·82	
From 12·60 on 15·60		

	10·50	to 11·35 on 16·80
	9·50	50
	8·85	
	8·70	30
	7·00	20
	4·00	20
	2·00	40
From 14·80 on 15·60	1·00	80

	15·60	to 43·30 on B. C.
10 5	14·80	Δ
30 20 20	14·60	
80	14·50	
	14·10	
80	12·20	
10	8·80	
400	3·70	
50	3·35	
	3·20	180
From 15·00 on 35·90		

	16·10	Δ
	15·80	60
	15·50	
100	15·00	
263	14·00	
180	13·00	
	12·20	
70	12·00	
Δ	11·30	
55	11·20	130
10	11·10	
15	10·80	
40	10·10	
50	10·00	
58	9·50	
90	8·00	
100	6·10	
55	3·50	
30	2·70	
100	2·50	
	2·10	

From 20·00 on B. C.

Road from Wigston	27·88	to 9·00 on 32·00 page 6
	27·20	
	26·20	190 to Newton Harcourt
	20·35	
	18·10	
	17·90	
Base Line 100	16·20	Crosses
100	13·00	
50	8·90	
110	8·80	
	5·35	
	1·50	140
	0·20	160

From 8·00 on 24·35

	24·35	to 32·10 on 32·10
	24·00	175
	16·00	45
	11·40	
	10·00	18
	8·00	25
	6·35	30
100	5·50	
95	4·75	
	3·60	
Occ	3·40	Road
40	1·00	
50	0·00	

From 43·30 on B. C.

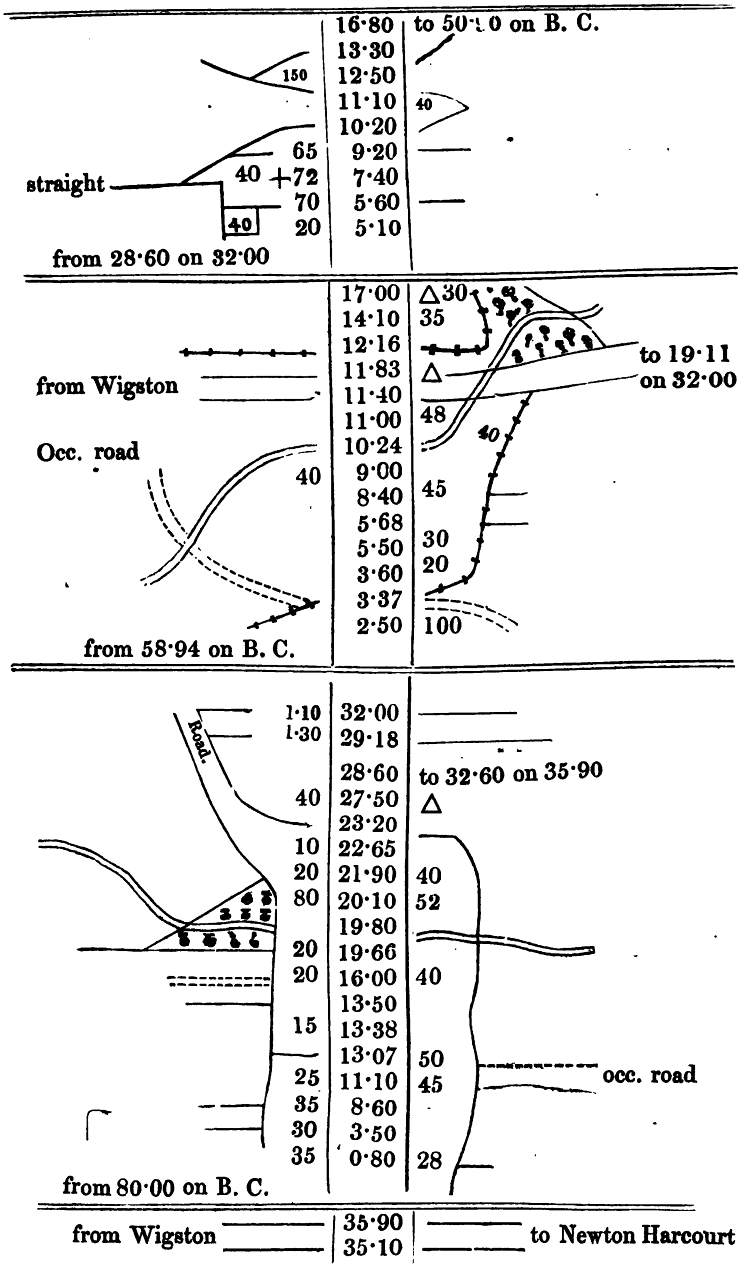
	Δ	32·10		
		30·32	30	
LEFT		26·00	30	BASE
		21·50	20	
		19·80		
SIDE OF		19·70	20	LINE.
		15·80	30	
Brook		12·70		
		11·00		
	80	5·00		
	120	0·60		
		0·15		
From 80·00 on B. C.				

		7·80	to 10·00 on 14·20
		6·00	
	80	3·50	
	100	2·00	
	90	1·20	
		0·50	
From 11·40 on 11·40			

	Δ	11·40	to 31·40 on B. C.
	45	11·00	
	35	8·22	
	0	6 00	
	20	4·00	
	40	0·82	
From 12·60 on 15·60			

		10·50	to 11·35 on 16·80
		9·50	50
		8·85	
		8·70	30
		7·00	20
		4·00	20
		2·00	40
From 14·80 on 15·60		1·00	80

		15·60	to 43·30 on B. C.
10	5	14·80	Δ
30 20	20	14·60	
	80	14·50	
		14·10	
	80	12·20	
	10	8·80	
400		3·70	
	50	3·35	
		3·20	180
From 15·00 on 35·90			


16·80
13·30
150
12·50
11·10
40
10·20
65
9·20
straight
40
+72
7·40
70
5·60
40
20
5·10
from 28·60 on 32·00
17·00
△30
14·10
35
12·16
from Wigston
11·83
△
to 19·11
on 32·00
11·40
11·00
48
Occ. road
10·24
40
40
9·00
8·40
45
5·68
5·50
30
3·60
20
3·37
2·50
100
from 58·94 on B. C.
1·10
32·00
1·30
29·18
Road.
28·60
to 32·60 on 35·90
40
27·50
△
23·20
10
22·65
20
21·90
40
80
20·10
52
19·80
20
19·66
20
16·00
40
13·50
15
13·38
13·07
50
25
11·10
45
occ. road
35
8·60
30
3·50
35
0·80
28
from 80·00 on B. C.
from Wigston
35·90
35·10
to Newton Harcourt

	32·60	△
—	31·90	—
	29·00	
100	28·50	
	20·90	
	17·20	52
△	15·00	
from 14·00 on 14·00 produced	11·73	—

	10·90	to 13·57 on 17·70
—	8·60	
	8·30	86
—	6·48	—
ditch	6·41	50
—	2·80	—
	2·30	
60	1·00	
from 16·43 on 16·43		

	17·70	—	
190	17·60	△	
	13·57	△	
	13·32	△ a	
path	12·40	- - - - - -	
	12·20	0	
	10·00	20	
	8·70	△	
8	7·00	50+40	11·50 on 26·80
—	5·75	90	
g	2·00	60	
from 0·32 on B. C.			

a	26·80	to 15·33 on B. C.
—	21·70	
a	21·50	△70
—	11·80	
g	11·50	△
	6·47	—
Check line 5	0·00	
from 16·38 on 1643		

	14·20	to 25·25 on B. C.
20	11·90	
30	11·00	
40	10·50	—
	10·20	
30	10·04	
40	10·00	△
60	6·00	
70	0·60	
from 14·00 on 14·00		

to 14·10 on 14·10

Road

	14·00	Δ
20	13.20	400
15	12·00	
	7·52	
20+12	6·39	Δ
15	5·00	12
	1·10	5
	0·80	
Occ.	0·55	

From 3·07 on 23·84

to 14·10 on 23·84

	14·10	Δ
	13·85	5
5	10·00	10
	8·00	
	7·00	15
	6·24	
	6·00	20
	0·00	20

From 20·00 on B. C.

to 25·57 on B. C.

	23·84	
	21·60	
Occ. Road	14·10	
	14·00	Δ
	13·80	5
	13·50	
	13·40	30
	3·50	st

From 17·00 on 20·30

to 9·70 on B. C.

	15·44	40
	14·00	78
	13·03	Δ
	12·00	25
	10·00	20
	7·60	65 st.
	2·00	60 a
	0·60	20 40
	0·00	12 a g

From 13·70 on 20·30.

Parish of Great Wigston

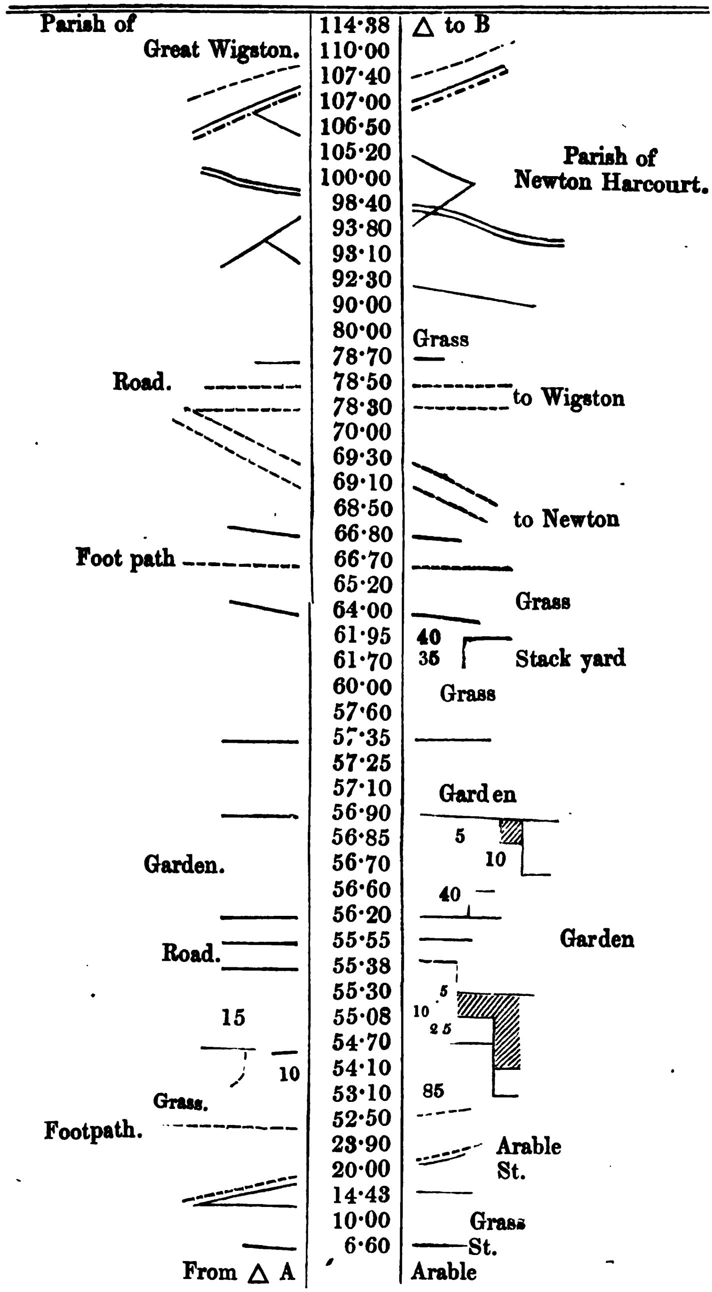
Parish of Newton Harcourt.

	20·30	Δ
	19·80	
50	18·80	
	18·40	
	17·40	1·10
Δ	17·00	80
	15·65	

	15·10	80
	14·17	
40 50 △	13·90	
	10·00	△
	2.76	15
	2·06	80
	I·06	
The last line		reversed
From 0·00 on B. C.		

	16.43	
	16.38	△
	10·00	4
	7·95	
	4·50	△
	4·40	
Going south		
From 0·00 on B. C.		

	81·07	
From Wigston	80.00	△ to Newton Harcourt
	79·60	
	79·00	△
	78·85	
	71·40	40+ pond
	70·80	25+(100)+80
	70·50	130
	70·00	△
	64·80	
	64·15	
Water	63·95	
	63·90	course
	62·08	△
	60·00	
	53·70	△
	42·60	△ Straight
	40·00	Grass
	33·93	△
	33·00	
	30·85	17 Plantation
	30·40	
	30·00	
	26·00	△
	22·50	△
	20·00	
Occ. Road	19·88	
	19·78	△
	10·10	
	9·90	
	0·80	
Between A.	227° 7′	and C.
At B.		



Between Newton Harcourt and Great Wigston.

TRAVERSE TABLES,

CALCULATED TO ANY NUMBER

OF

CHAINS, OR LINKS OF DISTANCE,

AND TO

THREE MINUTES OF THE ANGLE OF BEARING.*

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* *The principle and method of using these Tables will be found in the Third Part.*

DISTANCE IN CHAINS, &c.

Deg	1		2		3		4		5		6		7		8		9		10	
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep	Lat.	Dep.	Lat.	Dep.
3	1·00	·001	2·00	·002	3·00	·003	4·00	·003	5·00	·004	6·00	·005	7·00	·006	8·00	·007	9·00	·008	10·00	·009
6	1·00	·002	2·00	·003	3·00	·005	4·00	·007	5·00	·009	6·00	·010	7·00	·012	8·00	·014	9·00	·016	10·00	·017
9	1·00	·003	2·00	·005	3·00	·008	4·00	·010	5·00	·013	6·00	·016	7·00	·018	8·00	·021	9·00	·023	10·00	·026
12	1·00	·003	2·00	·007	3·00	·010	4·00	·014	5·00	·017	6·00	·021	7·00	·024	8·00	·028	9·00	·031	10·00	·035
15	1·00	·004	2·00	·009	3·00	·013	4·00	·017	5·00	·022	6·00	·026	7·00	·030	8·00	·035	9·00	·039	10·00	·044
18	1·00	·005	2·00	·010	3·00	·016	4·00	·021	5·00	·026	6·00	·031	7·00	·037	8·00	·042	9·00	·047	10·00	·052
21	1·00	·006	2·00	·012	3·00	·018	4·00	·024	5·00	·030	6·00	·037	7·00	·043	8·00	·049	9·00	·055	10·00	·061
24	1·00	·007	2·00	·014	3·00	·021	4·00	·028	5·00	·035	6·00	·042	7·00	·049	8·00	·056	9·00	·063	10·00	·070
27	1·00	·008	2·00	·016	3·00	·023	4·00	·031	5·00	·039	6·00	·047	7·00	·055	8·00	·063	9·00	·071	10·00	·079
30	1·00	·009	2·00	·017	3·00	·026	4·00	·035	5·00	·044	6·00	·052	7·00	·061	8·00	·070	9·00	·079	10·00	·087
33	1·00	·010	2·00	·019	3·00	·029	4·00	·038	5·00	·048	6·00	·058	7·00	·067	8·00	·077	9·00	·086	10·00	·096
36	1·00	·010	2·00	·021	3·00	·031	4·00	·042	5·00	·052	6·00	·063	7·00	·073	8·00	·084	9·00	·094	10·00	·105
39	1·00	·011	2·00	·023	3·00	·034	4·00	·045	5·00	·057	6·00	·068	7·00	·079	8·00	·091	9·00	·102	10·00	·113
42	1·00	·012	2·00	·024	3·00	·037	4·00	·049	5·00	·061	6·00	·073	7·00	·085	8·00	·098	9·00	·110	10·00	·122
45	1·00	·013	2·00	·026	3·00	·039	4·00	·052	5·00	·065	6·00	·079	7·00	·092	8·00	·105	9·00	·118	10·00	·131
48	1·00	·014	2·00	·028	3·00	·042	4·00	·056	5·00	·070	6·00	·084	7·00	·098	8·00	·112	9·00	·126	10·00	·140
51	1·00	·015	2·00	·030	3·00	·044	4·00	·059	5·00	·074	6·00	·089	7·00	·104	8·00	·119	9·00	·133	10·00	·148
54	1·00	·016	2·00	·031	3·00	·047	4·00	·063	5·00	·078	6·00	·094	7·00	·110	8·00	·126	9·00	·141	10·00	·157
57	1·00	·017	2·00	·033	3·00	·050	4·00	·066	5·00	·083	6·00	·100	7·00	·116	8·00	·133	9·00	·149	10·00	·166
1°	1·00	·017	2·00	·035	3·00	·052	4·00	·070	5·00	·087	6·00	·105	7·00	·122	8·00	·140	9·00	·157	10·00	·175
3	1·00	·018	2·00	·037	3·00	·055	4·00	·073	5·00	·092	6·00	·110	7·00	·128	8·00	·147	9·00	·165	10·00	·183
6	1·00	·019	2·00	·038	3·00	·057	4·00	·077	5·00	·096	6·00	·115	7·00	·134	8·00	·153	9·00	·173	10·00	·192
9	1·00	·020	2·00	·040	3·00	·060	4·00	·080	5·00	·100	6·00	·120	7·00	·140	8·00	·160	9·00	·181	10·00	·201
12	1.00	·021	2·00	·042	3·00	·063	4·00	·084	5·00	·105	6·00	·126	7·00	·147	8·00	·168	9·00	·188	10·00	·209
15	1·00	·022	2·00	·044	3·00	·065	4·00	·087	5·00	·109	6·00	·131	7·00	·153	8·00	·174	9·00	·196	10·00	·218
18	1·00	·023	2·00	·045	3·00	·068	4·00	·091	5·00	·113	6·00	·136	7·00	·159	8·00	·181	9·00	·204	10·00	·227
21	1·00	·023	2·00	·047	3·00	·071	4·00	·094	5·00	·118	6·00	·141	7·00	·165	8·00	·188	9·00	·212	10·00	·236
24	1·00	·024	2·00	·049	3·00	·073	4·00	·098	5·00	·122	6·00	·146	7·00	·171	8·00	·195	9·00	·220	10·00	·244
27	1·00	·025	2·00	·051	3·00	·076	4·00	·101	5·00	·127	6·00	·152	7·00	·177	8·00	·202	9·00	·228	10·00	·253
30	1·00	·026	2·00	·052	3·00	·079	4·00	·105	5·00	·131	6·00	·157	7·00	·183	8·00	·209	9 00	·236	10·00	·262
33	1·00	·027	2·00	·054	3·00	·081	4·00	·108	5·00	·135	6·00	·162	7·00	·189	8·00	·216	9·00	·243	10·00	·270
36	1·00	·028	2·00	·056	3·00	·084	4·00	·112	5·00	·140	6·00	·168	7·00	·195	8·00	·223	9·00	·251	10·00	·280
39	1·00	·029	2·00	·058	3·00	·086	4·00	·115	5·00	·144	6·00	·173	7·00	·202	8·00	·230	9·00	·259	10·00	·288
42	1·00	·030	2·00	·059	3·00	·089	4·00	·119	5·00	·148	6·00	·178	7·00	·208	8·00	·237	9·00	·267	10·00	·297
45	1·00	·031	2·00	·061	3·00	·092	4·00	·122	5·00	·153	6·00	·183	7·00	·214	8·00	·244	9·00	·275	10·00	·305
48	1·00	·031	2·00	·063	3·00	·094	4·00	·126	5·00	·157	6·00	·188	7·00	·220	8·00	·251	9·00	·283	10·00	·314
51	1·00	·032	2·00	·065	3·00	·097	4·00	·129	5·00	·161	6·00	·194	7·00	·226	8·00	·258	8·99	·291	9·99	·323
54	1·00	·033	2·00	·066	3·00	·099	4·00	·133	5·00	·166	6·00	·199	7·00	·232	8·00	·265	8·99	·298	9·99	·331
57	1·00	·034	2·00	·068	3·00	·102	4·00	·136	5·00	·170	6·00	·204	7·00	·238	7·99	·272	8·99	·306	9·99	·340
2°	·999	·035	2·00	·070	3·00	·105	4·00	·140	5·00	·175	6·00	·209	7·00	·244	7·99	·279	8·99	·314	9·99	·349
3	·999	·036	2·00	·072	3·00	·107	4·00	·143	5·00	·179	6·00	·215	7·00	·251	7·99	·286	8·99	·322	9·99	·358
6	·999	·037	2·00	·073	3·00	·110	4·00	·146	5·00	·183	6·00	·220	6·99	·256	7·99	·293	8·99	·330	9·99	·366
9	·999	·037	2·00	·075	3·00	·112	4·00	·150	5·00	·188	6·00	·225	6·99	·263	7·99	·300	8·99	·338	9·99	·375
12	·999	·038	2·00	·077	3·00	·115	4·00	·154	5·00	·192	6·00	·230	6·99	·269	7·99	·307	8·99	·345	9·99	·384
15	·999	·039	2·00	·078	3·00	·118	4·00	·157	5·00	·196	5·99	·235	6·99	·275	7·99	·314	8·99	·353	9·99	·392
18	·999	·040	2·00	·080	3·00	·120	4·00	·160	5·00	·201	5·99	·241	6·99	·281	7·99	·321	8·99	·361	9·99	·401
21	·999	·041	2·00	·082	3·00	·123	4·00	·164	5·00	·205	5·99	·246	6·99	·287	7·99	·328	8·99	·369	9·99	·410
24	·999	·042	2·00	·084	3·00	·126	4·00	·168	5·00	·210	5·99	·251	6·99	·293	7·99	·335	8·99	·377	9·99	·419
27	·999	·043	2·00	·085	3·00	·128	4·00	·171	4·99	·214	5·99	·256	6·99	·299	7·99	·342	8·99	·385	9·99	·427
30	·999	·044	2·00	·087	3·00	·131	4·00	·174	4·99	·218	5·99	·262	6·99	·305	7·99	·349	8·99	·392	9·99	·436
33	·999	·044	2·00	·089	3·00	·133	4·00	·178	4·99	·222	5·99	·267	6·99	·311	7·99	·356	8·99	·400	9·99	·445
36	·999	·045	2·00	·091	3·00	·136	4·00	·182	4·99	·227	5·99	·272	6·99	·318	7·99	·363	8·99	·408	9·99	·454
39	·999	·046	2·00	·092	3·00	·139	4·00	·185	4·99	·231	5·99	·277	6·99	·324	7·99	·370	8·99	·416	9·99	·462
42	·999	·047	2·00	·094	3·00	·141	4·00	·188	4·99	·236	5·99	·283	6·99	·330	7·99	·377	8·99	·424	9·99	·471
45	·999	·048	2 00	·096	3·00	·144	3·99	·192	4·99	·240	5·99	·288	6·99	·336	7·99	·384	8·99	·432	9·99	·480
48	·999	·049	2·00	·098	3·00	·147	3·99	·195	4·99	·244	5·99	·293	6·99	·342	7·99	·391	8·99	·440	9·99	·489
51	·999	·050	2·00	·099	3·00	·149	3·99	·199	4·99	·249	5·99	·298	6·99	·348	7·99	·398	8·99	·447	9·99	·497
54	·999	·051	2·00	·101	3·00	·152	3·99	·202	4·99	·253	5·99	·304	6·99	·354	7·99	·405	8·99	·455	9·99	·506
57	·999	·051	2·00	·103	3·00	·154	3·99	·206	4·99	·257	5·99	·309	6·99	·360	7·99	·412	8·99	·463	9·99	·515
Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.

DISTANCE IN CHAINS, &c.

Deg.	1		2		3		4		5		6		7		8		9		10		Deg.
'	Lat.	Dep	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	D p.	Lat.	Dep.	Lat.	Dep	'
3°	·999	·052	2·00	·105	3·00	·157	3·99	·209	4·99	·262	5·99	·314	6·99	·366	7·99	·419	8·99	·471	9·99	·523	**87°**
3	·999	·053	2·00	·106	3·00	·160	3·99	·213	4·99	·266	5·99	·319	6·99	·372	7·99	·426	8·99	·479	9·99	·532	57
6	·998	·054	2·00	·108	3·00	·162	3·99	·216	4·99	·270	5·99	·324	6·99	·379	7·99	·433	8·99	·487	9·99	·541	54
9	·998	·055	2·00	·110	3·00	·165	3·99	·220	4·99	·275	5·99	·330	6·99	·385	7·99	·440	8·99	·495	9·98	·550	51
12	·998	·056	2·00	·112	3·00	·167	3·99	·223	4·99	·279	5·99	·335	6·99	·391	7·99	·446	8·99	·502	9·98	·558	48
15	·998	·057	2·00	·113	2·99	·170	3·99	·227	4·99	·283	5·99	·340	6·99	·397	7·99	·454	8·99	·510	9·98	·567	45
18	·998	·058	2·00	·115	2·99	·173	3·99	·230	4·99	·288	5·99	·345	6·99	·403	7·99	·460	8·99	·518	9·98	·576	42
21	·998	·058	2·00	·117	2·99	·175	3·99	·234	4·99	·292	5·99	·350	6·99	·409	7·99	·467	8·98	·526	9·98	·584	39
24	·998	·059	2·00	·119	2·99	·178	3·99	·237	4·99	·297	5·99	·356	6·99	·415	7·99	·474	8·98	·534	9·98	·593	36
27	·998	·060	2·00	·120	2·99	·180	3·99	·241	4·99	·301	5·99	·361	6·99	·421	7·99	·481	8·98	·542	9·98	·602	33
30	·998	·061	2·00	·122	2·99	·183	3·99	·244	4·99	·305	5·99	·366	6·99	·427	7·99	·488	8·98	·549	9·98	·610	30
33	·998	·062	2·00	·124	2·99	·186	3·99	·248	4·99	·310	5·99	·371	6·99	·433	7·98	·495	8·98	·557	9·98	·619	27
36	·998	·063	2·00	·126	2·99	·188	3·99	·251	4·99	·314	5·99	·377	6·99	·440	7·98	·502	8·98	·565	9·98	·628	24
39	·998	·064	2·00	·127	2·99	·191	3.99	·255	4·99	·318	5·99	·382	6·99	·446	7·98	·509	8·98	·573	9·98	·636	21
42	·998	·064	2·00	·129	2·99	·194	3.99	·258	4·99	·323	5·99	·387	6·99	·452	7·98	·516	8·98	·581	9·93	·645	18
45	·998	·065	2.00	·131	2.99	·196	3·99	·262	4·99	·327	5·99	·392	6·99	·458	7·98	·523	8·98	·589	9·98	·654	15
48	·998	·066	2·00	·132	2.99	·199	3·99	·265	4·99	·331	5·99	·407	6·99	·464	7·98	·530	8·98	·596	9·98	·663	12
51	·998	·067	2·00	·134	2·99	·201	3·99	·268	4·99	·336	5·99	·403	6·98	·470	7·98	·537	8·98	·604	9·98	·671	9
54	·998	·068	2·00	·136	2·99	·204	3·99	·272	4·99	·340	5·99	·408	6·98	·476	7·98	·544	8·98	·612	9·98	·680	6
57	·998	·069	1·99	·138	2·99	·207	3·99	·276	4·99	·344	5·99	·413	6·98	·482	7·98	·551	8·98	·620	9·98	·689	3
4°	·998	·070	1·99	·139	2·99	·209	3·99	·279	4·99	·349	5·99	·418	6·98	·488	7·98	·558	8·98	·628	9·98	·697	**86°**
3	·998	·071	1·99	·141	2·99	·212	3·99	·282	4·99	·353	5·99	·424	6·98	·494	7·98	·565	8·98	·636	9·98	·706	57
6	·997	·071	1·99	·143	2·99	·214	3·99	·286	4·99	·357	5·99	·429	6·98	·500	7·98	·572	8·98	·643	9·97	·715	54
9	·997	·072	1·99	·145	2·99	·217	3·99	·289	4·99	·362	5·98	·434	6·98	·507	7·98	·579	8·98	·651	9·97	·724	51
12	·997	·073	1·99	·146	2·99	·220	3·99	·293	4·99	·366	5·98	·439	6·98	·513	7·98	·586	8·98	·659	9·97	·732	48
15	·997	·074	1·99	·148	2·99	·222	3·99	·296	4·99	·371	5·98	·445	6·98	·519	7·98	·593	8·98	·667	9·97	·741	45
18	·997	.075	1·99	·150	2·99	·225	3·99	·300	4·99	·375	5·98	·450	6·98	·525	7·98	·600	8·97	·675	9·97	·750	42
21	·997	·076	1·99	·152	2.99	·227	3·99	·303	4·99	·379	5·98	·455	6·98	·531	7·98	·607	8·97	·683	9·97	·758	39
24	·997	·077	1·99	·153	2·99	·230	3·99	·307	4·99	·384	5·98	·460	6·98	·537	7·98	·614	8·97	·690	9·97	·767	36
27	·997	·078	1·99	·155	2·99	·233	3·99	·310	4·98	·388	5·98	·466	6·98	·543	7·98	·621	8·97	·698	9·97	·776	33
30	·997	.078	1·99	·157	2·99	·235	3·99	·314	4·98	·392	5·98	·470	6·98	·549	7·98	·627	8·97	·706	9·97	·784	30
33	·997	·079	1·99	·159	2·99	·238	3·99	·317	4·98	·397	5·98	·476	6·98	·555	7·98	·634	8·97	·714	9·97	·793	27
36	·997	·080	1·99	·160	2·99	·241	3·99	·321	4·98	·401	5·98	·481	6·98	·561	7·97	·642	8·97	·722	9·97	·802	24
39	·997	·081	1·99	·162	2·99	·243	3·99	·324	4·98	·405	5·98	·486	6·98	·567	7·97	·649	8·97	·730	9·97	·811	21
42	·997	·082	1·99	·164	2·99	·246	3·99	·328	4·98	·410	5·98	·491	6·98	·573	7·97	·655	8·97	·737	9·97	·819	18
45	·997	·083	1·99	·166	2·99	·248	3·99	·331	4·98	·414	5·98	·497	6·98	·580	7·97	·662	8·97	·745	9·97	·828	15
48	·996	·084	1·99	·167	2·99	·251	3·99	·335	4·98	·418	5·98	·502	6·98	·586	7·97	·669	8·97	·753	9·96	·837	12
51	·996	·085	1·99	·169	2·99	·254	3·99	·338	4·98	·423	5·98	·507	6·97	·592	7·97	·676	8·97	·761	9·96	·845	9
54	·996	·085	1.99	·171	2·99	·256	3·99	·342	4·98	·427	5·98	·512	6·97	·598	7·97	·683	8·97	·769	9·96	·854	6
57	·996	·086	1·99	·172	2·99	·259	3·99	·345	4·98	·431	5·98	·517	6·97	·604	7·97	·690	8·97	·776	9·96	·863	3
5°	·996	·087	1·99	·174	2·99	·261	3·98	·349	4·98	·436	5·98	·523	6·97	·610	7·97	·697	8·97	·784	9·96	·872	**85°**
3	·996	·088	1·99	·176	2·99	·264	3·98	·352	4·98	·440	5·98	·528	6·97	·616	7·97	·704	8·96	·792	9·96	·880	57
6	·996	·089	1·99	·178	2·99	·267	3·98	·356	4·98	·444	5·98	·533	6·97	·622	7·97	·711	8·96	·800	9·96	·889	54
9	·996	·090	1·99	·179	2·99	·269	3·98	·359	4·98	·449	5·98	·539	6·97	·628	7·97	·718	8·96	·808	9·96	·898	51
12	·996	·091	1·99	·181	2·99	·272	3·98	·362	4·98	·453	5·97	·544	6·97	·634	7·97	·725	8·96	·816	9·96	·906	48
15	·996	·091	1·99	·183	2·99	·274	3·98	·366	4·98	·457	5·97	·549	6·97	·640	7·97	·732	8·96	·823	9·96	·915	45
18	·996	·092	1·99	·185	2·99	·277	3·98	·369	4·98	·462	5·97	·554	6·97	·647	7·97	·739	8·96	·831	9·96	·924	42
21	·996	·093	1·99	·186	2·99	·280	3·98	·373	4·98	·466	5·97	·559	6·97	·653	7·96	·746	8·96	·839	9·96	·932	39
24	·996	·094	1·99	·188	2·99	·282	3·98	·376	4·98	·470	5·97	·565	6·97	·659	7·96	·753	8·96	·847	9·96	·941	36
27	·995	·095	1·99	·190	2·99	·285	3·98	·380	4·98	·475	5·97	·570	6·97	·665	7·96	·760	8·96	·855	9·95	·950	33
30	·995	·096	1·99	·192	2·99	·287	3·98	·383	4·98	·479	5·97	·575	6·97	·671	7·96	·767	8·96	·862	9·95	·958	30
33	·995	·097	1·99	·193	2·99	·290	3·98	·387	4·98	·483	5·97	·580	6·97	·677	7·96	·774	8·96	·870	9·95	·967	27
36	·695	·098	1·99	·195	2·99	·293	3·98	·390	4·98	·488	5·97	·585	6·97	·683	7·96	·781	8·96	·878	9·95	·976	24
39	·995	·098	1·99	·197	2·99	·295	3·98	·393	4·97	·492	5·97	·591	6·97	·689	7·96	·788	8·96	·886	9·95	·984	21
42	·995	·099	1·99	·199	2·99	·298	3·98	·397	4·97	·497	5·97	·596	6·96	·695	7·96	·794	8·95	·894	9·95	·993	18
45	·995	·100	1·99	·200	2·98	·300	3·98	·401	4·97	·501	5·97	·601	6·96	·701	7·96	·801	8·95	·902	9·95	1·00	15
48	·995	·101	1·99	·202	2·98	·303	3·98	·404	4·97	·505	5·97	·606	6·96	·707	7·96	·808	8·95	·909	9·95	1·01	12
51	·995	·102	1·99	·204	2·98	·306	3·98	·408	4·97	·510	5·97	·612	6·96	·713	7·96	·815	8·95	·917	9·95	1·02	9
54	·895	·103	1·99	·205	2·98	·308	3·98	·411	4·97	·513	5·97	·616	6·96	·719	7·96	·822	8·95	·924	9·95	1·03	6
57	·995	·104	1·99	·207	2·98	·311	3·98	·415	4·97	·518	5·97	·622	6·96	·726	7·96	·830	8·95	·933	9·95	1·04	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.

DISTANCE IN CHAINS, &c.

Deg.	1		2		3		4		5		6		7		8		9		10		Deg.
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	La·.	Dep.	Lat.	Dep.	Lat.	Dep.	′
6°	·995	·104	1·99	·209	2·98	·313	3·98	·418	4·97	·523	5·97	·627	6·96	·732	7·96	·836	8·95	·941	9·95	1·04	84°
3	·994	·105	1·99	·211	2·98	·316	3·98	·421	4·97	·527	5·97	·632	6·96	·738	7·96	·843	8·95	·948	9·94	1·05	57
6	·994	·106	1·99	·213	2·98	·319	3·98	·425	4·97	·531	5·97	·638	6·96	·744	7·95	·850	8·95	·956	9·94	1·06	54
9	·994	·107	1·99	·214	2·98	·321	3·98	·428	4·97	·536	5·97	·643	6·96	·750	7·95	·857	8·95	·964	9·94	1·07	51
12	·994	·108	1·99	·216	2·98	·324	3·98	·432	4·97	·540	5·96	·648	6·96	·756	7·95	·864	8·95	·972	9·94	1·08	48
15	·994	·109	1·99	·218	2·98	·327	3·98	·435	4·97	·544	5·96	·653	6·96	·762	7·95	·871	8·95	·980	9·94	1·09	45
18	·994	·110	1·99	·219	2·98	·329	3·98	·439	4·97	·549	5·96	·658	6·96	·768	7·95	·878	8·95	·987	9·94	1·10	42
21	·994	·111	1·99	·221	2·98	·332	3·98	·442	4·97	·553	5·96	·664	6·96	·774	7·95	·885	8·94	·995	9·94	1·11	39
24	·994	·111	1·99	·223	2·98	·334	3·98	·446	4·97	·557	5·96	·669	6·96	·780	7·95	·892	8·94	1·00	9·94	1·11	36
27	·994	·112	1·99	·224	2·98	·337	3·97	·449	4·97	·562	5·96	·674	6·96	·786	7·95	·899	8·94	1·01	9·94	1·12	33
30	·994	·113	1·99	·226	2·98	·340	3·97	·453	4·97	·566	5·96	·679	6·95	·792	7·95	·906	8·94	1·02	9·94	1·13	30
33	·993	·114	1·99	·228	2·98	·342	3·97	·456	4·97	·570	5·96	·684	6·95	·798	7·95	·913	8·94	1·03	9·93	1·14	27
36	·993	·115	1·99	·230	2·98	·345	3·97	·460	4·97	·575	5·96	·690	6·95	·804	7·95	·919	8·94	1·03	9·93	1·15	24
39	·993	·116	1·99	·232	2·98	·347	3·97	·463	4·97	·579	5·96	·695	6·95	·811	7·95	·926	8·94	1·04	9·93	1·16	21
42	·993	·117	1·99	·233	2·98	·350	3·97	·467	4·97	·583	5·96	·700	6·95	·817	7·95	·933	8·94	1·05	9·93	1·17	18
45	·993	·117	1·99	·235	2·98	·353	3·97	·470	4·96	·588	5·96	·705	6·95	·823	7·94	·940	8·94	1·06	9·93	1·17	15
48	·993	·118	1·99	·237	2·98	·355	3·97	·474	4·96	·592	5·96	·710	6·95	·829	7·94	·947	8·94	1·07	9·93	1·18	12
51	·993	·119	1·99	·238	2·98	·358	3·97	·477	4·96	·596	5·96	·716	6·95	·835	7·94	·954	8·94	1·07	9·93	1·19	9
54	·993	·120	1·99	·240	2·98	·360	3·97	·480	4·96	·600	5·96	·721	6·95	·841	7·94	·961	8·93	1·08	9·93	1·20	6
57	·993	·121	1·99	·242	2·98	·363	3·97	·484	4·96	·605	5·96	·726	6·95	·847	7·94	·968	8·93	1·09	9·93	1·21	3
7°	·993	·122	1·99	·244	2·98	·366	3·97	·487	4·96	·609	5·96	·731	6·95	·853	7·94	·975	8·93	1·10	9·93	1·22	83°
3	·992	·123	1·98	·245	2·98	·368	3·97	·491	4·96	·614	5·95	·736	6·95	·859	7·94	·981	8·93	1·10	9·92	1·23	57
6	·992	·124	1·98	·247	2·98	·371	3·97	·494	4·96	·618	5·95	·741	6·95	·865	7·94	·989	8·93	1·11	9·92	1·24	54
9	·992	·124	1·98	·249	2·98	·373	3·97	·498	4·96	·622	5·95	·747	6·95	·871	7·94	·996	8·93	1·12	9·92	1·24	51
12	·992	·125	1·98	·251	2·98	·376	3·97	·501	4·96	·627	5·95	·752	6·94	·877	7·94	1·00	8·93	1·13	9·92	1·25	48
15	·992	·126	1·98	·252	2·98	·379	3·97	·505	4·96	·631	5·95	·757	6·94	·883	7·94	1·01	8·93	1·13	9·92	1·26	45
18	·992	·127	1·98	·254	2·98	·381	3·97	·508	4·96	·635	5·95	·762	6·94	·889	7·94	1·02	8·93	1·14	9·92	1·27	42
21	·992	·128	1·98	·256	2·98	·384	3·97	·512	4·96	·640	5·95	·768	6·94	·896	7·93	1·02	8·93	1·15	9·92	1·28	39
24	·992	·129	1·98	·258	2·98	·386	3·97	·515	4·96	·644	5·95	·773	6·94	·902	7·93	1·03	8·93	1·16	9·92	1·29	36
27	·992	·130	1·98	·259	2·97	·389	3·97	·519	4·96	·648	5·95	·778	6·94	·908	7·93	1·04	8·92	1·17	9·92	1·30	33
30	·991	·130	1·98	·261	2·97	·391	3·97	·522	4·96	·653	5·95	·783	6·94	·914	7·93	1·04	8·92	1·17	9·91	1·30	30
33	·991	·131	1·98	·263	2·97	·394	3·97	·525	4·96	·657	5·95	·788	6·94	·920	7·93	1·05	8·92	1·18	9·91	·31	27
36	·991	·132	1·98	·264	2·97	·397	3·96	·529	4·96	·661	5·95	·793	6·94	·926	7·93	1·06	8·92	1·19	9·91	1·32	24
39	·991	·133	1·98	·266	2·97	·399	3·96	·532	4·96	·666	5·95	·799	6·94	·932	7·93	1·06	8·92	1·20	9·91	1·33	21
42	·991	·134	1·98	·268	2·97	·402	3·96	·536	4·95	·670	5·95	·804	6·94	·938	7·93	1·07	8·92	1·21	9·91	1·34	18
45	·991	·135	1·98	·270	2·97	·405	3·96	·539	4·95	·674	5·95	·809	6·94	·944	7·93	1·08	8·92	1·21	9·91	1·35	15
48	·991	·136	1·98	·271	2·97	·407	3·96	·543	4·95	·678	5·94	·814	6·93	·250	7·93	1·09	8·92	1·22	9·91	1·36	12
51	·991	·137	1·98	·273	2·97	·410	3·96	·546	4·95	·683	5·94	·819	6·93	·956	7·92	1·09	8·92	1·23	9·91	1·37	9
54	·991	·137	1.98	·275	2·97	·412	3·96	·550	4·95	·687	5·94	·825	6·93	·962	7·92	1·10	8·91	1·24	9·90	1·37	6
57	·990	·138	1·98	·276	2·97	·415	3·96	·553	4·95	·691	5·94	·830	6·93	·968	7·92	1·11	8·91	1·24	9·90	1·38	3
8°	·990	·139	1·98	·278	2·97	·417	3·96	·557	4·95	·696	5·94	·835	6·93	·974	7·92	1·11	8·91	1·25	9·90	1·39	82°
3	·990	·140	1·98	·280	2·97	·420	3·96	·560	4·95	·700	5·94	·840	6·93	·980	7·92	1·12	8·91	1·26	9·90	1·40	57
6	·990	·141	1·98	·282	2·97	·423	3·96	·564	4·95	·705	5·94	·845	6·93	·986	7·92	1·13	8·91	1·27	9·90	1·41	54
9	·990	·142	1·98	·284	2·97	·425	3·96	·567	4·95	·709	5·94	·851	6·93	·992	7·92	1·13	8·91	1·28	9·90	1·42	51
12	·990	·143	1·98	·285	2·97	·428	3·96	·570	4·95	·713	5·94	·856	6·93	·998	7·92	1·14	8·91	1·28	9·90	1·43	48
15	·990	·143	1·98	·287	2·97	·430	3·96	·574	4·95	·717	5·94	·861	6·93	1·00	7·92	1·15	8·91	1·29	9·90	1·43	45
18	·989	·144	1·98	·289	2·97	·433	3·96	·577	4·95	·722	5·94	·866	6·93	1·01	7·92	1·15	8·91	1·30	9·90	1·44	42
21	·989	·145	1·98	·290	2·97	·436	3·96	·581	4·95	·726	5·94	·871	6·93	1·02	7·91	1·16	8·90	1·31	9·89	1·45	39
24	·989	·146	1·98	·292	2·97	·438	3·96	·584	4·95	·730	5·94	·876	6·92	1·02	7·91	1·17	8·90	1·31	9·89	1·46	36
27	·989	·147	1·98	·294	2·97	·441	3·96	·588	4·94	·735	5·93	·882	6·92	1·03	7·91	1·18	8·90	1·32	9·89	1·47	33
30	·989	·148	1·98	·296	2·97	·443	3·96	·591	4·94	·739	5·93	·887	6·92	1·03	7·91	1·18	8·90	1·33	9·89	1·48	30
33	·989	·149	1·98	·298	2·97	·446	3·95	·595	4·94	·743	5·93	·892	6·92	1·04	7·91	1·19	8·90	1·34	9·89	1·49	27
36	·989	·149	1·98	·299	2·97	·449	3·95	·598	4·94	·748	5·93	·897	6·92	1·05	7·91	1·20	8·90	1·34	9·89	1·49	24
39	·989	·150	1·98	·301	2·96	·451	3·95	·601	4·94	·752	5·93	·902	6·92	1·05	7·91	1·20	8·90	1·35	9·89	1·50	21
42	·988	·151	1·98	·302	2·96	·454	3·95	·605	4·94	·756	5·93	·908	6·92	1·06	7·91	1·21	8·90	1·36	9·88	·51	18
45	·988	·152	1·98	·304	2·96	·456	3·95	·608	4·94	·761	5·93	·913	6·92	1·06	7·91	1·22	8·89	1·37	9·88	1·52	15
48	·988	·153	1·98	·306	2·96	·459	3·95	·612	4·94	·765	5·93	·918	6·92	1·07	7·91	1·22	8·89	1·38	9·88	1·53	12
51	·988	·154	1·98	·308	2·96	·461	3·95	·615	4·94	·769	5·93	·923	6·92	1·08	7·90	1·23	8·89	1·38	9·88	1·54	9
54	·988	·155	1·98	·309	2·96	·464	3·95	·619	4·94	·773	5·93	·928	6·92	1·08	7·90	1·24	8·89	1·39	9·88	1·55	6
57	·988	·156	1·97	·311	2·96	·467	3·95	·622	4·94	·778	5·93	·933	6·91	1·09	7·90	1·24	8·89	1·40	9·88	1·56	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Deg

DISTANCE IN CHAINS, &c.

Deg.	1		2		3		4		5		6		7		8		9		10		
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
9°	·988	·156	1·97	·313	2·96	·469	3·95	·626	4·94	·782	5·93	·939	6·91	1·09	7·90	1·25	8·89	1·41	9·88	1·56	**81°**
3	·988	·157	1·97	·314	2·96	·472	3·95	·629	4·94	·786	5·93	·944	6·91	1·10	7·90	1·26	8·89	1·41	9·88	1·57	57
6	·987	·158	1·97	·316	2·96	·474	3·95	·633	4·94	·791	5·92	·949	6·91	1·11	7·90	1·26	8·89	1·42	9·87	1·58	54
9	·987	·159	1·97	·318	2·96	·477	3·95	·636	4·94	·795	5·92	·954	6·91	1·11	7·90	1·27	8·89	1·43	9·87	1·59	51
12	·987	·160	1·97	·320	2·96	·480	3·95	·640	4·94	·800	5·92	·960	6·91	1·12	7·90	1·28	8·88	1·44	9·87	1·60	48
15	·987	·161	1·97	·321	2·96	·482	3·95	·643	4·93	·804	5·92	·964	6·91	1·12	7·90	1·28	8·88	1·45	9·87	1·61	45
18	·987	·162	1·97	·323	2·96	·485	3·95	·646	4·93	·808	5·92	·970	6·91	1·13	7·89	1·29	8·88	1·45	9·87	1·62	42
21	·987	·162	1·97	·325	2·96	·487	3·95	·650	4·93	·812	5·92	·975	6·91	1·14	7·89	1·30	8·88	1·46	9·87	1·62	39
24	·987	·163	1·97	·327	2·96	·490	3·95	·653	4·93	·817	5·92	·980	6·91	1·14	7·89	1·31	8·88	1·47	9·87	1·63	36
27	·986	·164	1·97	·328	2·96	·492	3·95	·657	4·93	·821	5·92	·985	6·91	1·15	7·89	1·31	8·88	1·48	9·86	1·64	33
30	·986	·165	1·97	·329	2·96	·495	3·94	·660	4·93	·825	5·92	·990	6·90	1·15	7·89	1·32	8·88	1·48	9·86	1·65	30
33	·986	·166	1·97	·332	2·96	·498	3·94	·664	4·93	·830	5·92	·995	6·90	1·16	7·89	1·33	8·87	1·49	9·86	1·66	27
36	·986	·167	1·97	·334	2·96	·500	3·94	·667	4·93	·834	5·92	1·00	6·90	1·17	7·89	1·33	8·87	1·50	9·86	1·67	24
39	·986	·168	1·97	·335	2·96	·503	3·94	·670	4·93	·838	5·91	1·01	6·90	1·17	7·89	1·34	8·87	1·51	9·86	1·68	21
42	·986	·168	1·97	·837	2·96	·505	3·94	·674	4·93	·842	5·91	1·01	6·90	1·18	7·89	1·35	8·87	1·52	9·86	1·68	18
45	·986	·169	1·97	·339	2·96	·508	3·94	·677	4·93	·847	5·91	1·02	6·90	1·18	7·88	1·35	8·87	1·52	9·86	1·69	15
48	·985	·170	1·97	·340	2·96	·511	3·94	·681	4·93	·851	5·91	1·02	6·90	1·19	7·88	1·36	8·87	1·53	9·85	1·70	12
51	·985	·171	1·97	·342	2·96	·513	3·94	·684	4·93	·855	5·91	1·03	6·90	1·20	7·88	1·37	8·87	1·54	9·85	1·71	9
54	·995	·172	1·97	·344	2·95	·516	3·94	·688	4·92	·860	5·91	1·03	6·90	1·20	7·88	1·37	8·87	1·55	9·85	1·72	6
57	·985	·173	1·97	·346	2·95	·518	3·94	·691	4·92	·864	5·91	1·04	6·89	1·21	7·88	1·38	8·86	1·55	9·85	1·73	3
10°	·985	·174	1·97	·347	2·95	·521	3·94	·695	4·92	·868	5·91	1·04	6·89	1·21	7·88	1·39	8·86	1·56	9·85	1·74	**80°**
3	·985	·174	1·97	·349	2·95	·523	3·94	·698	4·92	·872	5·91	1·05	6·89	1·22	7·88	1·40	8·86	1·57	9·85	1·74	57
6	·985	·175	1·97	·351	2·95	·526	3·94	·701	4·92	·877	5·91	1·05	6·89	1·23	7·88	1·40	8·86	1·58	9·85	1·75	54
9	·984	·176	1·97	·352	2·95	·529	3·94	·705	4·92	·881	5·91	1·06	6·89	1·23	7·87	1·41	8·86	1·59	9·84	1·76	51
12	·984	·177	1·97	·354	2·95	·531	3·94	·708	4·92	·885	5·90	1·06	6·89	1·24	7·87	1·42	8·86	1·59	9·84	1·77	48
15	·984	·178	1·97	·356	2·95	·534	3·94	·712	4·92	·890	5·90	1·07	6·89	1·25	7·87	1·42	8·86	1·60	9·84	1·78	45
18	·984	·179	1·97	·358	2·95	·536	3·94	·715	4·92	·894	5·90	1·07	6·89	1·25	7·87	1·43	8·85	1·61	9·84	1·79	42
21	·984	·180	1·97	·359	2·95	·539	3·93	·719	4·92	·898	5·90	1·08	6·89	1·26	7·87	1·44	8·85	1·62	9·84	1·80	39
24	·984	·180	1·97	·361	2·95	·541	3·93	·722	4·92	·902	5·90	1·08	6·88	1·26	7·87	1·44	8·85	1·62	9·84	1·80	36
27	·983	·181	1·97	·363	2·95	·544	3·93	·725	4·92	·907	5·90	1·09	6·88	1·27	7·87	1·45	8·85	1·62	9·83	1·81	33
30	·983	·182	1·97	·364	2·95	·547	3·93	·729	4·92	·911	5·90	1·09	6·88	1·27	7·87	1·46	8·85	1·64	9·83	1·82	30
33	·983	·183	1·97	·366	2·95	·549	3·93	·732	4·91	·915	5·90	1·10	6·88	1·28	7·86	1·46	8·85	1·65	9·83	1·83	27
36	·983	·184	1·97	·368	2·95	·552	3·93	·736	4·91	·920	5·90	1·10	6·88	1·29	7·86	1·47	8·85	1·65	9·83	1·84	24
39	·983	·185	1·97	·370	2·95	·554	3·93	·739	4·91	·924	5·90	1·11	6·88	1·29	7·86	1·48	8·84	1·66	9·83	1·85	21
42	·983	·186	1·96	·371	2·95	·557	3·93	·743	4·91	·928	5·90	1·11	6·88	1·30	7·86	1·48	8·84	1·67	9·83	1·86	18
45	·982	·186	1·96	·373	2·95	·559	3·93	·746	4·91	·932	5·89	1·12	6·88	1·30	7·86	1·49	8·84	1·68	9·82	1·86	15
48	·982	·187	1·96	·375	2·95	·562	3·93	·749	4·91	·937	5·89	1·12	6·88	1·31	7·86	1·50	8·84	1·69	9·82	1·87	12
51	·982	·188	1·96	·376	2·95	·565	3·93	·753	4·91	·941	5·89	1·13	6·87	1·32	7·86	1·51	8·84	1·69	9·82	1·88	9
54	·982	·189	1·96	·378	2·95	·567	3·93	·756	4·91	·945	5·89	1·13	6·87	1·32	7·86	1·51	8·84	1·70	9·82	1·89	6
57	·982	·190	1·96	·380	2·94	·570	3·93	·760	4·91	·950	5·89	1·14	6·87	1·33	7·85	1·52	8·84	1·71	9·82	1·90	3
11°	·982	·191	1·96	·382	2·94	·572	3·93	·763	4·91	·954	5·89	1·14	6·87	1·34	7·85	1·53	8·83	1·72	9·82	1·91	**79°**
3	·982	·192	1·96	·383	2·94	·575	3·93	·767	4·91	·958	5·89	1·15	6·87	1·34	7·85	1·53	8·83	1·72	9·81	1·92	57
6	·981	·192	1·96	·385	2·94	·577	3·93	·770	4·91	·963	5·89	1·15	6·87	1·35	7·85	1·54	8·83	1·73	9·81	1·92	54
9	·981	·193	1·96	·387	2·94	·580	3·92	·773	4·91	·967	5·89	1·16	6·87	1·35	7·85	1·55	8·83	1·74	9·81	1·93	51
12	·981	·194	1·96	·388	2·94	·583	3·92	·777	4·90	·971	5·89	1·16	6·87	1·36	7·85	1·55	8·83	1·75	9·81	1·94	48
15	·981	·195	1·96	·390	2·94	·585	3·92	·780	4·90	·975	5·88	1·17	6·87	1·36	7·85	1·56	8·83	1·76	9·81	1·95	45
18	·981	·196	1·96	·392	2·94	·588	3·92	·784	4·90	·980	5·88	1·18	6·86	1·37	7·84	1·57	8·83	1·76	9·81	1·96	42
21	·980	·197	1·96	·394	2·94	·590	3·92	·787	4·90	·984	5·88	1·18	6·86	1·38	7·84	1·57	8·82	1·77	9·80	1·97	39
24	·980	·198	1·96	·395	2·94	·593	3·92	·791	4·90	·988	5·88	1·19	6·86	1·38	7·84	1·58	8·82	1·78	9·80	1·98	36
27	·980	·198	1·96	·397	2·94	·595	3·92	·794	4·90	·993	5·88	1·19	6·86	1·39	7·84	1·59	8·82	1·79	9·80	1·98	33
30	·980	·199	1·96	·399	2·94	·598	3·92	·797	4·90	·997	5·88	1·20	6·86	1·39	7·84	1·59	8·82	1·79	9·80	1·99	30
33	·980	·200	1·96	·400	2·94	·601	3·92	·801	4·90	1·00	5·88	1·20	6·86	1·40	7·84	1·60	8·82	1·80	9·80	2·00	27
36	·980	·201	1·96	·402	2·94	·603	3·92	·804	4·90	1·00	5·88	1·21	6·86	1·41	7·84	1·61	8·82	1·81	9·80	2·01	24
39	·979	·202	1·96	·404	2·94	·606	3·92	·808	4·90	1·01	5·88	1·21	6·86	1·41	7·84	1·61	8·81	1·82	9·79	2·02	21
42	·979	·203	1·96	·406	2·94	·608	3·92	·811	4·90	1·01	5·88	1·22	6·85	1·42	7·83	1·62	8·81	1·82	9·79	2·03	18
45	·979	·204	1·96	·407	2·94	·611	3·92	·815	4·90	1·02	5·87	1·22	6·85	1·43	7·83	1·63	8·81	1·83	9·79	2·04	15
48	·979	·204	1·96	·409	2·94	·613	3·91	·818	4·89	1·02	5·87	1·23	6·85	1·43	7·83	1·64	8·81	1·84	9·79	2·04	12
51	·979	·205	1·96	·411	2·94	·616	3·91	·821	4·89	1·03	5·87	1·23	6·85	1·44	7·83	1·64	8·81	1·85	9·79	2·05	9
54	·979	·206	1·96	·412	2·93	·619	3·91	·825	4·89	1·03	5·87	1·24	6·85	1·44	7·83	1·65	8·81	1·86	9·79	2·06	6
57	·978	·207	1·96	·414	2·93	·621	3·91	·828	4·89	1·03	5·87	1·24	6·85	1·45	7·83	1·66	8·80	1·86	9·78	2·07	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.

	DISTANCE IN CHAINS, &c.																				
Deg.	1		2		3		4		5		6		7		8		9		10		Deg.
′	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	′
12°	·978	·208	1·96	·416	2·93	·624	3·91	·832	4·89	1·04	5·87	1·25	6·85	1·46	7·82	1·66	8·80	1·87	9·78	2·08	78°
3	·978	·209	1·96	·418	2·93	·626	3·91	·835	4·89	1·04	5·87	1·25	6·85	1·46	7·82	1·67	8·80	1·88	9·78	2·09	57
6	·978	·210	1·96	·419	2·93	·629	3·91	·838	4·89	1·05	5·87	1·26	6·84	1·47	7·82	1·68	8·80	1·89	9·78	2·10	54
9	·978	·210	1·96	·421	2·93	·632	3·91	·842	4·89	1·05	5·87	1·26	6·84	1·47	7·82	1·68	8·80	1·90	9·78	2·10	51
12	·977	·211	1·95	·423	2·93	·634	3·91	·845	4·89	1·06	5·86	1·27	6·84	1·48	7·82	1·69	8·80	1·90	9·77	2·11	48
15	·977	·212	1·95	·424	2·93	·637	3·91	·849	4·89	1·06	5·86	1·27	6·84	1·49	7·82	1·70	8·80	1·91	9·77	2·12	45
18	·977	·213	1·95	·426	2·93	·639	3·91	·852	4·89	1·07	5·86	1·28	6·84	1·49	7·82	1·70	8·79	1·92	9·77	2·13	42
21	·977	·214	1·95	·428	2·93	·642	3·91	·856	4·88	1·07	5·86	1·28	6·84	1·50	7·81	1·71	8·79	1·92	9·77	2·14	39
24	·977	·215	1·95	·429	2·93	·644	3·91	·859	4·88	1·07	5·86	1·29	6·84	1·50	7·81	1·72	8·79	1·93	9·77	2·15	36
27	·977	·216	1·95	·431	2·93	·647	3·91	·862	4·88	1·08	5·86	1·29	6·84	1·51	7·81	1·72	8·79	1·94	9·76	2·16	33
30	·976	·216	1·95	·433	2·93	·649	3·91	·866	4·88	1·08	5·86	1·30	6·83	1·52	7·81	1·73	8·79	1·95	9·76	2·16	30
33	·976	·217	1·95	·435	2·93	·651	3·90	·869	4·88	1·09	5·86	1·30	6·83	1·52	7·81	1·74	8·78	1·96	9·76	2·17	27
36	·976	·218	1·95	·436	2·93	·654	3·90	·873	4·88	1·09	5·86	1·31	6·83	1·53	7·81	1·75	8·78	1·96	9·76	2·18	24
39	·976	·219	1·95	·438	2·93	·657	3·90	·876	4·88	1·09	5·85	1·31	6·83	1·53	7·81	1·75	8·78	1·97	9·76	2·19	21
42	·976	·220	1·95	·440	2·93	·660	3·90	·879	4·88	1·10	5·85	1·32	6·83	1·54	7·80	1·76	8·78	1·98	9·76	2·20	18
45	·975	·221	1·95	·441	2·93	·662	3·90	·883	4·88	1·10	5·85	1·32	6·83	1·54	7·80	1·77	8·78	1·99	9·75	2·21	15
48	·975	·222	1·95	·443	2·93	·665	3·90	·886	4·88	1·11	5·85	1·33	6·83	1·55	7·80	1·77	8·78	1·99	9·75	2·22	12
51	·975	·222	1·95	·446	2·92	·667	3·90	·890	4·87	1·11	5·85	1·33	6·82	1·56	7·80	1·78	8·77	2·00	9·75	2·22	9
54	·975	·223	1·95	·447	2·92	·670	3·90	·893	4·87	1·12	5·85	1·34	6·82	1·56	7·80	1·79	8·77	2·01	9·75	2·23	6
57	·975	·224	1·95	·448	2·92	·672	3·90	·896	4·87	1·12	5·85	1·34	6·82	1·57	7·80	1·79	8·77	2·02	9·75	2·24	3
13°	·974	·225	1·95	·450	2·92	·675	3·90	·900	4·87	1·12	5·85	1·35	6·82	1·57	7·79	1·80	8·77	2·02	9·74	2·25	77°
3	·974	·226	1·95	·452	2·92	·677	3·90	·903	4·87	1·13	5·84	1·35	6·82	1·58	7·79	1·81	8·77	2·03	9·74	2·26	57
6	·974	·227	1·95	·453	2·92	·680	3·90	907	4·87	1·13	5·84	1·36	6·82	1·59	7·79	1·81	8·76	2·04	9·74	2·27	54
9	·974	·227	1·95	·455	2·92	·682	3·90	·910	4·87	1·14	5·84	1·36	6·82	1·59	7·79	1·82	8·76	2·05	9·74	2·27	51
12	·974	·228	1·95	·457	2·92	·685	3·89	·913	4·87	1·14	5·84	1·37	6·81	1·60	7·79	1·83	8·76	2·06	9·74	2·28	48
15	·973	·229	1·95	·458	9·92	·688	3·89	·917	4·87	1·15	5·84	1·38	6·81	1·60	7·78	1·83	8·76	2·06	9·73	2·29	45
18	·973	·230	1·95	·460	9·92	·690	3·89	·920	4·87	1·15	5·84	1·38	6·81	1·61	7·78	1·84	8·76	2·07	9·73	2·30	42
21	.973	·231	1·95	·462	2·92	·693	3·89	·924	4·86	1·15	5·84	1·38	6·81	1·62	7·78	1·85	8·76	2·08	9·73	2·31	39
24	·973	·232	1·95	·463	2·92	·695	3·89	·927	4·86	1.16	5·84	1·38	6·81	1·62	7·78	1·85	8·75	2.09	9·73	2·32	36
27	·973	·233	1·94	·465	2·92	·698	3·89	·930	4·86	1·16	5·83	1·40	6·81	1·63	7·78	1·86	8·75	2·09	9·73	2·33	33
30	·972	·233	1·94	·467	2·92	·700	3·89	·934	4·86	1·17	5·83	1·40	6·81	1·63	7·78	1·87	8·75	2·10	9·72	2·33	30
33	·972	·234	1·94	·468	2·92	·703	3·89	·937	4·86	1·17	5·83	1·41	6·81	1·64	7·78	1·87	8·75	2·11	9·72	2·34	27
36	·972	·235	1·94	·470	2·92	·705	4·89	·940	4·86	1·18	5·83	1·41	6·80	1·65	7·78	1·88	8·75	2·12	9·72	2·35	24
39	·972	·236	1·94	·472	2·91	·708	3·89	·944	4·86	1.18	5·83	1·42	6·80	1·65	7·77	1·89	8·75	2·12	9·72	2·36	21
42	·972	·237	1·94	·474	2·91	·710	3·89	·947	4·86	1·18	5·83	1·42	6·80	1·66	7·77	1·89	8·74	2·13	9·72	2·37	18
45	·971	·238	1·94	·475	2·91	·713	3·89	·951	4·86	1·19	5·83	1·43	6·80	1·66	7·77	1·90	8·74	2·14	9·71	2·38	15
48	·971	·239	1·94	·478	2·91	·716	3·88	·954	4·86	1·19	5·83	1·43	6·80	1·67	7·77	1·91	8·74	2·15	9·71	2·39	12
51	·971	·239	1·94	·479	2·91	·718	3·88	·957	4·85	1·20	5·82	1·44	6·80	1·68	7·77	1·91	8·74	2·15	9·71	2·39	9
54	·971	·240	1·94	·480	2·91	·721	3·88	·961	4·85	1·20	5·82	1·44	6·79	1·68	7·77	1·92	8·74	2·16	9·71	2·40	6
57	·971	·241	1·94	·482	2·91	·723	3·88	·964	4·85	1·21	5·82	1·45	6·79	1·69	7·76	1·93	8·73	2·17	9·70	2·41	3
14°	·970	·242	1·94	·484	2·91	·726	3·88	·968	4·85	1·21	5·82	1·45	6·79	1·69	7·76	1·93	8·73	2·18	9·70	2·42	76°
3	·970	·243	1·94	·485	2·91	·728	3·88	·971	4·85	1·21	5·82	1·46	6·79	1·70	7·76	1·94	8·73	2·18	9·70	2·43	57
6	·970	·243	1·94	·487	2·91	·730	3·88	·974	4·85	1·22	5·82	1·46	6·79	1·70	7·76	1·95	8·73	2·19	9·70	2·44	54
9	·970	·244	1·94	·489	2·91	·733	3·88	·978	4·85	1·22	5·82	1·47	6·79	1·71	7·76	1·96	8·73	2·20	9·70	2·44	51
12	·970	·245	1·94	·491	2·91	·736	3·88	·981	4·85	1·23	4·82	1·47	6·79	1·72	7·76	1·96	8·73	2·21	9·69	2·45	48
15	·969	·246	1·94	·492	2·91	·738	3·88	·985	4·85	1·23	5·82	1·48	6·78	1·72	7·75	1·97	8·72	2·22	9·69	2·46	45
18	·969	·247	1·94	·494	2·91	·741	3·88	·988	4·85	1·23	5·81	1·48	6·78	1·73	7·75	1·98	8·72	2·22	9·69	2·47	42
21	·969	·248	1·94	·496	2·91	·743	3·87	·991	4·84	1·24	5·81	1·49	6·78	1·73	7·75	1·98	8·72	2·23	9·69	2·48	39
24	·969	·249	1·94	·497	2·91	·746	3·87	·995	4·84	1·24	5·81	1·49	6·78	1·74	7·75	1·99	8·72	2·24	0·69	2·49	36
27	·968	·250	1·94	·499	2·90	·749	3·87	·998	4·84	1·25	5·81	1·50	6·78	1·75	7·75	2·00	8·72	2·25	9·68	2·50	33
30	·968	·250	1·94	·501	2·90	·751	3·87	1·00	4·84	1·25	5·81	1·50	6·78	1·75	7·75	2·00	8·71	2·25	8·68	2·50	30
33	·968	·251	1·94	·502	2·90	·754	3·87	1·00	4·84	1·26	5·81	1·51	6·78	1·76	7·74	2·01	8·71	2·26	9·68	2·51	27
36	·968	·252	1·94	·504	2·90	·756	3·87	1·01	4·84	1·26	5·81	1·51	6·77	1·76	7·74	2·02	8·71	2·27	9·68	2·52	24
39	·967	·253	1·94	·506	2·90	·759	3·87	1·01	4·84	1·26	5·80	1·52	6·77	1·77	7·74	2·02	8·71	2·28	9·67	2·53	21
42	·967	·254	1·93	·507	2·90	·761	3·87	1·01	4·84	1·27	5·80	1·52	6·77	1·78	7·74	2·03	8·71	2·28	9·67	2·54	18
45	·967	·255	1·93	·509	2·90	·764	3·87	1·02	4·84	1·27	5·80	1·53	6·77	1·78	7·74	2·04	8·70	2·29	9·67	2·55	15
48	·967	·255	1·93	·511	2·90	·766	3·87	1·02	4·83	1·28	5·80	1·53	6·77	1·79	7·73	2·04	8·70	2·30	9·67	2·55	12
51	·967	·256	1·93	·513	2·90	·769	3·87	·103	4·83	1·28	5·80	1·54	6·77	1·79	7·73	2·05	8·70	2·31	9·67	2·56	9
54	·966	·257	1·93	·514	2·90	·771	3·87	·103	4·83	1·29	5·80	1·54	6·76	1·80	7·73	2·06	8·70	2·31	9·67	2·57	6
57	·966	·258	1·93	·516	2·90	·774	3·86	·103	4·83	1·29	5·80	1·55	6·76	1·81	7·73	2·06	8·70	2·32	9·66	2·58	3
Deg.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.

DISTANCE IN CHAINS, &c.

Deg	1		2		3		4		5		6		7		8		9		10		Deg.
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	′
15°	·966	·259	1·93	·518	2·90	·776	3·86	1·04	4·83	1·29	5·80	1·55	6·76	1·81	7·73	2·07	8·69	2·33	9·66	2·59	**75°**
3	·966	·260	1·93	·519	2·90	·779	3·86	1·04	4·83	1·30	5·79	1·56	6·76	1·82	7·73	2·08	8·69	2·34	9·66	2·60	57
6	·965	·260	1·93	·521	2·90	·781	3·86	1·04	4·83	1·30	5·79	1·56	6·76	1·82	7·72	2·08	8·69	2·34	9·65	2·60	54
9	·965	·261	1·93	·523	2·90	·784	3·86	1·05	4·83	1·31	5·79	1·57	6·76	1·83	7·72	2·09	8·69	2·35	9·65	2·61	51
12	·965	·262	1·93	·524	2·89	·787	3·86	1·05	4·82	1·31	5·79	1·57	6·75	1·83	7·72	2·10	8·68	2·36	9·65	2·62	48
15	·965	·263	1·93	·526	2·89	·789	3·86	1·05	4·82	1·32	5·79	1·58	6·75	1·84	7·72	2·10	8·68	2·37	9·65	2·63	45
18	·965	·264	1·93	·528	2·89	·792	3·86	1·06	4·82	1·32	5·79	1·58	6·75	1·85	7·72	2·12	8·68	2·37	9·65	2·64	42
21	·964	·265	1·93	·529	2·89	·794	3·86	1·06	4·82	1·32	5·79	1·59	6·75	1·85	7·71	2·12	8·68	2·38	9·64	2·65	39
24	·964	·266	1·93	·531	2·89	·797	3·86	1·06	4·82	1·33	5·78	1·59	6·75	1·86	7·71	2·12	8·68	2·39	9·64	2·66	36
27	·964	·266	1·93	·533	2·89	·799	3·86	1·07	4·82	1·33	5·78	1·60	6·75	1·86	7·71	2·13	8·67	2·40	9·64	2·66	33
30	·964	·267	1·93	·534	2·89	·802	3·85	1·07	4·82	1·34	5·78	1·60	6·75	1·87	7·71	2·14	8·67	2·41	9·64	2·67	30
33	·963	·268	1·93	·536	2·89	·804	3·85	1·07	4·82	1·34	5·78	1·61	6·74	1·88	7·71	2·14	8·67	2·41	9·63	2·68	27
36	·963	·270	1·93	·538	2·89	·807	3·85	1·08	4·82	1·34	5·78	1·61	6·74	1·88	7·71	2·15	8·67	2·42	9·63	2·69	24
39	·963	·270	1·93	·540	2·89	·809	3·85	1·08	4·81	1·35	5·78	1·62	6·74	1·89	7·70	2·16	8·67	2·43	9·63	2·70	21
42	·963	·271	1·93	·541	2·89	·812	3·85	1·08	4·81	1·35	5·78	1·62	6·74	1·89	7·70	2·16	8·66	2·44	9·63	2·71	18
45	·962	·271	1·92	·543	2·89	·814	3·85	1·09	4·81	1·36	5·77	1·63	6·74	1·90	7·70	2·17	8·66	2·44	9·62	2·71	15
48	·962	·272	1·92	·545	2·89	·817	3·85	1·09	4·81	1·36	5·77	1·63	6·74	1·91	7·70	2·18	8·66	2·45	9·62	2·72	12
51	·962	·273	1·92	·546	2·89	·819	3·85	1·09	4·81	1·37	5·77	1·64	6·73	1·91	7·70	2·18	8·66	2·46	9·62	2·73	9
54	·962	·274	1·92	·548	2·89	·822	3·85	1·10	4·81	1·37	5·77	1·64	6·73	1·92	7·69	2·19	8·66	2·47	9·62	2·74	6
57	·961	·275	1·92	·550	2·88	·824	3·85	1·10	4·81	1·37	5·77	1·65	6·73	1·92	7·69	2·20	8·65	2·47	9·61	2·75	3
16°	·961	·276	1·92	·551	2·88	·827	3·85	1·10	4·81	1·38	5·77	1·65	6·73	1·93	7·69	2·20	8·65	2·48	9·61	2·76	**74°**
3	·961	·276	1·92	·553	2·88	·829	3·84	1·10	4·80	1·38	5·77	1·66	6·73	1·94	7·69	2·21	8·65	2·49	9·61	2·76	57
6	·961	·277	1·92	·555	2·88	·832	3·84	1·11	4·80	1·39	5·76	1·66	6·73	1·94	7·69	2·22	8·65	2·50	9·61	2·77	54
9	·961	·278	1·92	·556	2·88	·834	3·84	1·11	4·80	1·39	5·76	1·67	6·72	1·95	7·68	2·22	8·64	2·50	9·61	2·78	51
12	·960	·279	1·92	·558	2·88	·837	3·84	1·12	4·80	1·39	5·76	1·67	6·72	1·95	7·68	2·23	8·64	2·51	9·60	2·79	48
15	·960	·280	1·92	·560	2·88	·839	3·84	1·12	4·80	1·40	5·76	1·68	6·72	1·96	7·68	2·24	8·64	2·52	9·60	2·80	45
18	·960	·281	1·92	·561	2·88	·842	3·84	1·12	4·80	1·40	5·76	1·68	6·72	1·96	7·68	2·24	8·64	2·53	9·60	2·81	42
21	·960	·281	1·92	·563	2·88	·844	3·84	1·13	4·80	1·41	5·76	1·69	6·72	1·97	7·68	2·25	8·64	2·53	9·60	2·81	39
24	·959	·282	1·92	·565	2·88	·847	3·84	1·13	4·80	1·41	5·76	1·69	6·72	1·98	7·67	2·26	8·63	2·54	9·59	2·82	36
27	·959	·283	1·92	·566	2·88	·850	3·84	1·13	4·80	1·42	5·75	1·70	6·71	1·98	7·67	2·26	8·63	2·55	9·59	2·83	33
30	·959	·284	1·92	·568	2·88	·852	3·84	1·14	4·79	1·42	5·75	1·70	6·71	1·99	7·67	2·27	8·63	2·56	9·59	2·84	30
33	·959	·285	1·92	·570	2·88	·855	3·83	1·14	4·79	1·42	5·75	1·71	6·71	1·99	7·67	2·28	8·63	2·56	9·59	2·85	27
36	·958	·286	1·92	·571	2·87	·857	3·83	1·14	4·79	1·43	5·75	1·71	6·71	2·00	7·67	2·28	8·62	2·57	9·58	2·86	24
39	·958	·287	1·92	·573	2·87	·860	3·83	1·15	4·79	1·43	5·75	1·72	6·71	2·01	7·66	2·29	8·62	2·58	9·58	2·87	21
42	·958	·287	1·92	·575	2·87	·862	3·83	1·15	4·79	1·44	5·75	1·72	6·70	2·01	7·66	2·30	8·62	2·59	9·58	2·87	18
45	·958	·288	1·92	·576	2·87	·865	3·83	1·15	4·79	1·44	5·75	1·73	6·70	2·02	7·66	2·31	8·62	2·59	9·58	2·88	15
48	·957	·289	1·91	·578	2·87	·867	3·83	1·16	4·79	1·44	5·74	1·73	6·70	2·02	7·66	2·31	8·62	2·60	9·57	2·89	12
51	·957	·290	1·91	·580	2·87	·870	3·83	1·16	4·79	1·45	5·74	1·74	6·70	2·03	7·66	2·32	8·61	2·61	9·57	2·90	9
54	·957	·291	1·91	·581	2·87	·872	3·83	1·16	4·78	1·45	5·74	1·74	6·70	2·03	7·65	2·32	8·61	2·62	9·57	2·91	6
57	·957	·291	1·91	·583	2·87	·875	3·83	1·17	4·78	1·46	5·74	1·75	6·70	2·04	7·65	2·33	8·61	2·62	9·57	2·91	3
17°	·956	·292	1·91	·585	2·87	·877	3·83	1·17	4·78	1·46	5·74	1·75	6·69	2·05	7·65	2·34	8·61	2·63	9·56	2·92	**73°**
3	·956	·293	1·91	·586	2·87	·880	3·82	1·17	4·78	1·47	5·73	1·76	6·69	2·05	7·65	2·35	8·60	2·64	9·56	2·93	57
6	·956	·294	1·91	·588	2·87	·882	3·82	1·18	4·78	1·47	5·73	1·76	6·69	2·06	7·65	2·35	8·60	2·65	9·56	2·94	54
9	·956	·295	1·91	·590	2·87	·885	3·82	1·18	4·78	1·47	5·73	1·77	6·69	2·06	7·64	2·36	8·60	2·65	9·56	2·95	51
12	·955	·296	1·91	·591	2·87	·887	3·82	1·18	4·78	1·48	5·73	1·77	6·69	2·07	7·64	2·37	8·60	2·66	9·55	2·96	48
15	·955	·297	1·91	·593	2·87	·890	3·82	1·19	4·78	1·48	5·73	1·78	6·69	2·08	7·64	2·37	8·60	2·67	9·55	2·97	45
18	·955	·297	1·91	·595	2·86	·892	3·82	1·19	4·77	1·49	5·73	1·78	6·68	2·08	7·54	2·38	8·59	2·68	9·55	2·97	42
21	·955	·298	1·91	·596	2·86	·895	3·82	1·19	4·77	1·49	5·73	1·79	6·68	2·09	7·64	2·39	8·59	2·68	9·55	2·98	39
24	·954	·299	1·91	·598	2·86	·897	3·82	1·20	4·77	1·49	5·73	1·79	6·68	2·09	7·63	2·39	8·59	2·69	9·54	2·99	36
27	·954	·300	1·91	·600	2·86	·900	3·82	1·20	4·77	1·50	5·72	1·80	6·68	2·10	7·63	2·40	8·59	2·70	9·54	3·00	33
30	·954	·301	1·91	·601	2·86	·902	3·81	1·20	4·77	1·50	5·72	1·80	6·68	2·10	7·63	2·40	8·58	2·71	9·54	3·01	30
33	·953	·301	1·91	·603	2·86	·905	3·81	1·21	4·77	1·51	5·72	1·81	6·67	2·11	7·63	2·41	8·58	2·71	9·53	3·01	27
36	·953	·302	1·91	·605	2·86	·907	3·81	1·21	4·77	1·51	5·72	1·81	6·67	2·12	7·63	2·42	8·58	2·72	9·53	3·02	24
39	·953	·303	1·91	·606	2·86	·910	3·81	1·21	4·76	1·52	5·72	1·82	6·67	2·12	7·62	2·43	8·58	2·73	9·53	3·03	21
42	·953	·304	1·91	·608	2·86	·912	3·81	1·22	4·76	1·52	5·72	1·82	6·67	2·13	7·62	2·43	8·57	2·74	9·53	3·04	18
45	·952	·305	1·90	·610	2·86	·915	3·81	1·22	4·76	1·52	5·71	1·83	6·67	2·13	7·62	2·44	8·57	2·74	9·52	3·05	15
48	·952	·306	1·90	·611	2·86	·917	3·81	1·22	4·76	1·53	5·71	1·83	6·66	2·14	7·62	2·45	8·57	2·75	9·52	3·06	12
51	·952	·306	1·90	·613	2·86	·920	3·81	1·23	4·76	1·53	5·71	1·84	6·66	2·15	7·61	2·45	8·57	2·76	9·52	3·07	9
54	·952	·307	1·90	·615	2·85	·922	3·81	1·23	4·76	1·54	5·71	1·84	6·66	2·15	7·61	2·46	8·56	2·77	9·52	3·07	6
57	951	·308	1·90	·616	2·85	·925	3·81	1·23	4·76	1·54	5·71	1·85	6·66	2·16	7·61	2·46	8·56	2·77	9·51	3·08	3
Deg.	Dep	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.

DISTANCE IN CHAINS, &c.

Deg.	1		2		3		4		5		6		7		8		9		10		Deg.
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	′
18°	·951	·309	1·90	·618	2·85	·927	3·80	1·24	4·76	1·55	5·71	1·85	6·66	2·16	7·61	2·47	8·56	2·78	9·51	3·09	72°
3	·951	·310	1·90	·620	2·85	·930	3·80	1·24	4·75	1·55	5·70	1·86	6·66	2·17	7·61	2·48	8·56	2·79	9·51	3·10	57
6	·951	·311	1·90	·621	2·85	·932	3·80	1·24	4·75	1·55	5·70	1·86	6·65	2·17	7·60	2·48	8·55	2·80	9·51	3·11	54
9	·950	·312	1·90	·623	2·85	·935	3·80	1·25	4·75	1·56	5·70	1·87	6·65	2·18	7·60	2·49	8·55	2·80	9·50	3·12	51
12	·950	·312	1·90	·625	2·85	·937	3·80	1·25	4·75	1·56	5·70	1·87	6·65	2·19	7·60	2·50	8·55	2·81	9·50	3·12	48
15	·950	·313	1·90	·626	2·85	·939	3·80	1·25	4·75	1·57	5·70	1·88	6·65	2·19	7·60	2·51	8·55	2·82	9·50	3·13	45
18	·949	·314	1·90	·628	2·85	·942	3·80	1·26	4·75	1·57	5·70	1·88	6·65	2·20	7·60	2·51	8·54	2·83	9·49	3·14	42
21	·949	·315	1·90	·630	2·85	·944	3·80	1·26	4·75	1·57	5·69	1·89	6·64	2·20	7·59	2·52	8·54	2·83	9·49	3·15	39
24	·949	·316	1·90	·631	2 85	·947	3·80	1·26	4·74	1·58	5·69	1·89	6·64	2·21	7·59	2·52	8·54	2·84	9·49	3·16	36
27	·949	·316	1·90	·633	2·85	·949	3·79	1·27	4·74	1·58	5·69	1·90	6·64	2·22	7·59	2·53	8·54	2·85	9·49	3·16	33
30	·948	·317	1·90	·635	2·84	·952	3·79	1·27	4·74	1·59	5·69	1·90	6·64	2·22	7·59	2·54	8·53	2·86	9·48	3·17	30
33	·948	·318	1·90	·636	2·84	·954	3·79	1·27	4·74	1·59	5·69	1·91	6·64	2·23	7·58	2·55	8·53	2·86	9·48	3·18	27
36	·948	·319	1·90	·638	2·84	·957	3·79	1·28	4·74	1·59	5·69	1·91	6·63	2·23	7·58	2·55	8·53	2·87	9·48	3·19	24
39	·947	·320	1·89	·640	2·84	·959	3·79	1·28	4·74	1·60	5·68	1·92	6·63	2·24	7·58	2·56	8·53	2·88	9·47	3·20	21
42	·947	·321	1·89	·641	2·84	·962	3·79	1·28	4·74	1·60	5·68	1·92	6 63	2·24	7·58	2·56	8·52	2·88	9·47	3·21	18
45	·947	·321	1·89	·643	2·84	·964	3·79	1·29	4·73	1·61	5·68	1·93	6·63	2·25	7·58	2·57	8·52	2·89	9·47	3·21	15
48	·947	·322	1·89	·645	2·84	·967	3·79	1·29	4·73	1·61	5·68	1·93	6·63	2·26	7·57	2·58	8·52	2·90	9·47	3·22	12
51	·946	·323	1·89	·646	2·84	·969	3·78	1·29	4·73	1·62	5·68	1·94	6·62	2·26	7·57	2·58	8·52	2·91	9·46	3·23	9
54	·946	·324	1·89	·648	2·84	·972	3·78	1·30	4·73	1·62	5·68	1·94	6·62	2·27	7·57	2·59	8·51	2·91	9·46	3·24	6
57	·946	·325	1·89	·649	2·84	.974	3·78	1·30	4·73	1·62	5·67	1·95	6·62	2·27	7·57	2·60	8·51	2·92	9·46	3·25	3
19°	·946	·326	1·89	·651	2·84	·977	3·78	1·30	4·73	1·63	5·67	1·95	6·62	2·28	7·56	2·60	8·51	2·93	9·46	3·26	71
3	·945	·326	1·89	·653	2·84	·971	3·78	1·31	4·73	1·63	5·67	1·96	6·62	2·28	7·56	2·61	8·51	2·94	9·45	3·26	57
6	·945	·327	1·89	·654	2·83	·982	3·78	1·31	4·72	1·64	5·67	1·96	6·61	2·29	7·56	2·62	8·50	2·95	9·45	3·27	54
9	·945	·328	1·89	·656	2·83	·984	3·78	1·31	4·72	1·64	5·67	1·97	6·61	2·30	7·56	2·62	8·50	2·95	9·45	3·28	51
12	·944	·329	1·89	·658	2·83	·987	3·78	1·31	4·72	1·64	5·67	1·97	6·61	2·30	7·56	2·63	8·50	2·96	9·44	3·29	48
15	·944	·330	1·89	·659	2·83	·989	3·78	1·32	4·72	1·65	5·66	1·98	6·61	2·31	7·55	2·64	8·50	2·97	9·44	3·30	45
18	·944	·330	1·89	·661	2·83	·992	3·78	1·32	4·72	1·65	5·66	1·98	6·61	2·31	7·55	2·64	8·49	2·97	9·44	3·30	42
21	·943	·331	1·89	·663	2·83	·994	3·77	1·32	4·72	1·66	5·66	1·99	6·60	2·32	7·55	2·65	8·49	2·98	9·43	3·31	39
24	·943	·332	1·89	·664	2·83	·996	3·77	1·33	4·72	1·66	5·66	1·99	6·60	2·33	7·55	2·66	8·49	2·99	9·43	3·32	36
27	·943	·333	1·89	·666	2·83	·999	3·77	1·33	4·71	1·66	5·66	2·00	6·60	2·33	7·54	2·66	8·49	3·00	9·43	3·33	33
30	·943	·334	1·89	·668	2·83	·100	3·77	1·34	4·71	1·67	5·66	2·00	6·60	2·34	7·54	2·67	8·48	3·00	9·43	3·34	30
33	·942	·335	1·88	·669	2·83	·100	3·77	1·34	4·71	1·67	5·65	2·01	6·60	2·34	7·54	2·68	8·48	3·01	9·42	3·35	27
36	·942	·335	1·88	·671	2·83	·101	3·77	1·34	4·71	1·68	5·65	2·01	6·59	2·35	7·54	2·68	8·48	3·02	9·42	3·35	24
39	·942	·336	1·88	·673	2·83	·101	3·77	1·35	4·71	1·68	5·65	2·02	6·59	2·35	7·53	2·69	8·48	3·03	9·42	3·36	21
42	·941	·337	1·88	·674	2·82	·101	3·77	1·35	4·71	1·68	5·65	2·02	6·59	2·36	7·53	2·70	8·47	3·03	9·41	3·37	18
45	·941	·338	1·88	·676	2·82	·101	3·76	1·35	4·71	1·69	5·65	2·03	6·59	2·37	7·53	2·70	8·47	3·04	9·41	3·38	15
48	·941	·339	1·88	·677	2·82	·102	3·76	1·35	4·70	1·69	5·64	2·03	6·59	2·37	7·53	2·71	8·47	3·05	9·41	3·39	12
51	·941	·340	1·88	·679	2·82	·102	3·76	1·36	4·70	1·70	5·64	2·04	6·58	2·38	7·52	2·72	8·47	3·06	9·41	3·40	9
54	·940	·340	1·88	·681	2·82	·102	3·76	1·36	4·70	1·70	5·64	2·04	6·58	2·38	7·52	2·72	8·46	3·06	9·40	3·40	6
57	·940	·341	1·88	·682	2·82	·102	3·76	1·36	4·70	1·71	5·64	2 05	6·58	2·39	7·52	2·73	8·46	3·07	9·40	3·41	3
20°	·940	·342	1·88	·684	2·82	·103	3·76	1·37	4·70	1·71	5·64	2·05	6·58	2·39	7·52	2·74	8·46	3·08	9·40	3·42	70
3	·939	·343	1·88	·686	2·82	·103	3·76	1·37	4·70	1·71	5·64	2·06	6·58	2·40	7·52	2·74	8·45	3·09	9·39	3·43	57
6	·939	·344	1·88	·687	2·82	·103	3·76	1·37	4·70	1·72	5·63	2·06	6·57	2·41	7·51	2·75	8·45	3·09	9·39	3·44	54
9	·939	·344	1·88	·689	2·82	·103	3·76	1·38	4·69	1·72	5·63	2·07	6·57	2·41	7·51	2·76	8·45	3·10	9·39	3·44	51
12	·938	·345	1·88	·691	2·82	·104	3·75	1·38	4·69	1·73	5·63	2·07	6·57	2·42	7·51	2·76	8·45	3·11	9·38	3·45	48
15	·938	·346	1·88	·692	2·81	·104	3·75	1·38	4·69	1·73	5·63	2·08	6·57	2·42	7·51	2·77	8·44	3·12	9·38	3·46	45
18	·938	·347	1·88	·694	2·81	·104	3·75	1·39	4·69	1·73	5·63	2·08	6·57	2·43	7·50	2·77	8·44	3·12	9·38	3·47	42
21	·938	·348	1·88	·695	2·81	·104	3·75	1·39	4·69	1·74	5·63	2·09	6·56	2·43	7·50	2·78	8·44	3·13	9·38	3·48	39
24	·937	·349	1·87	·697	2·81	·105	3·75	1·39	4·69	1·74	5·62	2·09	6·56	2·44	7·50	2·79	8·44	3·14	9·37	3·49	36
27	·937	·349	1·87	·699	2·81	·105	3·75	1·40	4·68	1·75	5·62	2·10	6·56	2·45	7·50	2·80	8·43	3·14	9·37	3·49	33
30	·937	·350	1·87	·700	2·81	·105	3·75	1·40	4·68	1·75	5·62	2·10	6·56	2·45	7·49	2·80	8·43	3·15	9·37	3·50	30
33	·936	·351	1·87	·702	2·81	·105	3·75	1·40	4·68	1·75	5·62	2·11	6·55	2·46	7·49	2·81	8·43	3·16	9·36	3·51	27
36	·936	·352	1·87	·704	2·81	·106	3·74	1·41	4·68	1·76	5·62	2·11	6·55	2·46	7·49	2·81	8·42	3·17	9·36	3·52	24
39	·936	·353	1·87	·705	2·81	·106	3·74	1·41	4·68	1·76	5·61	2·12	6·55	2·47	7·49	2·82	8·42	3·17	9·36	3·53	21
42	·935	·353	1·87	·707	2·81	·106	3·74	1·41	4·68	1·77	5·61	2·12	6·55	2·47	7·48	2·83	8·42	3·18	9·35	3·53	18
45	·935	·354	1·87	·709	2·81	·106	3·74	1·42	4·68	1·77	5·61	2·13	6·55	2·48	7·48	2·83	8·42	3·19	9·35	3·54	15
48	·935	·355	1·87	·710	2·80	·107	3·74	1·42	4·67	1·78	5·61	2·13	6·54	2·49	7·48	2·84	8·41	3·20	9·35	3·55	12
51	·935	·356	1·87	·712	2·80	·107	3·74	1·42	4·67	1·78	5·61	2·14	6·54	2·49	7·48	2·85	8·41	3·20	9·35	3·56	9
54	·934	·357	1·87	·713	2·80	·107	3·74	1·43	4·67	1·78	5·61	2·14	6·54	2·50	7·47	2·85	8·41	3·21	9·34	3·57	6
57	·934	·358	1·87	·715	2·80	·107	3·74	1·43	4·67	1·79	5·60	2·15	6·54	2·50	7·47	2·86	8·41	3·22	9·34	3·58	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Deg

DISTANCE IN CHAINS, &c.

Deg.	1		2		3		4		5		6		7		8		9		10		Deg.
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	′
21°	·934	·358	1·87	·717	2·80	1·07	3·73	1·43	4·67	1·79	5·60	2·15	6·54	2·51	7·47	2·87	8·40	3·23	9·34	3·58	**69°**
3	·933	·359	1·87	·718	2·80	1·08	3·73	1·44	4·67	1·80	5·60	2·16	6·53	2·51	7·47	2·87	8·40	3·23	9·33	3·59	57
6	·933	·360	1·87	·720	2·80	1·08	3·73	1·44	4·66	1·80	5·60	2·16	6·53	2·52	7·46	2·88	8·40	3·24	9·33	3·60	54
9	·933	·361	1·87	·722	2·80	1·08	3·73	1·44	4·66	1·80	5·60	2·16	6·53	2·53	7·46	2·89	8·39	3·25	9·33	3·61	51
12	·932	·361	1·86	·723	2·80	1·08	3·73	1·45	4·66	1·81	5·59	2·17	6·53	2·53	7·46	2·89	8·39	3·25	9·32	3·62	48
15	·932	·362	1,86	·725	2·80	1·09	3·73	1·45	4·66	1·81	5·59	2·17	6·52	2·54	7·46	2·90	8·39	3·26	9·32	3·62	45
18	·932	·363	1·86	·726	2·80	1·09	3·73	1·45	4·66	1·82	5·59	2·18	6·52	2·54	7·45	2·91	8·39	3·27	9·32	3·63	42
21	·931	·364	1·86	·728	2·79	1·09	3·73	1·46	4·66	1·82	5·59	2·18	6·52	2·55	7·45	2·91	8·38	3·28	9·31	3·64	39
24	·931	·365	1·86	730	2·79	1·09	3·72	1·46	4·66	1·82	5·59	2·19	6·52	2·55	7·45	2·92	8·38	3·28	9·31	3·65	36
27	·931	·366	1·86	731	2·79	1·10	3·72	1·46	4·65	1·83	5·58	2·19	6·51	2·56	7·45	2·93	8·38	3·29	9·31	3·66	33
30	·930	·366	1·86	·733	2·79	1·10	3·72	1·47	4·65	1·83	5·58	2·20	6·51	2·57	7·44	2·93	8·37	3·30	9·30	3·66	30
33	·930	·367	1·86	·735	2·79	1·10	3·72	1·47	4·65	1·84	5·58	2·20	6·51	2·57	7·44	2·94	8·37	3·31	9·30	3·67	27
36	·930	·368	1·86	·736	2·79	1·10	3·72	1·47	4·65	1·84	5·58	2·21	6·51	2·58	7·44	2·94	8·37	3·31	9·30	3·68	24
39	·929	·369	1·86	·738	2·79	1·11	3·72	1·48	4·65	1·84	5·58	2·21	6·51	2·58	7·44	2·95	8·37	3·32	9·29	3·69	21
42	·929	·370	1·86	·739	2·79	1·11	3·72	1·48	4·65	1·85	5·57	2·22	6·50	2·59	7·43	2·96	8·37	3·33	9·29	3·70	18
45	·929	·371	1·86	·741	2·79	1·11	3·72	1·48	4·64	1·85	5·57	2·22	6·50	2·59	7·43	2·96	8·36	3·33	9·29	3·71	15
48	·928	·371	1,86	·743	2·79	1·11	3·71	1·49	4·64	1·86	5·57	2·23	6·50	2·60	7·43	2·97	8·36	3·34	9·28	3·71	12
51	·928	·372	1·86	·744	2·78	1·12	3·71	1·49	4·64	1·86	5·57	2·23	6·50	2·61	7·43	2·98	8·35	3·35	9·28	3·72	9
54	·928	·373	1·86	·746	2·78	1·12	3·71	1·49	4·64	1·86	5·57	2·24	6·49	2·61	7·42	2·98	8·35	3·36	9·28	3·73	6
57	·928	·373	1·86	·748	2·78	1·12	3·71	1·50	4·64	1·87	5·57	2·24	6·49	2·62	7·42	2·99	8·35	3·36	9·28	3·74	3
22°	·927	·375	1·85	.749	2·78	1·12	3·71	1·50	4·64	1·87	5·56	2·25	6·49	2·62	7·42	3·00	8·34	3·37	9·27	3·75	**68°**
3	·927	·375	1·85	·751	2·78	1·13	3·71	1·50	4·63	1·88	5·56	2·25	6·49	2·63	7·41	3·00	8·34	3·38	9·27	3·75	57
6	·927	·376	1·85	·752	2·78	1·13	3·71	1·50	4·63	1·88	5·56	2·26	6·49	2·63	7·41	3·01	8·34	3·39	9·27	3·76	54
9	·926	·377	1·85	·754	2·78	1·13	3·70	1·51	4·63	1·89	5·56	2·26	6·48	2·64	7·41	3·02	8·34	3·39	9·26	3·77	51
12	·926	·378	1·85	·756	2·78	1·13	3·70	1·51	4·63	1·89	5·56	2·27	6·48	2·64	7·41	3·02	8·33	3·40	9·26	3·78	48
15	·926	·379	1·85	·757	2·78	1·14	3·70	1·51	4·63	1·89	5·55	2·27	6·48	2·65	7·40	3·03	8·33	3·41	9·26	3·79	45
18	·925	·379	1·85	·759	2·78	1·14	3·70	1·52	4·63	1·90	5·55	2·28	6·48	2·66	7·40	3·04	8·33	3·42	9·25	3·79	42
21	·925	·380	1·85	·761	2·77	1·14	3·70	1·52	4·62	1·90	5·55	2·28	6·47	2·66	7·40	3·04	8·32	3·42	9·25	3·80	39
24	·925	·381	1·85	·762	2·77	1·14	3·70	1·52	4·62	1·91	5·55	2·29	6·47	2·67	7·40	3·05	8·32	3·43	9·25	3·81	36
27	·924	·382	1·85	·764	2·77	1·15	3·70	1·53	4·62	1·91	5·55	2·29	6·47	2·67	7·40	3·06	8·32	3·44	9·24	3·82	33
30	·924	·383	1·85	·765	2·77	1·15	3·70	1·53	4·62	1·91	5·54	2·30	6·47	2·68	7·39	3·06	8·32	3·44	9·24	3 83	30
33	·924	·383	1·85	·767	2·77	1·15	3·69	1·53	4·62	1·92	5·54	2·30	6·46	2·68	7·39	3·07	8·31	3·45	9·24	3·83	27
36	·923	·384	1·85	·769	2·77	1·15	3·69	1·54	4·62	1·92	5·54	2·31	6·46	2·69	7·39	3 07	8·31	3·46	9·23	3·84	24
39	·923	·385	1·85	·770	2·77	1·16	3·69	1·54	4·61	1·93	5·54	2·31	6·46	2·70	7·38	3·08	8·31	3·47	9·23	3·85	21
42	·923	·386	1·85	·772	2·77	1·16	3·69	1·54	4·61	1·93	5·54	2·32	6·46	2·70	7·38	3·09	8·30	3·47	9·23	3·86	18
45	·922	·387	1·84	·773	2·77	1·16	3·69	1·55	4·61	1·93	5·53	2·32	6·46	2·71	7·38	3·09	8·30	3·48	9·22	3·87	15
48	·922	·388	1·84	·775	2·77	1·16	3·69	1·55	4·61	1·94	5·53	2·33	6·45	2·71	7·37	3·10	8·30	3·49	9·22	3·88	12
51	·922	·388	1·84	·777	2·76	1·16	3·69	1·55	4·61	1·94	5·53	2·33	6·45	2·72	7·37	3·11	8·29	3·49	9·22	3·88	9
54	·921	·389	1.84	·778	2·76	1·17	3·68	1·56	4·61	1·95	5·53	2·33	6·45	2·72	7·37	3·11	8·29	3·50	9·21	3·89	6
57	·921	·390	1·84	·780	2·76	1·17	3·68	1·56	4·60	1·95	5·52	2·34	6·45	2·73	7·37	3·12	8·29	3·51	9·21	3·90	3
23°	·921	·391	1·84	·781	2·76	1·17	3·68	1·56	4·60	1·95	5·52	2·34	6·44	2·74	7·36	3·13	8·28	3·52	9·21	3·91	**67°**
3	·920	·392	1·84	·783	2·76	1·17	3·68	1·57	4·60	1·96	5 52	2·35	6·44	2·74	7·36	3·13	8·28	3·52	9·20	3·92	57
6	·920	·392	1·84	·785	2·76	1·18	3·68	1·57	4·60	1·96	5·52	2·35	6·44	2·75	7·36	3·14	8·28	3·53	9·20	3·92	54
9	·919	·393	1·84	·786	2·76	1·18	3·68	1·57	4·60	1·97	5·52	2·36	6·44	2·75	7·36	3·15	8·28	3·54	9·19	3·93	51
12	·919	·394	1·84	·788	2·76	1·18	3·68	1·58	4·60	1·97	5·51	2·36	6·43	2·76	7·35	3·15	8·27	3·55	9·19	3·94	48
15	·919	·395	1·84	·790	2·76	1·18	3·68	1·58	4·59	1·97	5·51	2·37	6·43	2·76	7·35	3·16	8·27	3·55	9·19	3·95	45
18	·918	·396	1·84	·791	2·76	1·19	3·67	1·58	4·59	1·98	5·51	2·37	6·43	2·77	7·35	3·16	8·27	3·56	9·18	3·96	42
21	·918	·396	1·84	·793	2·75	1·19	3·67	1·59	4·59	1·98	5·51	2·38	6·43	2·77	7·34	3·17	8·26	3·57	9·18	3·96	39
24	·918	·397	1·84	·794	2·75	1·19	3·67	1·59	4·59	1·99	5·51	2·38	6·42	2·78	7·34	3·18	8·26	3·57	9·18	3·97	36
27	·917	·398	1·83	·796	2·75	1·19	3·67	1·59	4·59	1·99	5·50	2·39	6·42	2·79	7·34	3·18	8·26	3·58	9·17	3·98	33
30	·917	·399	1·83	·797	2·75	1·20	3·67	1·59	4·59	1·99	5·50	2·39	6·42	2·79	7·34	3·19	8·25	3·59	9·17	3·99	30
33	·917	·400	1·83	·800	2·75	1·20	3·67	1·60	4·58	2·00	5·50	2·40	6·42	2·80	7·33	3·20	8·25	3·60	9·17	4·00	27
36	·916	·400	1·83	·801	2·75	1·20	3·67	1·60	4·58	2·00	5·50	2·40	6·41	2·80	7·33	3·20	8·25	3·60	9·16	4·00	24
39	·916	·401	1·83	·802	2·75	1·20	3·66	1·60	4·58	2·01	5·50	2·41	6·41	2·81	7·33	3·21	8·24	3·61	9·16	4·01	21
42	·916	·402	1·83	·804	2·75	1·21	3·66	1·61	4·58	2·01	5·49	2·41	6·41	2·81	7·33	3·22	8·24	3·62	9·16	4·02	18
45	·915	·403	1·83	·805	2·75	1·21	3·66	1·61	4·58	2·01	5·49	2·42	6·41	2·82	7·32	3·22	8·24	3·62	9·15	4·03	15
48	915	·404	1·83	·807	2·74	1·21	3·66	1·61	4·57	2·02	5·49	2·42	6·40	2·82	7·32	3·23	8·23	3·63	9·15	4·04	12
51	·915	·404	1·83	·809	2·74	1·21	3·66	1·61	4·57	2·02	5·49	2·43	6·40	2·83	7·32	3·23	8·23	3·64	9·15	4·04	9
54	·914	·406	1·83	·810	2·74	1·22	3·66	1·62	4·57	2·03	5 49	2·43	6·40	2·84	7·31	3·24	8·23	3·65	9·14	4·05	6
57	·914	·406	1·83	·812	2·74	1·22	3·66	1·62	4·57	2·03	5·48	2·44	6·40	2·84	7·31	3·25	8·23	3·65	9·14	4·06	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.

Deg.	DISTANCE IN CHAINS, &c.																				Deg.
	1		2		3		4		5		6		7		8		9		10		
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	′
24°	·914	·407	1·83	·813	2·74	1·22	3·65	1·63	4·57	2·03	5·48	2·44	6·40	2·85	7·31	3·25	8·22	3·66	9·14	4·07	66°
3	·913	·408	1·83	·815	2·74	1·22	3·65	1·63	4·57	2·04	5·48	2·45	6·39	2·85	7·31	3·26	8·22	3·67	9·13	4·08	57
6	·913	·408	1·83	·817	2·74	1·22	3·65	1·63	4·56	2·04	5·48	2·45	6·39	2·86	7·30	3·27	8·22	3·67	9·13	4·08	54
9	·912	·409	1·82	·818	2·74	1·23	3·65	1·64	4·56	2·05	5·47	2·45	6·39	2·86	7·30	3·27	8·21	3·68	9·12	4·09	51
12	·912	·410	1·82	·820	2·74	1·23	3·65	1·64	4·56	2·05	5·47	2·46	6·38	2·87	7·30	3·28	8·21	3·69	9·12	4·10	48
15	·912	·411	1·82	·821	2·74	1·23	3·65	1·64	4·56	2·05	5·47	2·46	6·38	2·87	7·29	3·29	8·21	3·70	9·12	4·11	45
18	·911	·412	1·82	·823	2·73	1·23	3·65	1·65	4·56	2·06	5·47	2·47	6·38	2·88	7·29	3·29	8·20	3·70	9·11	4·12	42
21	·911	·412	1·82	·825	2·73	1·24	3·64	1·65	4·56	2·06	5·47	2·47	6·38	2·89	7·29	3·30	8·20	3·71	9·11	4·12	39
24	·911	·413	1·82	·826	2·73	1·24	3·64	1·65	4·55	2·07	5·46	2·48	6·37	2·89	7·29	3·30	8·20	3·72	9·11	4·13	36
27	·910	·414	1·82	·828	2·73	1·24	3·64	1·66	4·55	2·07	5·46	2·48	6·37	2·90	7·28	3·31	8·19	3·72	9·10	4·14	33
30	·910	·415	1·82	·829	2·73	1·24	3·64	1·66	4·55	2·07	5·46	2·49	6·37	2·90	7·28	3·32	8·19	3·73	9·10	4·15	30
33	·910	·415	1·82	·831	2·73	1·25	3·64	1·66	4·55	2·08	5·46	2·49	6·37	2·91	7·28	3·32	8·19	3·74	9·10	4·15	27
36	·909	·416	1·82	·833	2·73	1·25	3·64	1·66	4·55	2·08	5·46	2·50	6·36	2·91	7·27	3·33	8·18	3·75	9·09	4·16	24
39	·909	·417	1·82	·834	2·73	1·25	3·64	1·67	4·54	2·09	5·45	2·50	6·36	2·92	7·27	3·34	8·18	3·75	9·09	4·17	21
42	·909	·418	1·82	·836	2·73	1·25	3·63	1·67	4·54	2·09	5·45	2·51	6·36	2·93	7·27	3·34	8·18	3·76	9·09	4·18	18
45	·908	·419	1·82	·837	2·72	1·26	3·63	1·67	4·54	2·09	5·45	2·51	6·36	2·93	7·27	3·35	8·17	3·77	9·08	4·19	15
48	·908	·419	1·82	·839	2·72	1·26	3·63	1·68	4·54	2·10	5·45	2·52	6·35	2·94	7·26	3·36	8·17	3·78	9·08	4·19	12
51	·907	·420	1·81	·840	2·72	1·26	3·63	1·68	4·54	2·10	5·44	2·52	6·35	2·94	7·26	3·36	8·17	3·78	9·07	4·20	9
54	·907	·421	1·81	·842	2·72	1·26	3·63	1·68	4·54	2·11	5·44	2·53	6·35	2·95	7·26	3·37	8·16	3·79	9·07	4·21	6
57	·907	·422	1·81	·844	2·72	1·27	3·63	1·69	4·53	2·11	5·44	2·53	6·35	2·95	7·25	3·37	8·16	3·80	9·07	4·22	3
25°	·906	·423	1·81	·845	2·72	1·27	3·63	1·69	4·53	2·11	5·44	2·54	6·34	2·96	7·25	3·38	8·16	3·80	9·06	4·23	65°
3	·906	·423	1·81	·847	2·72	1·27	3·62	1·69	4·53	2·12	5·44	2·54	6·34	2·96	7·25	3·39	8·15	3·81	9·06	4·23	57
6	·906	·424	1·81	·848	2·72	1·27	3·62	1·70	4·53	2·12	5·43	2·55	6·34	2·97	7·24	3·39	8·15	3·82	9·06	4·24	54
9	·905	·425	1·81	·850	2·72	1·27	3·62	1·70	4·53	2·12	5·43	2·55	6·34	2·97	7·24	3·40	8·15	3·82	9·05	4·25	51
12	·905	·426	1·81	·852	2·71	1·28	3·62	1·70	4·52	2·13	5·43	2·55	6·33	2·98	7·24	3·41	8·14	3·83	9·05	4·26	48
15	·905	·427	1·81	·853	2·71	1·28	3·62	1·71	4·52	2·13	5·43	2·56	6·33	2·99	7·24	3·41	8·14	3·84	9·04	4·27	45
18	·904	·427	1·81	·855	2·71	1·28	3·62	1·71	4·52	2·14	5·42	2·56	6·33	2·99	7·23	3·42	8·14	3·85	9·04	4·27	42
21	·904	·428	1·81	·856	2·71	1·28	3·61	1·71	4·52	2·14	5·42	2·57	6·33	3·00	7·23	3·43	8·13	3·85	9·04	4·28	39
24	·903	·429	1·81	·858	2·71	1·29	3·61	1·72	4·52	2·14	5·42	2·57	6·32	3·00	7·23	3·43	8·13	3·86	9·03	4·29	36
27	·903	·430	1·81	·860	2·71	1·29	3·61	1·72	4·51	2·15	5·42	2·58	6·32	3·01	7·22	3·44	8·13	3·87	9·03	4·30	33
30	·903	·430	1·81	·861	2·71	1·29	3·61	1·72	4·51	2·15	5·42	2·58	6·32	3·01	7·22	3·44	8·12	3·87	9·03	4·30	30
33	·902	·431	1·80	·863	2·71	1·29	3·61	1·73	4·51	2·16	5·41	2·59	6·32	3·02	7·22	3·45	8·12	3·88	9·02	4·31	27
36	·902	·432	1·80	·864	2·71	1·30	3·61	1·73	4·51	2·16	5·41	2·59	6·31	3·02	7·21	3·46	8·12	3·89	9·02	4·32	24
39	·901	·433	1·80	·866	2·70	1·30	3·61	1·73	4·51	2·16	5·41	2·60	6·31	3·03	7·21	3·46	8·11	3·90	9·01	4·33	21
42	·901	·434	1·80	·867	2·70	1·30	3·60	1·73	4·51	2·17	5·41	2·60	6·31	3·04	7·21	3·47	8·11	3·90	9·01	4·34	18
45	·901	·434	1·80	·869	2·70	1·30	3·60	1·74	4·50	2·17	5·40	2·61	6·30	3·04	7·21	3·48	8·11	3·91	9·01	4·34	15
48	·900	·435	1·80	·870	2·70	1·31	3·60	1·74	4·50	2·18	5·40	2·61	6·30	3·05	7·20	3·48	8·10	3·92	9·00	4·35	12
51	·900	·436	1·80	·872	2·70	1·31	3·60	1·74	4·50	2·18	5·40	2·62	6·30	3·05	7·20	3·49	8·10	3·92	9·00	4·36	9
54	·900	·437	1·80	·874	2·70	1·31	3·60	1·75	4·50	2·18	5·40	2·62	6·30	3·06	7·20	3·49	8·10	3·93	9·00	4·37	6
57	·899	·438	1·80	·875	2·70	1·31	3·60	1·75	4·50	2·19	5·40	2·63	6·29	3·06	7·19	3·50	8·09	3·94	8·99	4·38	3
26°	·899	·438	1·80	·877	2·70	1·32	3·60	1·75	4·49	2·19	5·39	2·63	9·29	3·07	7·19	3·51	8·09	3·95	8·99	4·38	64°
3	·898	·439	1·80	·878	2·70	1·32	3·59	1·76	4·49	2·20	5·39	2·63	6·29	3·07	7·19	3·51	8·09	3·95	8·98	4·39	57
6	·898	·440	1·80	·880	2·69	1·32	3·59	1·76	4·49	2·20	5·39	2·64	6·29	3·08	7·18	3·52	8·08	3·96	8·98	4·40	54
9	·898	·441	1·80	·881	2·69	1·32	3·59	1·76	4·49	2·20	5·39	2·64	6·28	3·08	7·18	3·53	8·08	3·97	8·98	4·41	51
12	·897	·441	1·79	·883	2·69	1·32	3·59	1·77	4·49	2·21	5·38	2·65	6·28	3·09	7·18	3·53	8·08	3·97	8·97	4·41	48
15	·897	·442	1·79	·885	2·69	1·33	3·59	1·77	4·48	2·21	5·38	2·65	6·28	3·10	7·17	3·54	8·07	3·98	8·97	4·42	45
18	·896	·443	1·79	·886	2·69	1·33	3·59	1·77	4·48	2·22	5·38	2·66	6·28	3·10	7·17	3·54	8·07	3·99	8·96	4·43	42
21	·896	·444	1·79	·888	2·69	1·33	3·58	1·78	4·48	2·22	5·38	2·66	6·27	3·11	7·17	3·55	8·06	3·99	8·96	4·44	39
24	·896	·444	1·79	·889	2·69	1·33	3·58	1·78	4·48	2·22	5·37	2·67	6·27	3·11	7·17	3·56	8·06	4·00	8·96	4·45	36
27	·895	·445	1·79	·891	2·69	1·34	3·58	1·78	4·48	2·23	5·37	2·67	6·27	3·12	7·16	3·56	8·06	4·01	8·95	4·45	33
30	·895	·446	1·79	·892	2·68	1·34	3·58	1·78	4·47	2·23	5·37	2·68	6·26	3·12	7·16	3·57	8·05	4·02	8·95	4·46	30
33	·895	·447	1·79	·894	2·68	1·34	3·58	1·79	4·47	2·23	5·37	2·68	6·26	3·13	7·16	3·58	8·05	4·02	8·95	4·47	27
36	·894	·448	1·79	·896	2·68	1·34	3·58	1·79	4·47	2·24	5·36	2·69	6·26	3·13	7·15	3·58	8·05	4·03	8·94	4·48	24
39	·894	·449	1·79	·897	2·68	1·35	3·58	1·79	4·47	2·24	5·36	2·69	6·26	3·14	7·15	3·59	8·04	4·04	8·94	4·49	21
42	·893	·449	1·79	·899	2·68	1·35	3·57	1·80	4·47	2·25	5·36	2·70	6·25	3·15	7·15	2·59	8·04	4·04	8·93	4·49	18
45	·893	·450	1·79	·900	2·68	1·35	3·57	1·80	4·46	2·25	5·36	2·70	6·25	3·15	7·14	3·60	8·04	4·05	8·93	4·50	15
48	·893	·451	1·79	·902	2·68	1·35	3·57	1·80	4·46	2·25	5·36	2·71	6·25	3·16	7·14	3·61	8·03	4·06	8·93	4·51	12
51	·892	·452	1·78	·903	2·68	1·35	3·57	1·81	4·46	2·26	5·35	2·71	6·25	3·16	7·14	3·61	8·03	4·06	8·92	4·52	9
54	·892	·452	1·78	·905	2·68	1·36	3·57	1·81	4·46	2·26	5·35	2·71	6·24	3·17	7·14	3·62	8·03	4·07	8·92	4·52	6
57	·891	·453	1·78	·906	2·67	1·36	3·57	1·81	4·46	2·27	5·35	2·72	6·24	3·17	7·13	3·63	8·02	4·08	8·91	4·23	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.

DISTANCE IN CHAINS, &c.

Deg.	1		2		3		4		5		6		7		8		9		10		Deg.
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat	Dep.	′
7°	.891	.454	1.78	.908	2.67	1.36	3.56	1.82	4.46	2.27	5.35	2.72	6.24	3.18	7.13	3.63	8.02	4.09	8.91	4.54	63°
3	.891	.455	1.78	.910	2.67	1.36	3.56	1.82	4.45	2.27	5.34	2.73	6.23	3.18	7.12	3.64	8.02	4.09	8.91	4.55	57
6	.890	.456	1.78	.911	2.67	1.37	3.56	1.82	4.45	2.28	5.34	2.73	6.23	3.19	7.12	3.64	8.01	4.10	8.90	4.56	54
9	.890	.456	1.78	.913	2.67	1.37	3.56	1.83	4.45	2.28	5.34	2.74	6.23	3.19	7.12	3.65	8.01	4.11	8.90	4.56	51
12	.889	.457	1.78	.914	2.67	1.37	3.56	1.83	4.45	2.29	5.34	2.74	6.23	3.20	7.12	3.66	8.00	4.11	8.89	4.57	48
15	.889	.458	1.78	.916	2.67	1.37	3.56	1.83	4.44	2.29	5.33	2.75	6.22	3.21	7.11	3.66	8.00	4.12	8.89	4.58	45
18	.889	.459	1.78	.917	2.67	1.38	3.55	1.83	4.44	2.29	5.33	2.75	6.22	3.21	7.11	3.67	8.00	4.13	8.89	4.59	42
21	.888	.459	1.78	.919	2.66	1.38	3.55	1.84	4.44	2.30	5.33	2.76	6.22	3.22	7.11	3.68	8.00	4.13	8.88	4.59	39
24	.888	.460	1.78	.920	2.66	1.38	3.55	1.84	4.44	2.30	5.33	2.76	6.21	3.22	7.10	3.68	7.99	4.14	8.88	4.60	36
27	.887	.461	1.77	.922	2.66	1.38	3.55	1.84	4.44	2.30	5.32	2.77	6.21	3.23	7.10	3.69	7.99	4.15	8.87	4.61	33
30	.887	.462	1.77	.923	2.66	1.38	3.55	1.85	4.44	2.31	5.32	2.77	6.21	3.23	7.10	3.69	7.98	4.16	8.87	4.62	30
33	.887	.463	1.77	.925	2.66	1.39	3.55	1.85	4.43	2.31	5.32	2.78	6.21	3.24	7.10	3.70	7.98	4.16	8.87	4.63	27
36	.886	.463	1.77	.927	2.66	1.39	3.54	1.85	4.43	2.32	5.32	2.78	6.20	3.24	7.09	3.71	7.98	4.17	8.86	4.63	24
39	.886	.464	1.77	.928	2.66	1.39	3.54	1.86	4.43	2.32	5.31	2.78	6.20	3.25	7.09	3.71	7.97	4.18	8.86	4.64	21
42	.885	.465	1.77	.930	2.66	1.39	3.54	1.86	4.43	2.32	5.31	2.79	6.20	3.25	7.08	3.72	7.97	4.18	8.85	4.65	18
45	.885	.466	1.77	.931	2.65	1.40	3.54	1.86	4.42	2.33	5.31	2.79	6.19	3.26	7.08	3.72	7.96	4.19	8.85	4.66	15
48	.885	.466	1.77	.933	2.65	1.40	3.54	1.87	4.42	2.33	5.31	2.80	6.19	3.26	7.08	3.73	7.96	4.20	8.85	4.66	12
51	.884	.467	1.77	.934	2.65	1.40	3.54	1.87	4.42	2.34	5.31	2.80	6.19	3.27	7.07	3.74	7.96	4.20	8.84	4.67	9
54	.884	.468	1.77	.936	2.65	1.40	3.54	1.87	4.42	2.34	5.30	2.81	6.19	3.28	7.07	3.74	7.95	4.21	8.84	4.68	6
57	.883	.469	1.77	.937	2.65	1.41	3.53	1.87	4.42	2.34	5.30	2.81	6.18	3.28	7.07	3.75	7.95	4.22	8.83	4.69	3
8°	.883	.469	1.77	.939	2.65	1.41	3.53	1.88	4.41	2.35	5.30	2.82	6.18	3.29	7.06	3.76	7.95	4.22	8.83	4.69	62°
3	.883	.470	1.77	.940	2.65	1.41	3.53	1.88	4.41	2.35	5.30	2.82	6.18	3.29	7.06	3.76	7.94	4.23	8.83	4.70	57
6	.882	.471	1.76	.942	2.65	1.41	3.53	1.88	4.41	2.36	5.29	2.83	6.17	3.30	7.06	3.77	7.94	4.24	8.82	4.71	54
9	.882	.472	1.76	.944	2.65	1.42	3.53	1.89	4.41	2.36	5.29	2.83	6.17	3.30	7.05	3.77	7.94	4.25	8.82	4.72	51
12	.881	.473	1.76	.945	2.64	1.42	3.53	1.89	4.41	2.36	5.29	2.84	6.17	3.31	7.05	3.78	7.93	4.25	8.81	4.73	48
15	.881	.473	1.76	.947	2.64	1.42	3.52	1.89	4.40	2.37	5.29	2.84	6.17	3.31	7.05	3.79	7.93	4.26	8.81	4.73	45
18	.880	.474	1.76	.948	2.64	1.42	3.52	1.90	4.40	2.37	5.28	2.84	6.16	3.32	7.04	3.79	7.92	4.27	8.80	4.74	42
21	.880	.475	1.76	.950	2.64	1.42	3.52	1.90	4.40	2.37	5.28	2.85	6.16	3.32	7.04	3.80	7.92	4.27	8.80	4.75	39
24	.880	.476	1.76	.951	2.64	1.43	3.52	1.90	4.40	2.38	5.28	2.85	6.16	3.33	7.04	3.80	7.92	4.28	8.80	4.76	36
27	.879	.476	1.76	.953	2.64	1.43	3.52	1.91	4.40	2.38	5.28	2.86	6.15	3.33	7.03	3.81	7.91	4.29	8.79	4.76	33
30	.879	.477	1.76	.954	2.64	1.43	3.52	1.91	4.39	2.39	5.27	2.86	6.15	3.34	7.03	3.82	7.91	4.29	8.79	4.77	30
33	.878	.478	1.76	.956	2.64	1.43	3.51	1.91	4.39	2.39	5.27	2.87	6.15	3.35	7.03	3.82	7.91	4.30	8.78	4.78	27
36	.878	.479	1.76	.957	2.63	1.44	3.51	1.91	4.39	2.39	5.27	2.87	6.15	3.35	7.02	3.83	7.90	4.31	8.78	4.79	24
39	.878	.479	1.76	.959	2.63	1.44	3.51	1.92	4.39	2.40	5.27	2.88	6.14	3.36	7.02	3.84	7.90	4.32	8.78	4.79	21
42	.877	.480	1.75	.960	2.63	1.44	3.51	1.92	4.39	2.40	5.26	2.88	6.14	3.36	7.02	3.84	7.89	4.32	8.77	4.80	18
45	.877	.481	1.75	.962	2.63	1.44	3.51	1.92	4.38	2.40	5.26	2.89	6.14	3.37	7.01	3.85	7.89	4.33	8.77	4.81	15
48	.876	.482	1.75	.964	2.63	1.45	3.51	1.93	4.38	2.41	5.26	2.89	6.13	3.37	7.01	3.85	7.89	4.34	8.76	4.82	12
51	.876	.483	1.75	.965	2.63	1.45	3.50	1.93	4.38	2.41	5.26	2.90	6.13	3.38	7.01	3.86	7.88	4.34	8.76	4.83	9
54	.875	.483	1.75	.967	2.63	1.45	3.50	1.93	4.38	2.42	5.25	2.90	6.13	3.38	7.00	3.87	7.88	4.35	8.75	4.83	6
57	.875	.484	1.75	.968	2.63	1.45	3.50	1.94	4.38	2.42	5.25	2.90	6.13	3.39	7.00	3.87	7.88	4.36	8.75	4.84	3
9°	.875	.485	1.75	.970	2.62	1.45	3.50	1.94	4.37	2.42	5.25	2.91	6.12	3.39	7.00	3.88	7.87	4.36	8.75	4.85	61°
3	.874	.486	1.75	.971	2.62	1.46	3.50	1.94	4.37	2.43	5.25	2.91	6.12	3.40	6.99	3.88	7.87	4.37	8.74	4.86	57
6	.874	.486	1.75	.973	2.62	1.46	3.50	1.95	4.37	2.43	5.24	2.92	6.12	3.40	6.99	3.89	7.86	4.38	8.74	4.86	54
9	.873	.487	1.75	.974	2.62	1.46	3.49	1.95	4.37	2.44	5.24	2.92	6.11	3.41	6.99	3.90	7.86	4.38	8.73	4.87	51
12	.873	.488	1.75	.976	2.62	1.46	3.49	1.95	4.36	2.44	5.24	2.93	6.11	3.42	6.98	3.90	7.86	4.39	8.73	4.88	48
15	.872	.489	1.74	.977	2.62	1.47	3.49	1.95	4.36	2.44	5.23	2.93	6.11	3.42	6.98	3.91	7.85	4.40	8.72	4.89	45
18	.872	.489	1.74	.979	2.62	1.47	3.49	1.96	4.36	2.45	5.23	2.94	6.10	3.43	6.98	3.92	7.85	4.40	8.72	4.89	42
21	.872	.490	1.74	.980	2.61	1.47	3.49	1.96	4.36	2.45	5.23	2.94	6.10	3.43	6.97	3.92	7.84	4.41	8.72	4.90	39
24	.871	.491	1.74	.982	2.61	1.47	3.48	1.96	4.36	2.45	5.23	2.95	6.10	3.44	6.97	3.93	7.84	4.42	8.71	4.91	36
27	.871	.492	1.74	.983	2.61	1.47	3.48	1.97	4.35	2.46	5.22	2.95	6.10	3.44	6.97	3.93	7.84	4.42	8.71	4.92	33
30	.870	.492	1.74	.985	2.61	1.48	3.48	1.97	4.35	2.46	5.22	2.95	6.09	3.45	6.96	3.94	7.83	4.43	8.70	4.92	30
33	.870	.493	1.74	.986	2.61	1.48	3.48	1.97	4.35	2.47	5.22	2.96	6.09	3.45	6.96	3.95	7.83	4.44	8.70	4.93	27
36	.869	.494	1.74	.988	2.61	1.48	3.48	1.98	4.35	2.47	5.22	2.96	6.09	3.46	6.96	3.95	7.83	4.45	8.69	4.94	24
39	.869	.495	1.74	.989	2.61	1.48	3.48	1.98	4.35	2.47	5.21	2.97	6.08	3.46	6.95	3.96	7.82	4.45	8.69	4.95	21
42	.869	.495	1.74	.991	2.61	1.49	3.47	1.98	4.34	2.48	5.21	2.97	6.08	3.47	6.95	3.96	7.82	4.46	8.69	4.95	18
45	.868	.496	1.74	.992	2.60	1.49	3.47	1.98	4.34	2.48	5.21	2.98	6.08	3.47	6.95	3.97	7.81	4.47	8.68	4.96	15
48	.868	.497	1.74	.994	2.60	1.49	3.47	1.99	4.34	2.48	5.21	2.98	6.07	3.48	6.94	3.98	7.81	4.47	8.68	4.97	12
51	.867	.498	1.73	.995	2.60	1.49	3.47	1.99	4.34	2.49	5.20	2.99	6.07	3.48	6.94	3.98	7.81	4.48	8.67	4.98	9
54	.867	.498	1.73	.997	2.60	1.50	3.47	1.99	4.33	2.49	5.20	2.99	6.07	3.49	6.94	3.99	7.80	4.49	8.67	4.98	6
57	.866	.499	1.73	.998	2.60	1.50	3.47	2.00	4.33	2.50	5.20	3.00	6.07	3.49	6.93	3.99	7.80	4.49	8.66	4.99	3
Deg.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.

Deg.	DISTANCE IN CHAINS, &c.																				Deg.
	1		2		3		4		5		6		7		8		9		10		
′	Lat.	Dep	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat	Dep.	Lat.	Dep	Lat.	Dep.	′
30°	·866	·500	1·73	1·00	2·60	1·50	3·46	2·00	4·33	2·50	5·20	3·00	6·06	3·50	6·93	4·00	7·79	4·50	8·66	5·00	60°
3	·866	·501	1·73	1·00	2·60	1·50	3·46	2·00	4·33	2·50	5·19	3·00	6·06	3·51	6·92	4·01	7·79	4·51	8·66	5·01	57
6	·865	·502	1·73	1·00	2·60	1·50	3·46	2·01	4·33	2·51	5·19	3·01	6·06	3·51	6·92	4·01	7·79	4·51	8·65	5·02	54
9	·865	·502	1·73	1·00	2·59	1·51	3·46	2·01	4·32	2·51	5·19	3·01	6·05	3·52	6·92	4·02	7·78	4·52	8·65	5·02	51
12	·864	·503	1·73	1·01	2·59	1·51	3·46	2·01	4·32	2·52	5·19	3·02	6·05	3·52	6·91	4·02	7·78	4·53	8·64	5·03	48
15	·864	·504	1·73	1·01	2·59	1·51	3·46	2·02	4·32	2·52	5·18	3·02	6·05	3·53	6·91	4·03	7·77	4·53	8·64	5·04	45
18	·863	·505	1·73	1·01	2·59	1·51	3·45	2·02	4·32	2·52	5·18	3·03	6·04	3·53	6·91	4·04	7·77	4·54	8·63	5·05	42
21	·863	·505	1·73	1·01	2·59	1·52	3·45	2·02	4·31	2·53	5·18	3·03	6·04	3·54	6·90	4·04	7·77	4·55	8·63	5·05	39
24	·862	·506	1·73	1·01	2 59	1·52	3·45	2·02	4·31	2·53	5·18	3·04	6·04	3·54	6·90	4·05	7·76	4·55	8·62	5·06	36
27	·862	·507	1·72	1·01	2·59	1·52	3·45	2·03	4·31	2·53	5·17	3·04	6·03	3·55	6·90	4·05	7·76	4·56	8·62	5·07	33
30	·862	·508	1·72	1·02	2·58	1·52	3·45	2·03	4·31	2·54	5·17	3·05	6·03	3·55	6·89	4·06	7·75	4·57	8·62	5·08	30
33	·861	·508	1·72	1·02	2·58	1·52	3·44	2·03	4·31	2·54	5·17	3·05	6·03	3·56	6·89	4·07	7·75	4·57	8·61	5·08	27
36	·861	·509	1·72	1·02	2·58	1·53	3·44	2·04	4·30	2·55	5·16	3·05	6·03	3·56	6·89	4·07	7·75	4·58	8·61	5·09	24
39	·860	·510	1·72	1·02	2·58	1·53	3·44	2·04	4·30	2·55	5·16	3·06	6·02	3·57	6·88	4·08	7·74	4·59	8·60	5·10	21
42	·860	·511	1·72	1·02	2·58	1·53	3·44	2·04	4·30	2·55	5·16	3·06	6·02	3·57	6·88	4·08	7·74	4·59	8·60	5·11	18
45	·860	·511	1·72	1·02	2·58	1·53	3·44	2·05	4·30	2·56	5·16	3·07	6·02	3·58	6·87	4·09	7·73	4·60	8·59	5·11	15
48	·859	·512	1·72	1·02	2·58	1·54	3·44	2·05	4·29	2·56	5·15	3·07	6·01	3·58	6·87	4·10	7·73	4·61	8·59	5·12	12
51	·859	·513	1·72	1·03	2·58	1·54	3·43	2·05	4·29	2·56	5·15	3·08	6·01	3·59	6·87	4·10	7·73	4·62	8·59	5·13	9
54	·858	·514	1·72	1·03	2.57	1·54	3·43	2·05	4·29	2·57	5·15	3·08	6·01	3·59	6·86	4·11	7·72	4·62	8·58	5·14	6
57	·858	·514	1·72	1·03	2·57	1.54	3·43	2·06	4·29	2·57	5·15	3·09	6·00	3·60	6·86	4·11	7·72	4·63	8·58	5·14	3
31°	·857	·515	1·71	1·03	2·57	1·55	3·43	2·06	4·29	2·58	5·14	3·09	6·00	3·61	6·86	4·12	7·71	4·64	8·57	5·15	59
3	·857	·516	1·71	1·03	2·57	1·55	3·43	2·06	4·28	2·58	5·14	3·09	6·00	3·61	6·85	4·13	7·71	4·64	8·57	5·16	57
6	·856	·517	1·71	1·03	2·57	1·55	3·43	2·07	4·28	2·58	5·14	3·10	5·99	3·62	6·85	4·13	7·71	4·65	8·56	5·17	51
9	·856	·517	1·71	1·03	2·57	1·55	3·42	2·07	4·28	2·59	5·13	3·10	5·99	3·62	6·85	4·14	7·70	4·66	8·56	5·17	51
12	·855	·518	1·71	1·04	2·57	1·55	3·42	2·07	4·28	2·59	5·13	3·11	5·99	3·63	6·84	4·14	7·70	4·66	8·55	5·18	48
15	·855	·519	1·71	1·04	2·56	1·56	3·42	2·08	4·27	2·59	5·13	3·11	5·98	3·63	6·84	4·15	7·69	4·67	8·55	5·19	45
18	·854	·520	1·71	1·04	2·56	1·56	3·42	2·08	4·27	2·60	5·13	3·12	5·98	3·64	6·84	4·16	7·69	4·68	8·54	5·20	42
21	·854	·520	1·71	1·04	2·56	1·56	3·42	2·08	4·27	2·60	5·12	3·12	5·98	3·64	6·83	4·16	7·69	4·68	8·54	5·20	39
24	·854	·521	1·71	1·04	2·56	1·56	3·41	2·08	4·27	2·61	5·12	3·13	5·97	3·65	6·83	4·17	7·68	4·69	8·54	5·21	36
27	·853	·522	1·71	1·04	2·56	1·57	3·41	2·09	4·27	2·61	5·12	3·13	5·97	3·65	6·82	4·17	7·68	4·70	8·53	5·22	33
30	·853	·522	1·71	1·04	2·56	1·57	3·41	2·09	4·26	2·61	5·12	3·13	5·97	3·66	6·82	4·18	7·67	4·70	8·53	5·22	30
33	·852	·523	1·70	1·05	2·56	1·57	3·41	2·09	4·26	2·62	5·11	3·14	5·97	3·66	6·82	4·19	7·67	4·71	8·52	5·23	27
36	·852	·524	1·70	1·05	2·56	1·57	3·41	2·10	4·26	2·62	5·11	3·14	5·96	3·67	6·81	4·19	7·67	4·72	8·52	5·24	24
39	·851	·525	1·70	1·05	2·55	1·57	3·41	2·10	4·26	2·62	5·11	3·15	5·96	3·67	6·81	4·20	7·66	4·72	8·51	5·25	21
42	·851	·525	1·70	1·05	2 55	1·58	3·40	2·10	4·25	2·63	5·10	3·15	5·96	3·68	6·81	4·20	7·66	4·73	8·51	5·25	18
45	·850	·526	1·70	1·05	2·55	1·58	3·40	2·10	4·25	2·63	5·10	3·16	5·95	3·68	6·80	4·21	7·65	4·74	8·50	5·26	15
48	·850	·527	1·70	1·05	2·55	1·58	3·40	2·11	4·25	2·63	5·10	3·16	5·95	3·69	6·80	4·22	7·65	4·74	8·50	5·27	12
51	·849	·528	1·70	1·06	2·55	1·58	3·40	2·11	4·25	2·64	5·10	3·17	5·95	3·69	6·80	4·22	7·64	4·75	8·49	5·28	9
54	·849	·528	1·70	1·06	2·55	1·59	3·40	2·11	4·24	2·64	5·10	3·17	5·94	3·70	6·79	4·23	7·64	4·76	8·49	5·28	6
57	·849	·529	1·70	1·06	2·55	1·59	3·39	2·12	4·24	2·65	5·09	3 18	5·94	3·70	6·79	4·23	7·64	4·76	8·49	5·29	3
32°	·848	·530	1·70	1·06	2·54	1·59	3·39	2·12	4·24	2·65	5·09	3·18	5·94	3·71	6·78	4·24	7·63	4·77	8·48	5·30	58
3	·848	·531	1·70	1·06	2·54	1·59	3·39	2·12	4·24	2·65	5·09	3·18	5·93	3·71	6·78	4·25	7·63	4·78	8·48	5·31	57
6	·847	·531	1·69	1·06	2·54	1·59	3·39	2·13	4·24	2·66	5·08	3·19	5·93	3·72	6·78	4·25	7·62	4·78	8·47	5·31	54
9	·847	·532	1·69	1·06	2·54	1·60	3·39	2·13	4·23	2·66	5·08	3·19	5·93	3·72	6·77	4·26	7·62	4·79	8·47	5·32	51
12	·846	·533	1·69	1·07	2·54	1·60	3·38	2·13	4·23	2·66	5·08	3·20	5·92	3·73	6·77	4·26	7·62	4·80	8·46	5·33	48
15	·846	·534	1·69	1·07	2·54	1·60	3·38	2·13	4·23	2·67	5·07	3·20	5 92	3·74	6·77	4·27	7·61	4·80	8·46	5·34	45
18	·845	·534	1·69	1·07	2·54	1·60	3·38	2·14	4·23	2·67	5·07	3·21	5·92	3·74	6·76	4·27	7·61	4·81	8·45	5·34	42
21	·845	·535	1·69	1·07	2·53	1·61	3·38	2·14	4·22	2·68	5·07	3·21	5·91	3·74	6·76	4·28	7·60	4·82	8·45	5·35	39
24	·844	·536	1·69	1·07	2·53	1·61	3·38	2·14	4·22	2·68	5·07	3·21	5·91	3·75	6·75	4·29	7·60	4·82	8·44	5·36	36
27	·844	·537	1·69	1·07	2·53	1·61	3·38	2·15	4·22	2·68	5·06	3·22	5·91	3·76	6·75	4·29	7·59	4·83	8·44	5·37	33
30	·843	·537	1·69	1·07	2·53	1·61	3·37	2·15	4·22	2·69	5·06	3·22	5·90	3·76	6·75	4·30	7·59	4·84	8·43	5·37	30
33	·843	·538	1·69	1·08	2·53	1·61	3·37	2·15	4·21	2·69	5·06	3·23	5·90	3·77	6·74	4·30	7·59	4·84	8·43	5·38	27
36	·842	·539	1·68	1·08	2·53	1·62	3·37	2·16	4·21	2·69	5·05	3·23	5·90	3·77	6·74	4·31	7·58	4·85	8·42	5·39	24
39	·842	·540	1·68	1·08	2·53	1·62	3·37	2·16	4·21	2·70	5·05	3·24	5·89	3·78	6·74	4·32	7·58	4·86	8·42	5·40	21
42	·842	·540	1·68	1·08	2·52	1·62	3·37	2·16	4·21	2·70	5·05	3·24	5·89	3·78	6·73	4·32	7·57	4·86	8·42	5·40	18
45	·841	·541	1·68	1·08	2·52	1·62	3·36	2·16	4·21	2·70	5·05	3·25	5·89	3·79	6·73	4·33	7·57	4·87	8·41	5·41	15
48	·841	·542	1·68	1·08	2·52	1·63	3·36	2·17	4·20	2·71	5·04	3·25	5·88	3·79	6·72	4·33	7·57	4·88	8·41	5·42	12
51	·840	·542	1·68	1·08	2·52	1·63	3·36	2·17	4·20	2·71	5·04	3·25	5·88	3·80	6·72	4·34	7·56	4·88	8·40	5·42	9
54	·840	·543	1·68	1·09	2·52	1·63	3·36	2·17	4·20	2·72	5·04	3·26	5·88	3·80	6·72	4·35	7·56	4·89	8·40	5·43	6
57	·839	·544	1·68	1·09	2·52	1·63	3·36	2·18	4·20	2·72	5·03	3·26	5·87	3·81	6·71	4·35	7·55	4·90	8·39	5·44	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Deg.

DISTANCE IN CHAINS, &c.

Deg.	1		2		3		4		5		6		7		8		9		10		Deg.
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	′
33°	·839	·545	1·68	1·09	2·52	1·63	3·35	2·18	4·19	2·72	5·03	3·27	5·87	3·81	6·71	4·36	7·55	4·90	8·39	5·45	57°
3	·838	·545	1·68	1·09	2·51	1·64	3·35	2·18	4·19	2·73	5·03	3·27	5·87	3·82	6·71	4·36	7·54	4·91	8·38	5·45	57
6	·838	·546	1·68	1·09	2·51	1·64	3·35	2·18	4·19	2·73	5·03	3·28	5·86	3·82	6·70	4·37	7·54	4·91	8·38	5·46	54
9	·837	·547	1·67	1·09	2·51	1·64	3·35	2·19	4·19	2·73	5·02	3·28	5·86	3·83	6·70	4·37	7·54	4·92	8·37	5·47	51
12	·837	·548	1·67	1·10	2·51	1·64	3·35	2·19	4·18	2·74	5·02	3·29	5·86	3·83	6·69	4·38	7·53	4·93	8·37	5·48	48
15	·836	·548	1·67	1·10	2·51	1·64	3·35	2·19	4·18	2·74	5·02	3·29	5·85	3·84	6·69	4·39	7·53	4·93	8·36	5·48	45
18	·836	·549	1·67	1·10	2·51	1·65	3·34	2·20	4·18	2·75	5·01	3·29	5·85	3·84	6·69	4·39	7·52	4·94	8·36	5·49	42
21	·835	·550	1·67	1·10	2·51	1·65	3·34	2·20	4·18	2·75	5·01	3·30	5·85	3·85	6·68	4·40	7·52	4·95	8·35	5·50	39
24	·835	·550	1·67	1·10	2·50	1·65	3·34	2·20	4·17	2·75	5·01	3·30	5·84	3·85	6·68	4·40	7·51	4·95	8·35	5·50	36
27	·834	·551	1·67	1·10	2·50	1·65	3·34	2·20	4·17	2·76	5·01	3·31	5·84	3·86	6·67	4·41	7·51	4·96	8·34	5·51	33
30	·834	·552	1·67	1·10	2·50	1·66	3·34	2·21	4·17	2·76	5·00	3·31	5·84	3·86	6·67	4·42	7·50	4·97	8·34	5·52	30
33	·833	·553	1·67	1·11	2·50	1·66	3·33	2·21	4·17	2·76	5·00	3·32	5·83	3·87	6·67	4·42	7·50	4·97	8·33	5·53	27
36	·833	·553	1·67	1·11	2·50	1·66	3·33	2·21	4·16	2·77	5·00	3·32	5·83	3·37	6·66	4·43	7·50	4·98	8·33	5·53	24
39	·832	·554	1·66	1·11	2·50	1·66	3·33	2·22	4·16	2·77	4·99	3·32	5·83	3·88	6·66	4·43	7·49	4·99	8·32	5·54	21
42	·832	·555	1·66	1·11	2·50	1·66	3·33	2·22	4·16	2·77	4·99	3·33	5·82	3·88	6·66	4·44	7·49	4·99	8·32	5·55	18
45	·831	·556	1·66	1·11	2·49	1·67	3·33	2·22	4·16	2·78	4·99	3·33	5·82	3·89	6·65	4·44	7·48	5·00	8·31	5·56	15
48	·831	·556	1·66	1·11	2·49	1·67	3·32	2·23	4·15	2·78	4·99	3·34	5·82	3·89	6·65	4·45	7·48	5·01	8·31	5·56	12
51	·830	·557	1·66	1·11	2·49	1·67	3·32	2·23	4·15	2·79	4·98	3·34	5·81	3·90	6·64	4·46	7·47	5·01	8·30	5·57	9
54	·830	·558	1·66	1·12	2·49	1·67	3·32	2·23	4·15	2·79	4·98	3·35	5·81	3·90	6·64	4·46	7·47	5·02	8·30	5·58	6
57	·830	·558	1·66	1·12	2·49	1.68	3·32	2·23	4·15	2·79	4·98	3·35	5·81	3·91	6·64	4·47	7·47	5·03	8·30	5·58	3
34°	·829	·559	1·66	1·12	2·49	1·68	3·32	2·24	4·15	2·80	4·97	3·36	5·80	3·91	6·63	4·47	7·46	5·03	8·29	5·59	56°
3	·829	·560	1·66	1·12	2·49	1·68	3·31	2·24	4·14	2·80	4·97	3·36	5·80	3·92	6·63	4·48	7·46	5·04	8·29	5·60	57
6	·828	·561	1·66	1·12	2·48	1·68	3·31	2·24	4·14	2·80	4·97	3·36	5·80	3·92	6·62	4·49	7·45	5·05	8·28	5·61	54
9	·828	·561	1·66	1·12	2·48	1·68	3·31	2·25	4·14	2·81	4·97	3·37	5·79	3·93	6·62	4·49	7·45	5·05	8·28	5·61	51
12	·827	·562	1·65	1·12	2·48	1·69	3·31	2·25	4·14	2·81	4·96	3·37	5·79	3·93	6·62	4·50	7·44	5 06	8·27	5·62	48
15	·827	·563	1·65	1·13	2·48	1·69	3·31	2·25	4·13	2·81	4·96	3·38	5·79	3·94	6·61	4·50	7·44	5·07	8·27	5·63	45
18	·826	·564	1·65	1·13	2·48	1·69	3·30	2·25	4·13	2·82	4·96	3·38	5·78	3·94	6·61	4·51	7·43	5·07	8·26	5·64	42
21	·826	·564	1·65	1·13	2·48	1·69	3·30	2·26	4·13	2·82	4·95	3·39	5·78	3·95	6·60	4·51	7·43	5·08	8·26	5·64	39
24	·825	·565	1·65	1·13	2·48	1·69	3·30	2·26	4·13	2·82	4·95	3·39	5·78	3·95	6·60	4·53	7·43	5·08	8·25	5·65	36
27	·825	·566	1·65	1·13	2·47	1·70	3·30	2·26	4·12	2·83	4·95	3·39	5·77	3·96	6·60	4·53	7·42	5·09	8·25	5·66	33
30	·824	·566	1·65	1·13	2·47	1·70	3·30	2·27	4·12	2·83	4·94	3·40	5·77	3·96	6·59	4·53	7·42	5·10	8·24	5·66	30
33	·824	·567	1·65	1·13	2·47	1·70	3·29	2·27	4·12	2·84	4·94	3·40	5·77	3·97	6·59	4·54	7·41	5·10	8·24	5·67	27
36	·823	·568	1·65	1·14	2·47	1·70	3·29	2·27	4·12	2·84	4·94	3·41	5·76	3·97	6·59	4·54	7·41	5·11	8·23	5·68	24
39	·823	·569	1·65	1·14	2·47	1·71	3·29	2·27	4·11	2·84	4·94	3·41	5·76	3·98	6·58	4·55	7·40	5·12	8·23	5·69	21
42	·822	·569	1·64	1·14	2·47	1·71	3·29	2·28	4·11	2·85	4·93	3·42	5·75	3·98	6·58	4·55	7·40	5·12	8·22	5·69	18
45	·822	·570	1·64	1·14	2·46	1·71	3·29	2·28	4·11	2·85	4·93	3·42	5·75	3·99	6·57	4·56	7·39	5·13	8·22	5·70	15
48	·821	·571	1·64	1·14	2·46	1·71	3·28	2·28	4·11	2·85	4·93	3·42	5·75	3·99	6·57	4·57	7·39	5·14	8·21	5·71	12
51	·821	·571	1·64	1·14	2·46	1·71	3·28	2·29	4·10	2·86	4·92	3·43	5·74	4·00	6·57	4·57	7·39	5·14	8·21	5·71	9
54	·820	·572	1·64	1·14	2·46	1·72	3·28	2·29	4·10	2·86	4·92	3·43	5·74	4·01	6·56	4·58	7·38	5·15	8·20	5·72	6
57	·820	·573	1·64	1· 5	2·46	1·72	3·28	2·29	4·10	2·86	4·92	3·44	5·74	4·01	6·56	4·58	7·38	5·16	8·20	5·73	3
35°	·819	·574	1·64	1·15	2·46	1·72	3·28	2·29	4·10	2·87	4·91	3·44	5·73	4·02	6·55	4·59	7·37	5·16	8·19	5·74	55°
3	·819	·574	1·64	1·15	2·46	1·72	3·27	2·30	4·09	2·87	4·91	3·45	5·73	4·02	6·55	4·59	7·37	5·17	8·19	5·74	57
6	·818	·575	1·64	1·15	2·45	1·73	3·27	2·30	4·09	2·88	4·91	3·45	5·73	4·03	6·55	4·60	7·36	5·18	8·18	5·75	54
9	·818	·576	1·64	1·15	2·45	1·73	3·27	2·30	4·09	2·88	4·91	3·45	5·72	4·03	6·54	4·61	7·36	5·18	8·18	5·76	51
2	·817	·576	1·63	1·15	2·45	1·73	3·27	2·31	4·09	2·88	4·90	3·46	5·72	4·04	6·54	4·61	7·35	5·19	8·17	5·76	48
15	·817	·577	1·63	1·15	2·45	1·73	3·27	2·31	4·08	2·89	4·90	3·46	5 72	4·04	6·53	4·62	7·35	5·19	8·17	5·77	45
18	·816	·578	1·63	1·16	2·45	1·73	3·26	2·31	4·08	2·89	4·90	3·47	5·71	4·04	6·53	4·62	7·35	5·20	8·16	5·78	42
21	·816	·579	1·63	1·16	2·45	1·74	3·26	2·31	4·08	2·89	4·89	3·47	5·71	4·05	6·53	4·63	7·34	5·21	8·16	5·79	39
24	·815	·579	1·63	1·16	2·45	1·74	3·26	2·32	4·08	2·90	4·89	3·48	5·71	4·05	6·52	4·63	7·34	5·21	8·15	5·79	36
27	·815	·580	1·63	1·16	2·44	1·74	3·26	2·32	4·07	2·90	4·89	3·48	5·70	4·06	6·52	4·64	7·33	5·22	8·15	5·80	33
30	·814	·581	1·63	1·16	2·44	1·74	3·26	2·32	4·07	2·90	4·88	3·48	5·70	4·06	6·51	4·65	7·33	5·23	8·14	5·81	30
33	·814	·581	1·63	1·16	2·44	1·74	3·25	2·33	4·07	2·91	4·88	3·49	5·70	4·07	6·51	4·65	7·32	5·23	8·14	5·81	27
36	·813	·582	1·63	1·16	2·44	1·75	3·25	2·33	4·07	2·91	4·88	3·49	5·69	4·07	6·50	4·66	7·32	5·24	8·13	5·82	24
39	·813	·583	1·63	1·17	2·44	1·75	3·25	2·33	4·06	2·91	4·88	3·50	5·69	4·08	6·50	4·66	7·31	5·25	8·13	5·83	21
42	·812	·584	1·62	1·17	2·44	1·75	3·25	2·33	4·06	2·92	4·87	3·50	5·68	4·08	6·50	4·67	7·31	5·25	8·12	5·84	18
45	·812	·584	1·62	1·17	2·43	1·75	3·25	2·34	4·06	2·92	4·87	3·51	5·68	4·09	6·49	4·67	7·30	5·26	8·12	5·84	15
48	·811	·585	1·62	1·17	2·43	1·75	3·24	2·34	4·06	2·92	4·87	3·51	5·68	4·10	6·49	4·68	7·30	5·26	8·11	5·85	12
51	·811	·586	1·62	1·17	2·43	1·76	3·24	2·34	4·05	2·93	4·86	3·51	5·67	4·10	6·48	4·69	7·29	5·27	8·11	5·86	9
54	·810	·586	1·62	1·17	2·43	1·76	3·24	2·35	4·05	2·93	4·86	3·52	5·67	4·10	6·48	4·69	7·29	5·28	8·10	5·86	6
57	·810	·587	1·62	1·17	2·43	1·76	3·24	2·35	4·05	2·94	4·86	3·52	5·67	4·11	6·48	4·70	7·29	5·28	8·10	5·87	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.

DISTANCE IN CHAINS, &c.

Deg	1		2		3		4		5		6		7		8		9		10		Deg.
	Lat.	Dep.	Lat.	Dep.	Lat	Dep	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	La .	Dep.	Lat.	Dep.	Lat.	Dep.	′
36°	·809	·588	1·62	1·18	2·43	1·76	3·24	2·35	4·05	2·94	4·85	3·53	5·66	4·11	6·47	4·70	7·28	5·29	8·09	5·88	54
3	·809	·588	1·62	1·18	2·43	1·77	3·23	2·35	4·04	2·94	4·85	3·53	5·66	4·12	6·47	4·71	7·28	5·30	8·09	5·88	57
6	·808	·589	1·62	1·18	2·42	1·77	3·23	2·36	4·04	2·95	4·85	3·54	5·66	3·12	6·46	4·71	7·27	5·30	8·08	5·89	54
9	·807	·590	1·61	1·18	2·42	1·77	3·23	2·36	4·04	2·95	4·84	3·54	5·65	4·13	6·46	4·72	7·27	5·31	8·07	5·90	51
12	·807	·591	1·61	1·18	2·42	1·77	3·23	2·36	4·03	2·95	4·84	3·54	5·65	4·13	6·46	4·72	7·26	5·32	8·07	5·91	48
15	·806	·591	1.61	1·18	2·42	1·77	3·23	2·37	4·03	2·96	4·84	3·55	5·65	4·14	6·45	4·73	7·26	5·32	8·06	5·91	45
18	·806	·592	1·61	1·18	2·42	1·78	3·22	2·37	4·03	2·96	4·84	3·55	5·64	4·14	6·45	4·74	7·25	5·33	8·06	5·92	42
21	·805	·593	1·61	1·19	2·42	1·78	3·22	2·37	4·03	2·96	4·83	3·56	5·64	4·15	6·44	4·74	7·25	5·33	8·05	5·93	39
24	·805	·593	1·61	1 19	2·41	1·78	3·22	2·37	4·02	2·97	4·83	3·56	5·63	4·15	6·44	4·75	7·24	5·34	8·05	5·93	36
27	·804	·594	1·61	1·19	2·41	1·78	3·22	2·38	4·02	2·97	4·83	3·56	5·63	4·16	6·44	4·75	7·24	5·35	8·04	5·94	33
30	·804	·595	1·61	1·19	2·41	1·78	3·22	2 38	4·02	2·97	4·82	3·57	5·63	4·16	6·43	4·76	7·23	5·35	8·04	5·95	30
33	·803	·596	1·61	1·19	2·41	1·79	3·21	2·38	4·02	2·98	4·82	3·57	5·62	4·17	6·43	4·76	7·23	5·36	8·03	5·96	27
36	·803	·596	1·61	1·19	2·41	1·79	3·21	2·38	4·01	2·98	4·82	3·58	5·62	4·17	6·42	4·77	7·23	5·37	8·03	5·96	24
39	·802	·597	1·60	1·19	2·41	1·79	3·21	2·39	4·01	2·98	4·81	3·58	5·62	4·18	6·42	4·78	7·22	5·37	8·02	5·97	21
42	·802	·598	1·60	1·20	2·41	1·79	3·21	2·39	4·01	2·99	4·81	3·59	5·61	4·18	6·41	4·78	7·22	5·38	8·02	5·98	18
45	·801	·598	1·60	1·20	2·40	1·79	3 21	2·39	4·01	2·99	4·81	3·59	5·61	4·19	6·41	4·79	7·21	5·38	8·01	5·98	15
48	·801	·599	1.60	1·20	2·40	1·80	3·20	2·40	4·00	3·00	4·80	3·59	5·61	4·19	6·41	4·79	7·21	5·39	8·01	5·99	12
51	·800	·600	1·60	1·20	2·40	1·80	3·20	2·40	4·00	3·00	4·80	3·60	5·60	4·20	6·40	4·80	7·20	5·40	8·00	6·00	9
54	·800	·600	1·60	1·20	2·40	1·80	3·20	2·40	4·00	3·00	4·80	3·60	5·60	4·20	6·40	4·80	7·20	5·40	8·00	6·00	6
57	·799	·601	1·60	1·20	2·40	1·80	3·20	2·40	4·00	3·01	4·79	3·61	5·59	4·21	6·39	4·81	7·19	5·41	7·99	6·01	3
37°	·799	·602	1·60	1·20	2·40	1·81	3·19	2·41	3·99	3·01	4·79	3·61	5·59	4·21	6·39	4·81	7·19	5·42	7·99	6·02	53
3	·798	·603	1·60	1·21	2·39	1·81	3·19	2·41	3·99	3·01	4·79	3·62	5·59	4·22	6·38	4·82	7·18	5·42	7·98	6·03	57
6	·798	·603	1·59	1·21	2·39	1·81	3·19	2·41	3·99	3·02	4·79	3·62	5·58	4·22	6·38	4·83	7·18	5·43	7·98	6·03	54
9	·797	·604	1·59	1·21	2·39	1·81	3·19	2·42	3·99	3·02	4·78	3·62	5·58	4·23	6·38	4·83	7·17	5·44	7·97	6·04	51
12	·797	·605	1·59	1·21	2·39	1·81	3·19	2·42	3·98	3·02	4·78	3·63	5·58	4·23	6·37	4·84	7·17	5·44	7·97	6·05	48
15	·796	·605	1·59	1·21	2·39	1·82	3·19	2·42	3·98	3·03	4·78	3·63	5·57	4·24	6·37	4·84	7·16	5·45	7·96	6·05	45
18	·795	·606	1·59	1·21	2·39	1·82	3·19	2·42	3·98	3·03	4·77	3·64	5·57	4·24	6·36	4·85	7·16	5·45	7·95	6·06	42
21	·795	·607	1·59	1·21	2·38	1·82	3·18	2·43	3·97	3·03	4·77	3·64	5·56	4·25	6·36	4·85	7·15	5·46	7·95	6·07	39
24	·794	·607	1·59	1·21	2·38	1·82	3·18	2·43	3·97	3·04	4·77	3·64	5·56	4·25	6·36	4·86	7·15	5·47	7·94	6·07	36
27	·794	·608	1·59	1·22	2·38	1·82	3·18	2·43	3·97	3·04	4·76	3·65	5·56	4·26	6·35	4·86	7·14	5·47	7·94	6·08	33
30	·793	·609	1·59	1·22	2·38	1·83	3·17	2·44	3·97	3·04	4·76	3·65	5·55	4·26	6·35	4·87	7 14	5·48	7·93	6 09	30
33	·793	·609	1·59	1·22	2·38	1·83	3·17	2·44	3·96	3·05	4·76	3·66	5·55	4·27	6·34	4·88	7·14	5·49	7 93	6·09	27
36	·792	·610	1·58	1·22	2·38	1·83	3·17	2·44	3·96	3·05	4·75	3·66	5·55	4·27	6·34	4·88	7·13	5·49	7·92	6·10	24
39	·792	·611	1·58	1·22	2·38	1·83	3·17	2·44	3·96	3·05	4·75	3·67	5·54	4·28	6·33	4·89	7·13	5·50	7·92	6·11	21
42	·791	·612	1·58	1·22	2·37	1·83	3·16	2·45	3·96	3·06	4·75	3·67	5·54	4·28	6·33	4·89	7·12	5·50	7·91	6·12	18
45	·791	·612	1·58	1·22	2·37	1·84	3 16	2·45	3·95	3·06	4·74	3·67	5·53	4·29	6·33	4·90	7·12	5·51	7·91	6·12	15
48	·790	·613	1·58	1·23	2·87	1·84	3·16	2·45	3·95	3·06	4·74	3·68	5·53	4·29	6·32	4·90	7·11	5·52	7·90	6·13	12
51	·790	·614	1·58	1·23	2·37	1·84	3·16	2·45	3·95	3·07	4·74	3·68	5·53	4·30	6·32	4·91	7·11	5·52	7·90	6·14	9
54	·789	·614	1·58	1·23	2·37	1·84	3·16	2·46	3·95	3·07	4·73	3·69	5·52	4·30	6·31	4·91	7·10	5·53	7·89	6·14	6
57	·789	·615	1·58	1·23	2·37	1·84	3·15	2·46	3·94	3·07	4·73	3·69	5·52	4·30	6·31	4·92	7·10	5·53	7·89	6·15	3
38°	·788	·616	1·58	1·23	2·36	1·85	3·15	2·46	3·94	3·08	4·73	3·69	5·52	4·31	6·30	4·93	7·09	5·54	7·88	6·16	52
3	·787	·616	1·57	1·23	2·36	1·85	3·15	2·47	3·94	3·08	4 72	3·70	5·51	4·31	6·30	4·93	7·09	5·55	7·87	6·16	57
6	·787	·617	1·57	1·23	2·36	1·85	3·15	2·47	3·93	3·09	4·72	3·70	5·51	4·32	6·30	4·94	7·08	5·55	7·87	6·17	54
9	·786	·618	1·57	1·24	2·36	1·85	3·15	2·47	3·93	3·09	4·72	3·71	5·50	4·32	6·29	4·94	7·08	5·56	7·86	6·18	51
12	·786	·618	1·57	1·24	2·36	1·86	3·14	2·47	3·93	3·09	4·72	3·71	5·50	4·33	6·29	4·95	7·07	5·57	7·86	6·18	48
15	·785	·619	1·57	1·24	2·36	1·86	3·14	2·48	3·93	3·10	4·71	3·71	5·50	4·33	6·28	4·95	7·07	5·57	7·85	6·19	45
18	·785	·620	1·57	1·24	2·35	1·86	3·14	2·48	3·92	3·10	4·71	3·72	5·49	4·34	6·28	4·96	7·06	5·58	7·85	6·20	42
21	·784	·621	1·57	1·24	2·35	1·86	3·14	2·48	3·92	3·10	4·71	3·72	5·49	4·34	6·27	4·96	7·06	5·58	7·84	6·20	39
24	·784	·622	1·57	1·24	2·35	1·86	3·13	2·48	3·92	3·11	4·70	3·73	5·49	4·35	6·27	4·97	7·05	5·59	7·84	6·21	36
27	·783	·622	1·57	1·24	2·35	1·87	3·13	2·49	3·92	3·11	4·70	3·73	5·48	4·35	6·27	4·97	7·05	5·60	7·83	6·22	33
30	·783	·623	1·57	1·25	2·35	1·87	3·13	2·49	3·91	3·11	4·70	3·74	5·48	4·36	6·26	4·98	7·04	5·60	7·83	6·23	30
33	·782	·623	1·56	1·25	2·35	1·87	3·13	2·49	3·91	3·12	4·69	3·74	5·47	4·36	6·26	4·99	7·04	5·61	7·82	6·23	27
36	·782	·624	1·56	1·25	2·34	1·87	3·13	2·50	3·91	3·12	4·69	3·74	5·47	4·37	6·25	4·99	7·03	5·61	7·82	6·24	24
39	·781	·625	1·56	1·25	2·34	1·87	3·12	2·50	3·90	3·12	4·69	3·75	5·47	4·37	6·25	5·00	7·03	5·62	7·81	6·25	21
42	·780	·625	1·56	1·25	2·34	1·88	3·12	2·50	3·90	3·13	4·68	3·75	5·46	4·38	6·24	5·00	7·02	5·63	7·80	6·25	18
45	·780	·626	1·56	1·25	2·34	1·88	3·12	2·50	3·90	3·13	4·68	3·76	5·46	4·38	6·24	5·01	7·02	5·63	7·80	6·26	15
48	·779	·627	1·56	1·25	2·34	1·88	3·12	2·51	3·90	3·13	4·68	3·76	5·46	4·39	6·23	5·01	7·01	5·64	7·79	6·27	12
51	·779	·627	1·56	1·25	2·34	1·88	3·12	2·51	3·89	3·14	4·67	3·76	5·45	4·39	6·23	5·02	7·01	5·65	7·79	6·27	9
54	·778	·628	1·56	1·26	2·33	1·88	3·11	2·51	3·89	3·14	4·67	3·77	5·45	4·40	6·23	5·02	7·00	5·65	7·78	6·28	6
57	·778	·629	1·56	1·26	2·33	1·89	3·11	2·51	3·89	3·14	4·67	3·77	5·44	4·40	6·22	5·03	7·00	5·66	7·78	6·29	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg

Deg.	DISTANCE IN CHAINS, &c.																				Deg.
	1		2		3		4		5		6		7		8		9		10		
′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	′
39°	·777	·629	1·55	1·26	2·33	1·89	3·11	2·52	3·89	3·15	4·66	3·78	5·44	4·41	6·22	5·03	6·99	5·66	7·77	6·29	**51°**
3	·777	·630	1·55	1·26	2·33	1·89	3·11	2·52	3·88	3·15	4·66	3·78	5·44	4·41	6·21	5·04	6·99	5·67	7·77	6·30	57
6	·776	·631	1·55	1·26	2·33	1·89	3·10	2·52	3·88	3·15	4·66	3·78	5·43	4·41	6·21	5·05	6·98	5·68	7·76	6·31	54
9	·775	·631	1·55	1·26	2·33	1·89	3·10	2·53	3·88	3·16	4·65	3·79	5·43	4·42	6·20	5·05	6·98	5·68	7·75	6·31	51
12	·775	·632	1·55	1·26	2·32	1·90	3·10	2·53	3·87	3·16	4·65	3·79	5·42	4·42	6·20	5·06	6·97	5·69	7·75	6·32	48
15	·774	·633	1·55	1·27	2·32	1·90	3·10	2·53	3·87	3·16	4·65	3·80	5·42	4·43	6·20	5·06	6·97	5·69	7·74	6·33	45
18	·774	·633	1·55	1·27	2·32	1·90	3·10	2·53	3·87	3·17	4·64	3·80	5·42	4·43	6·19	5·07	6·96	5·70	7·74	6·33	42
21	·773	·634	1·55	1·27	2·32	1·90	3·09	2·54	3·87	3·17	4·64	3·80	5·41	4·44	6·19	5·07	6·96	5·71	7·73	6·34	39
24	·773	·635	1·55	1·27	2·32	1·90	3·09	2·54	3·86	3·17	4·64	3·81	5·41	4·44	6·18	5·08	6·95	5·71	7·73	6·35	36
27	·772	·635	1·54	1·27	2·32	1·91	3·09	2·54	3·86	3·18	4·63	3·81	5·41	4·45	6·18	5·08	6·95	5·72	7·72	6·35	33
30	·772	·636	1·54	1·27	2·31	1·91	3·09	2·54	3·86	3·18	4·63	3·82	5·40	4·45	6·17	5·09	6·64	5·72	7·72	6·36	30
33	·771	·637	1·54	1·27	2·31	1·91	3·08	2·55	3·86	3·18	4·63	3·82	5·40	4·46	6·17	5·09	6·94	5·73	7·71	6·37	27
36	·771	·637	1·54	1·27	2·31	1·91	3·08	2·55	3·85	3·19	4·62	3·82	5·39	4·46	6·16	5·10	6·93	5·74	7·71	6·37	24
39	·770	·638	1·54	1·28	2·31	1·91	3·08	2·55	3·85	3·19	4·62	3·82	5·39	4·47	6·16	5·10	6·93	5·74	7·70	6·38	21
42	·769	·639	1·54	1·28	2·31	1·92	3·08	2·56	3·85	3·19	4·62	3·83	5·39	4·47	6·16	5·11	6·92	5·75	7·69	6·39	18
45	·769	·639	1·54	1·28	2·31	1·92	3·08	2·56	3·84	3·20	4·61	3·84	5·38	4·48	6·15	5·12	6·92	5·75	7·69	6·39	15
48	·768	·640	1·54	1·28	2·30	1·92	3·07	2·56	3·84	3·20	4·61	3·84	5·38	4·48	6·15	5·12	6·91	5·76	7·68	6·40	12
51	·768	·641	1·54	1·28	2·30	1·92	3·07	2·56	3·84	3·20	4·61	3·84	5·37	4·49	6·14	5·13	6·91	5·77	7·68	6·41	9
54	·767	·641	1·53	1·28	2·30	1·92	3·07	2·57	3·84	3·21	4·60	3·85	5·37	4·49	6·14	5·13	6·90	5·77	7·67	6·41	6
57	·767	·642	1·53	1·28	2·30	1·93	3·07	2·57	3·83	3·21	4·60	3·85	5·37	4·49	6·13	5·14	6·90	5·78	7·67	6·42	3
40°	·766	·643	1·53	1·29	2·30	1·93	3·06	2·57	3·83	3·21	4·60	3·86	5· 6	4·50	6·13	5·14	6·89	5·79	7·66	6·43	**50°**
3	·765	·643	1·53	1·29	2·30	1·93	3·06	2·57	3·83	3·22	4·59	3·86	5·36	4·50	6·12	5·15	6·89	5·79	7·65	6·43	57
6	·765	·644	1·53	1·29	2·29	1·93	3·06	2·58	3·82	3·22	4·59	3·86	5·35	4·51	6·12	5·15	6·88	5·80	7·65	6·44	54
9	·764	·645	1·53	1·29	2·29	1·93	3·06	2·58	3·82	3·22	4·59	3·87	5·35	4·51	6·11	5·16	6·88	5·80	7·64	6·45	51
12	·764	·645	1·53	1·29	2·29	1·94	3·06	2·58	3·82	3·23	4·58	3·87	5·35	4·52	6·11	5·16	6·87	5·81	7·64	6·45	48
15	·763	·646	1·53	1·29	2·29	1·94	3·05	2·58	3·82	3·23	4·58	3·88	5·34	4·52	6·11	5·17	6·87	5·82	7·63	6·46	45
18	·763	·647	1·53	1·29	2·29	1·94	3·05	2·59	3·81	3·23	4·58	3·88	5·34	4·53	6·10	5·17	6·86	5·82	7·63	6·47	42
21	·762	·647	1·52	1·29	2·29	1·94	3·05	2·59	3·81	3·24	4·57	3·88	5·33	4·53	6·10	5·18	6·86	5·83	7·62	6·47	39
24	·762	·648	1·52	1·30	2·28	1·94	3·05	2·59	3·81	3·24	4·57	3·89	5·33	4·54	6·09	5·18	6·85	5·88	7·62	6·48	36
27	·761	·649	1·52	1·30	2·28	1·95	3·04	2·60	3·80	3·24	4·57	3·89	5·33	4·54	6·09	5·19	6·85	5·84	7·61	6·49	33
30	·760	·549	1·52	1·30	2·28	1·95	3·04	2·60	3·80	3·25	4·56	3·90	5·32	4·55	6·08	5·20	5·84	5·85	7·60	6·49	30
33	·760	·650	1·52	1·30	2·28	1·95	3·04	2·60	3·80	3·25	4·56	3·90	5·32	4·55	6·08	5·20	6·84	5·85	7·60	6·50	27
36	·759	·651	1·52	1·30	2·28	1·95	3·04	2·60	3·80	3·25	4·56	3·90	5·31	4·56	6·07	5·21	6·83	5·86	7·59	6·51	24
39	·759	·651	1·52	1·30	2·28	1·95	3·03	2·61	3·79	3·26	4·55	3·91	5·31	4·56	6·07	5·21	6·83	5·86	7·59	6·51	21
42	·758	·652	1·52	1·30	2·27	1·96	3·03	2·61	3·79	3·26	4·55	3·91	5·31	4·56	6·07	5·22	6·82	5·87	7·58	6·52	18
45	·758	·653	1·52	1·31	2·27	1·96	3·03	2·61	3·79	3·26	4·55	3·92	5·30	4·57	6·06	5·22	6·82	5·87	7·58	6·53	15
48	·757	·653	1·51	1·31	2·27	1·96	3·03	2·61	3·78	3·27	4·54	3·92	5·30	4·57	6·06	5·23	6·81	5·88	7·57	6·53	12
51	·756	·654	1·51	1·31	2·27	1·96	3·03	2·62	3·78	3·27	4·54	3·92	5·29	4·58	6·05	5·23	6·81	5·89	7·56	6·54	9
54	·756	·655	1·51	1·31	2·27	1·96	3·02	2·62	3·78	3·27	4·54	3·93	5·29	4·58	6·05	5·24	6·80	5·89	7·56	6·55	6
57	·755	·655	1·51	1·31	2·27	1·97	3·02	2·62	3·78	3·28	4·53	3·93	5·29	4·59	6·04	5·24	6·80	5·90	7·55	6·55	3
41°	·755	·656	1·51	1·31	2·26	1·97	3·02	2·62	3·77	3·28	4·53	3·94	5·28	4·59	6·04	5·25	6·79	5·90	7·55	6·56	**49°**
3	·754	·657	1·51	1·31	2·26	1·97	3·02	2·63	3·77	3·28	4·52	3·94	5·28	4·60	6·03	5·25	6·79	5·91	7·54	6·57	57
6	·754	·657	1·51	1·31	2·26	1·97	3·01	2·63	3·77	3·29	4·52	3·94	5·27	4·60	6·03	5·26	6·78	5·92	7·54	6·57	54
9	·753	·658	1·51	1·32	2·26	1·97	3·01	2·63	3·76	3·29	4·52	3·95	5·27	4·61	6·02	5·26	6·78	5·92	7·53	6·58	51
12	·752	·659	1·50	1·32	2·26	1·98	3·01	2·63	3·76	3·29	4·51	3·95	5·27	4·61	6·02	5·27	6·77	5·93	7·52	6·59	48
15	·752	·659	1·50	1·32	2·26	1·98	3·01	2·64	3·76	3·30	4·51	3·96	5·26	4·62	6·01	5·27	6·77	5·93	7·52	6·59	45
18	·751	·660	1·50	1·32	2·25	1·98	3·01	2·64	3·76	3·30	4·51	3·96	5·26	4·62	6·01	5·28	6·76	5·94	7·51	6·60	42
21	·751	·661	1·50	1·32	2·25	1·98	3·00	2·64	3·75	3·30	4·50	3·96	5·25	4·62	6·01	5·29	6·76	5·95	7·51	6·61	39
24	·750	·661	1·50	1·32	2·25	1·98	3·00	2·65	3·75	3·31	4·50	3·97	5·25	4·63	6·00	5·29	6·75	5·95	7·50	6·61	36
27	·750	·662	1·50	1·32	2·25	1·99	3·00	2·65	3·75	3·31	4·50	3·97	5·25	4·63	6·00	5·30	6·75	5·96	7·50	6·62	33
30	·749	·663	1·50	1·33	2·25	1·99	3·00	2·65	3·74	3·31	4·49	3·98	5·24	4·64	5·99	5·30	6·74	5·96	7·49	6·63	30
33	·748	·663	1·50	1·33	2·25	1·99	2·99	2·65	3·74	3·32	4·49	3·98	5·24	4·64	5·99	5·31	6·74	5·97	7·48	6·63	27
36	·748	·664	1·50	1·33	2·24	1·99	2·99	2·66	3·74	3·32	4·49	3·98	5·23	4·65	5·98	5·31	6·73	5·98	7·48	6·64	24
39	·747	·665	1·49	1·33	2·24	1·99	2·99	2·66	3·74	3·32	4·48	3·99	5·23	4·65	5·98	5·32	6·73	5·98	7·47	6·65	21
42	·747	·665	1·49	1·33	2·24	2·00	2·99	2·66	3·73	3·33	4·48	3·99	5·23	4·66	5·97	5·32	6·72	5·99	7·47	6·65	18
45	·746	·666	1·49	1·33	2·24	2·00	2·98	2·66	3·73	3·33	4·48	4·00	5·22	4·66	5·97	5·33	6·71	5·99	7·46	6·66	15
48	·745	·667	1·49	1·33	2·24	2·00	2·98	2·67	3·73	3·33	4·47	4·00	5·22	4·67	5·96	5·33	6·71	6·00	7·45	6·67	12
51	·745	·667	1·49	1·33	2·23	2·00	2·98	2·67	3·72	3·34	4·47	4·00	5·21	4·67	5·96	5·34	6·70	6·00	7·45	6·67	9
54	·744	·668	1·49	1·34	2·23	2·00	2·98	2·67	3·72	3·34	4·47	4·01	5.21	4·67	5·95	5·34	6·70	6·01	7·44	6·68	6
57	·744	·668	1·49	1·34	2·23	2·01	2·97	2·67	3·72	3·34	4·46	4·01	5.21	4·68	5·95	5·35	6·69	6·02	7·44	6·68	3
Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.